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Analysis of accuracy of Williams series approximation of stress field in cracked body – influence of area of interest around crack-tip on multi-parameter regression performance

J. Sobek, P. Frantík, V. Veselý
Brno University of Technology, Faculty of Civil Engineering, Institute of Structural Mechanics, Brno, Czech Republic
sobek.j@fce.vutbr.cz, http://orcid.org/0000-0003-4215-1029
kitnarf@centrum.cz
vesely.v1@ fce.vutbr.cz, http://orcid.org/0000-0002-7723-971X

ABSTRACT. A study on the accuracy of an approximation of the stress field in a cracked body is presented. Crack-tip stress tensor is expressed using the linear elastic fracture mechanics (LEFM) theory in this work, more precisely via its multi-parameter formulation, i.e. by Williams power series (WPS). Determination of coefficients of terms of this series is performed using a least squares-based regression technique known as over-deterministic method (ODM) for which results from finite element (FE) method computation are usually taken as inputs. Main attention is paid to a detailed analysis of a suitable selection of FE nodes whose results serve as the inputs to the employed method. Two different ways of FE nodal selection are compared – nodes selected from the crack tip vicinity lying at a ring of a certain radius versus nodes selected more or less uniformly from a specified part of the test specimen body. Comparison of these approaches is made with the help of procedures developed by the authors which enable both the determination of the coefficients of terms of the analytical WPS approximation of the stress field based on the FE results and the backward reconstruction of the field (again using WPS) from those determined terms' coefficients/functions. The wedge-splitting test (WST) specimen with a crack is taken as example for the study.

KEYWORDS. Multi-parameter fracture mechanics; Williams power series; Crack-tip fields; Over-deterministic method; Higher order terms.

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INTRODUCTION

Determination of the higher order terms of Williams power series [1] approximating the fields of stress and displacements in a cracked body is of substantial interest of the presented work. Particular motivation of it is represented by the need for capturing the crack-tip fields in a farther distance from the crack tip.

Tensile failure of the quasi-brittle materials is accompanied with a crack propagation together with a nonlinear zone [2] (fracture process zone – FPZ) development, where the decrease of material integrity takes place. The size of the FPZ is not negligible in comparison with the rest of the body. Processes of failure therefore occur in a wider area around the crack tip. The description of failure mechanism must be in agreement with the stress and displacement field around the crack tip in an extended area. Recent works of many authors show the relevance of the topic of the crack tip fields accurate description [3–6], moreover, extending it into 3D and taking into account the effects of various loading modes [7–12]. Special focus on brittle and quasi-brittle fracture is summarized in [13]. Works [3,14,15] reported the fact, that for the description of the stress/displacement field in a cracked body in a more distant surroundings from the crack tip the necessity of usage of the several terms of Williams expansion (WE), not only the first or the first two terms, is crucial.

Procedures enabling the multi-parameter description of the near crack-tip fields (using e.g. hybrid crack elements [16] formulation, over-deterministic method [17] based on standard FE computation, or other techniques based e.g. on extrapolation of displacements of selected nodes of FE mesh) usually process results from FE nodes selected from the close vicinity of the crack tip. This is adequate for determination of the classical/two-parameter LEFM characteristics (SIF, *T*-stress). However, several questions arise if the area of accurate enough description of the stress and displacement field extends to a greater distance from the crack tip, where $K + T$ dominance vanishes? How many terms of the series should be taken into account? How to select the FE nodes considered for the regression technique? And how to optimize the mutual relationship between the area from where the nodes are considered for the regression (and how are they located/distributed in that area) and the extent where the approximation of the fields is of relevant accuracy? Answering these questions presents the actual motivation of this work.

This paper investigates the above-described issue via a parametric study evaluating the influence of the nodal selection on the quality of the obtained approximation of the field. The work presented here further builds on previous studies. A classical (common) way of nodal selection, where the nodes are selected from a ring in the vicinity of the crack tip was used in recent papers. Influences of several parameters on the description of stress/displacement fields in cracked bodies by Williams series were investigated. Reconstruction of the stress field with the help of a software tool developed by the author's team was shown in [14,15] where the accuracy of the approximation by WE was verified by a visual comparison (which was regarded as sufficient for its intended purpose) with the FE solution (which was regarded as the exact solution). However, the nodal selection that was used for obtaining the WE terms was performed from the ring around the crack tip (with the distance given by recommendation from [17,18]).

Published study [19] on WPS approximation with nodal selection from more than one ring around the crack tip resulted into next studies. Another type of nodal selection was considered in subsequent works by the authors [20,21,22]; the nodes were selected from specific parts of the test specimen body with specific distribution functions of their distance and angular position from the crack tip. This way was employed with expectations that the fields will be better (more accurately/efficiently) approximated.

An automatic utility to determine the values of coefficients of the higher order terms of WPS using the over-deterministic method was developed [20] to enable multi-parameter description of stress field, where the coefficients of WPS terms are calculated from several layers and angular sections. And the distance distribution of the nodal selection is governed by various functions. Comparison between those used variants was given by visual technique again [21].

Hence, a new detailed way for evaluation of the accuracy of the reconstruction was used in [22]. Method based on the plot of the relative deviation (percent difference) of the stress field between the correct solution (the FE solution) and the solution given by the approximation using WPS with a certain variant of nodal selection was introduced.

Main motivation of this study is to find the easiest way for the multi-parameter stress field description to obtain the sufficiently accurate solution valid for the as far as possible area from the crack tip.

METHODS

Multi-parameter linear elastic fracture mechanics (MP-LEFM)

The stress and displacement fields in a planar homogeneous isotropic cracked body can be formulated as an infinite power expansion – Williams series [1] by Eq. (1) and (2), respectively – for more details see [14,21,22,23].

$$\sigma_{ij} = \sum_{n=1}^{\infty} A_n \frac{n}{2} r^{\frac{n}{2}-1} f_{ij,\sigma}(n,\theta), \quad i,j \in \{x,y\} \qquad (1)$$

$$u_i = \sum_{n=1}^{\infty} A_n r^{\frac{n}{2}} f_{i,\mathrm{u}}(n,\theta,E,\nu), \qquad i \in \{x, y\} \tag{2}$$

In this study, the attention is paid to the mode I crack problem. MP-LEFM takes into account several initial terms of WE, i.e. n ranges from 1 to N (not ∞); coefficients of these terms are often expressed as dimensionless shape functions g_n (functions of the relative crack length $\alpha = a/W$).

Over-deterministic method

For determination of coefficients of the Williams series terms the so-called Over-deterministic method (ODM, [17]) is used. Based on the linear least-squares formulation, it solves a system of $2k$ equations, where k represents the number of selected nodes (in the original paper, they were selected from a nodal ring around the crack tip), for N chosen terms of the power series. Detailed analyses of this method can be found in works [18,19,24]. Displacements of selected nodes, together with their coordinates, serve as the input to that method. This issue was studied in detail in previous works which presented, among others, an implementation of this technique into an automatic numerical tool called *ODeMApp* [20] – the ODM analysis based on an arbitrary nodal selection and test specimen geometry variant is enabled.

ReFraPro approach

The *ReFraPro* (Reconstruction of Fracture Process [25]) is a Java application which allows an advanced determination of fracture characteristics of materials failing in a quasi-brittle manner (silicate-based materials). Estimation of the FPZ (its shape and size) is implemented by a technique developed by the authors combining MP-LEFM, classical non-linear models and plasticity approach. Reconstruction of the fracture process is made by this application generally based on the measured loading curves and basic mechanical properties of the material.
For the purpose of this study, a part of this program is used which provides the reconstruction of stress field from the available shape functions (corresponding to values of coefficients of terms of the WE) for the given test geometry. This part is accompanied with a special tool (a class called *percentdifference*) which allows the display of the deviance (percent difference) between the approximated stress field and the exact solution (for which the FE solution is regarded) to test the accuracy of the solution with nodal selection from an area of interest around the crack tip. The deviation itself is expressed via pixmap grid as the relative difference [22].

NUMERICAL STUDY

For the analysis of the accuracy of the WPS approximation, the wedge-splitting test specimen (WST) is used. Specimen loaded by eccentric tension through two steel platens with pins among which the steel wedge is impressed was developed by Linsbauer and Tscheg in [26]. Schema of the analysed test configuration is displayed in Fig. 1 left accompanied with drawings of its geometry in Fig. 1 right.
Computations of the stress and displacement fields were realized in the ANSYS finite element software [27]. Crack elements (PLANE82) were utilized for the FE solution. An automatic interconnection procedure has been developed between the computational tool and the *ODeMApp* technique for the shape functions determination. A WST variant with two supports and specimens width W = 100 mm (Fig. 1 right) was considered. The employed FE mesh is shown in Fig. 2a with the crack-tip details, where two basic nodal selections (serving as the reference nodal selections) are depicted. The first is taken from the ring at the distance of 5 mm from the crack tip (Fig. 2b) and the second selection is taken from the ring at the distance of 0.5 mm from the crack tip (see Fig. 2c). These variants are labelled as *ring* 5 mm and *ring* 0.5 mm further in the text.
Fig. 3a shows the FE mesh intended for models with a more general nodal selection. Variant from Fig. 3b labelled as *con* 180° (0°) represents a nodal selection governed by a constant distribution function (in the distance selection) taken from the whole area of test specimen (except of steel platens); *qua* 90° (45°) – a nodal selection with a quadratic distance distribution function from area of 90° angular section with the origin rotated by 45° angle (from the crack propagation direction) displayed in Fig. 3c; *exp* 90° (80°) – a nodal selection (from Fig. 3d with an exponential distribution function from the 90° section with initial rotation angle of 80°. Detailed description of the method for the conducted nodal selection, including several other variants, is carried out in [20,21,22]. The number of selected nodes is kept the same for each selection type, k = 49.

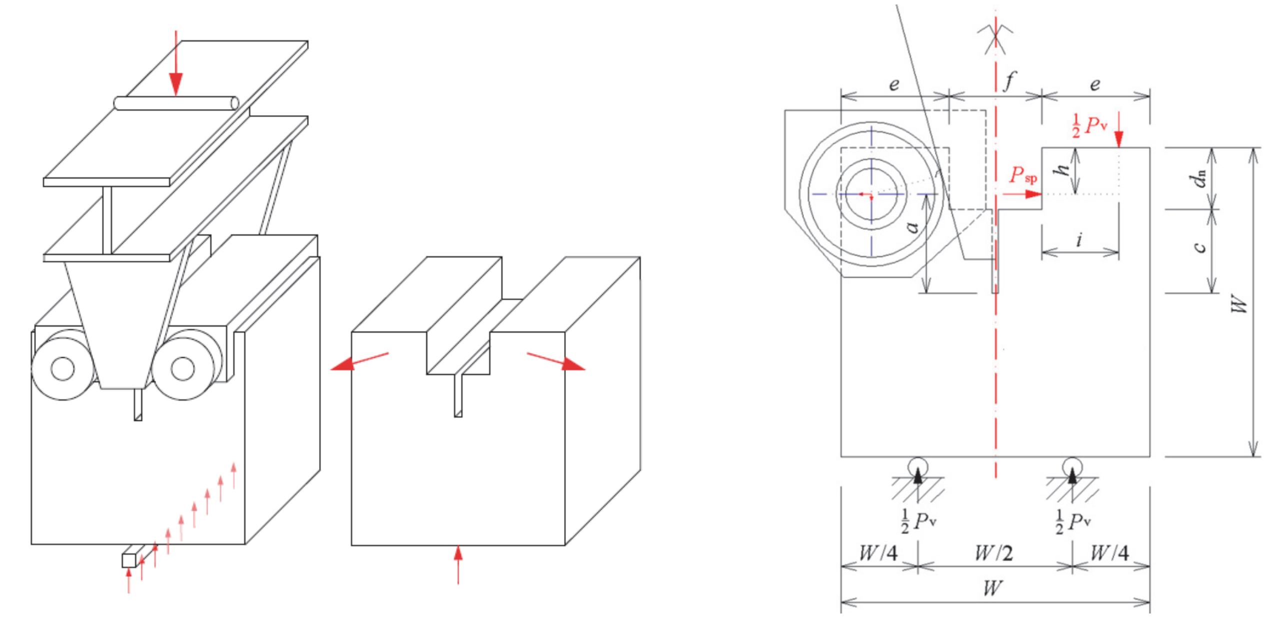

Figure 1: Wedge splitting test with the schema of loading imposition (left); geometry of the analysed wedge splitting test specimen (right)[15].

Shape functions, representing the non-dimensional expressions of the coefficients of the higher order terms of the WE as functions of the relative crack length α, corresponding to the mentioned nodal selection variants were determined by the *ODeMApp* procedure and then they were used as inputs for *ReFraPro* application (which allows displaying the relative deviation from the "exact" solution of the stress field distribution). Subsequently, a post-processing regarding the stress fields reconstructions based on one, two and even higher order terms of the WPS was conducted.

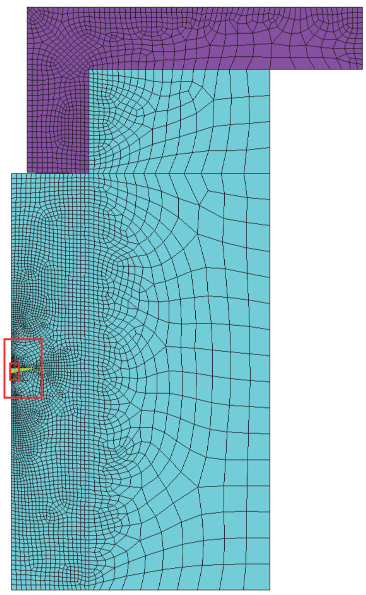

Figure 2a: Numerical model of the wedge splitting test, finite element mesh.

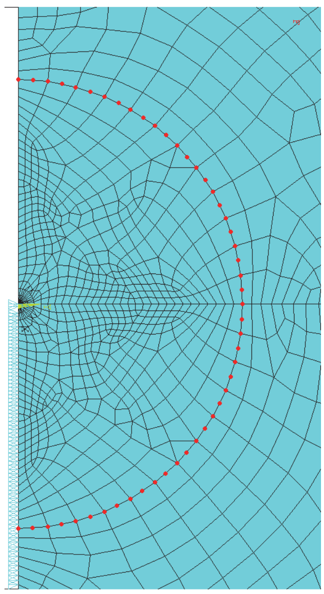

Figure 2b: Detail of the FE mesh for the *ring* 5 mm variant with nodal selection (red dots at the distance 5 mm from the crack tip).

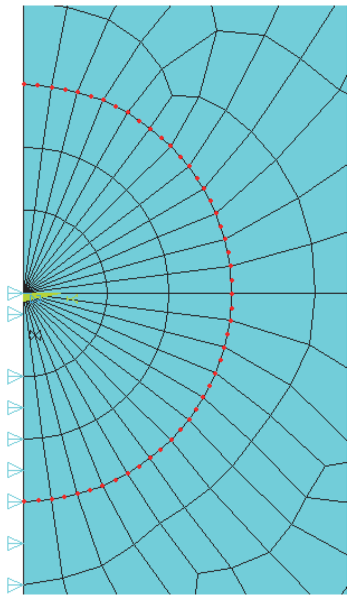

Figure 2c: Detail of the FE mesh for *ring* 0.5 variant with nodal selection (red dots at the distance 0.5 mm from the crack tip).

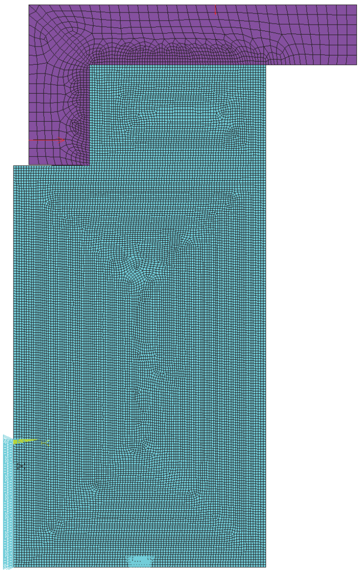
Figure 3a: Numerical model of the WST specimen, used finite element mesh [22].

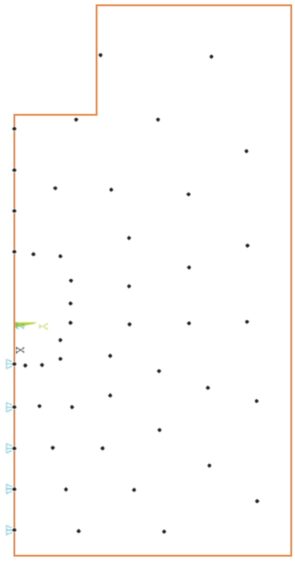
Figure 3b: Distribution of FE nodes for *con* 180° (0°) variant of nodal selection [22].

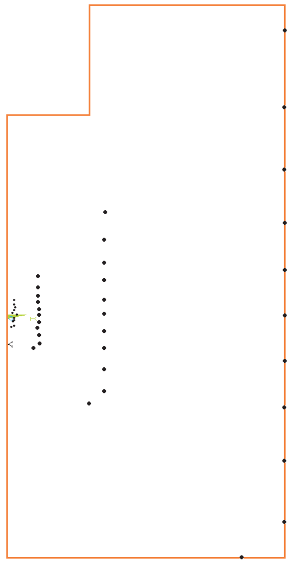
Figure 3c: Distribution of FE nodes for *qua* 90° (45°) variant of nodal selection [22].

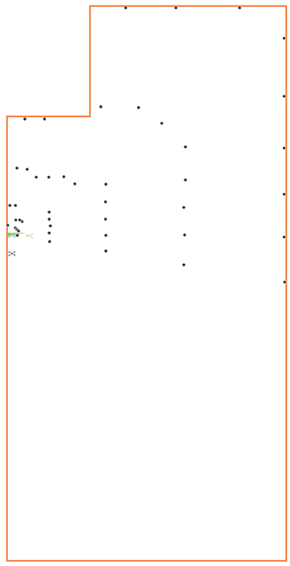
Figure 3d: Distribution of FE nodes for *exp* 90° (80°) variant of nodal selection [22].

RESULTS AND DISCUSSION

The reconstruction of the fields of the principal tensile stress σ_1 and the crack opening stress σ_{yy} was performed and compared to the exact solution. Figs. 4 and 5 display the relative differences between the approximation of the field corresponding to chosen variant of nodal selection and the FEM solution. Only few examples of the reconstruction (based on the number of used Williams series' terms equal to 1, 2, 4, 7 and 11 – positioned horizontally) are given for illustration. Cases with the relative crack length $\alpha = 0.5$ are shown here as an example.

It can be seen that low number of terms of the Williams series used provides very inaccurate approximation of the stress field (mainly for a wider area around the crack tip). Usage of only first two terms leads to a sufficient accuracy only in the very vicinity of the crack tip (where the classical and two-parameter LEFM holds). A sufficiently accurate solution can be provided by a usage of at least four terms of Williams expansion if an area of about 2 cm from the crack tip is requested.

Comparison between the used variants of nodal selection shows one important fact – in general [21,22], variant *con* 180° (0°) (uniform selection from the whole body of the test specimen with the constant distribution function of distance selection) comes out as the best variant of nodal selection (in Fig. 4 and 5 it is emphasized by a green coloured frame). As it can be seen, the greater is the share of the red colour in the percent difference diagram the larger is the relative error from the FE solution. That is why this variant appears in next analyses as the reference one for comparison with variants of nodal selection from just one ring; moreover, from a closer distance from the crack tip. In some cases the variant *qua* 90° (45°) looks also promising and is comparable with the *con* variant. Nevertheless, this is true only for the chosen $\alpha = 0.5$. Work [22] provides detailed views also at different α values.

Next figures compare results of two basic variants of nodal selection from the vicinity of the crack tip with the *con* 180° (0°) variant of the nodal selection, i.e. from the whole body of the test specimen.

Fig. 6 shows contour plots of the relative deviation of principal stress σ_1 for variants of nodal selection *con* 180° (0°), *ring* 5 mm and *ring* 0.5 mm for the WST specimen with relative crack length $\alpha = 0.5$. As far as four terms of Williams expansion for approximation is used the same trend in the relative deviation can be observed. With increasing N the results of *ring* 0.5 mm variant appear to be very inaccurate which can be seen on the extension of the red colour area (denoting a large proportion of 40% difference from the FE solution).

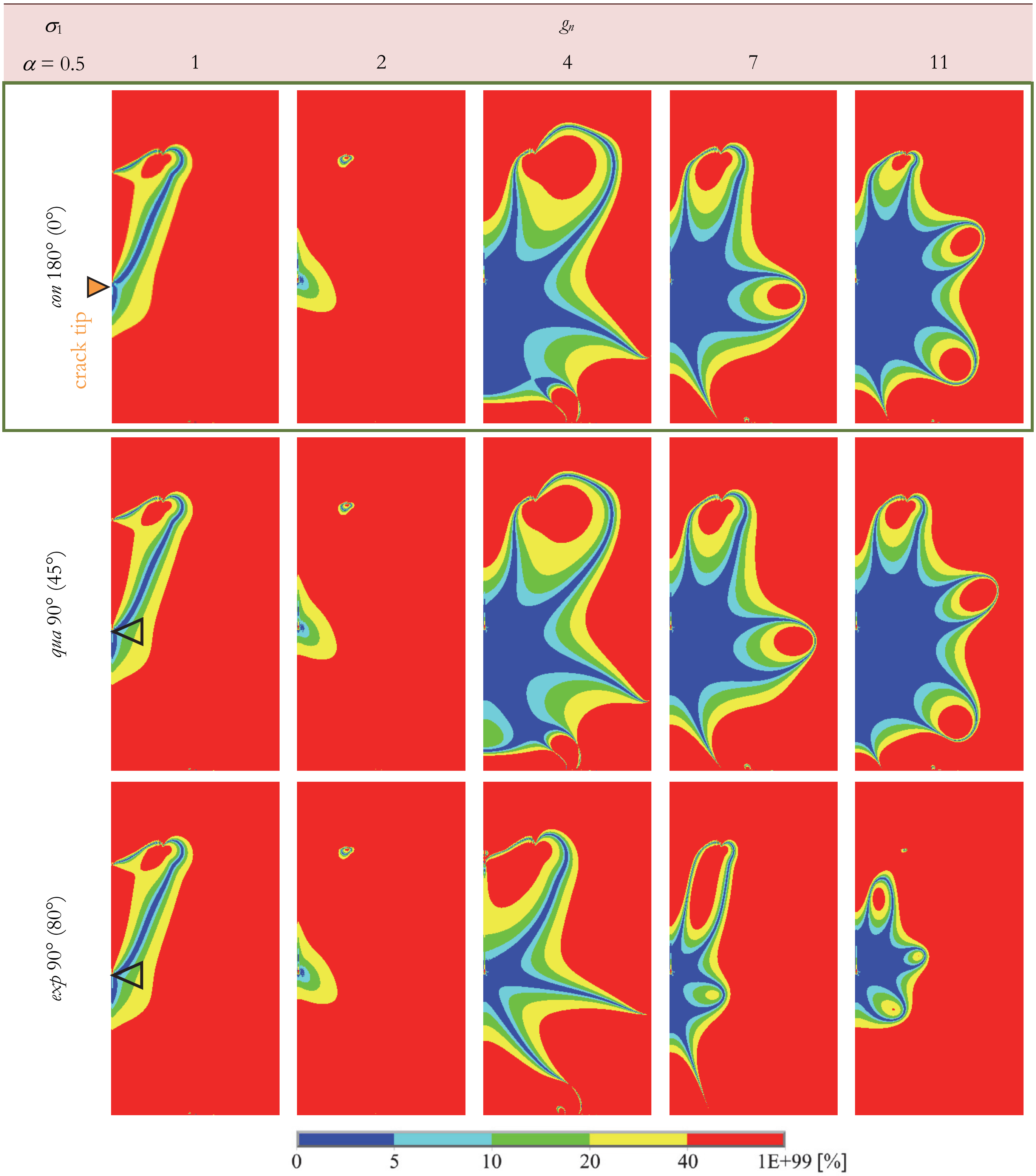

Figure 4: Relative error of principal tensile stress σ_1 field approximation for chosen variants of nodal selection.

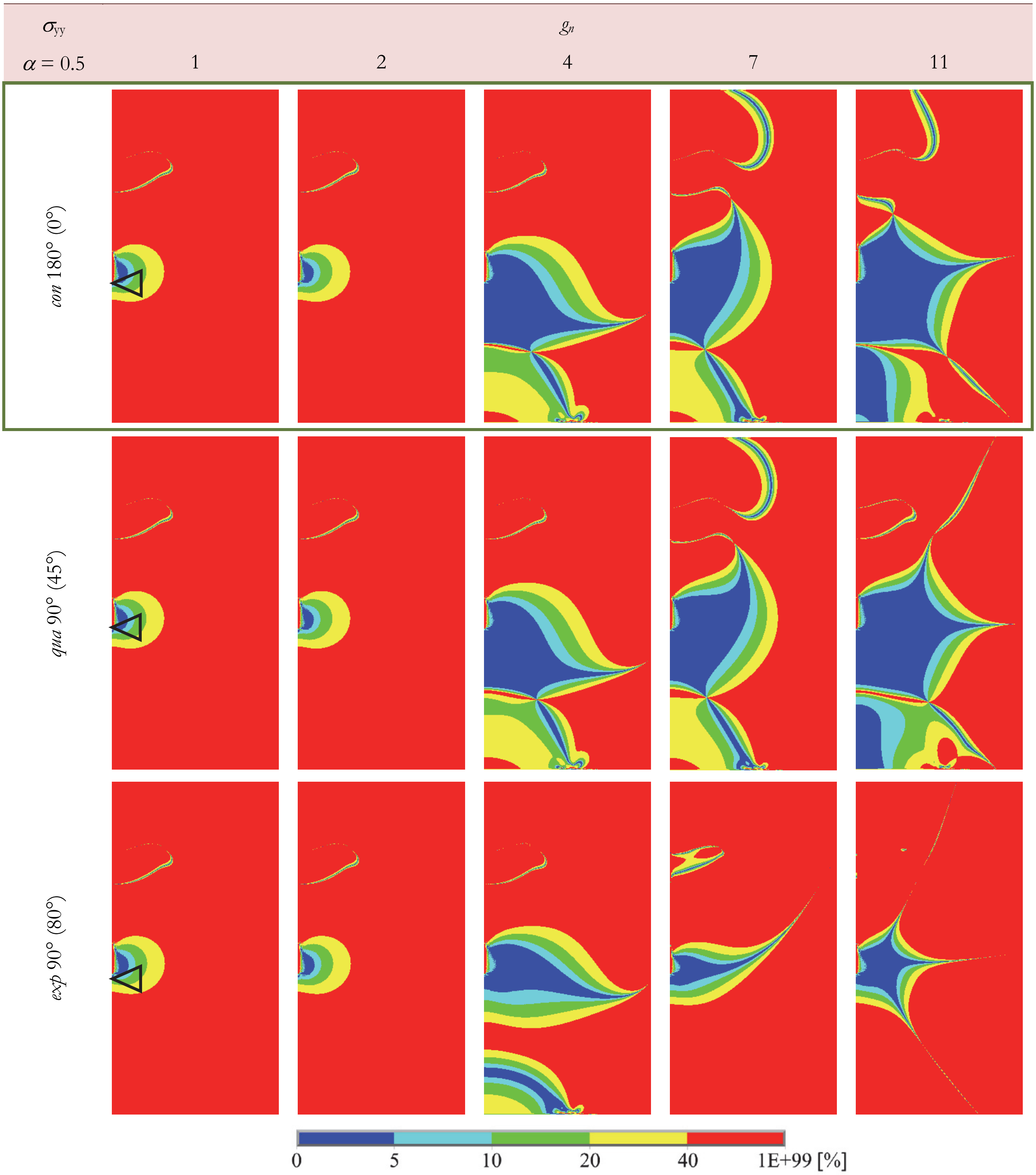

Figure 5: Relative error of crack opening stress σ_{yy} field approximation for chosen variants of nodal selection.

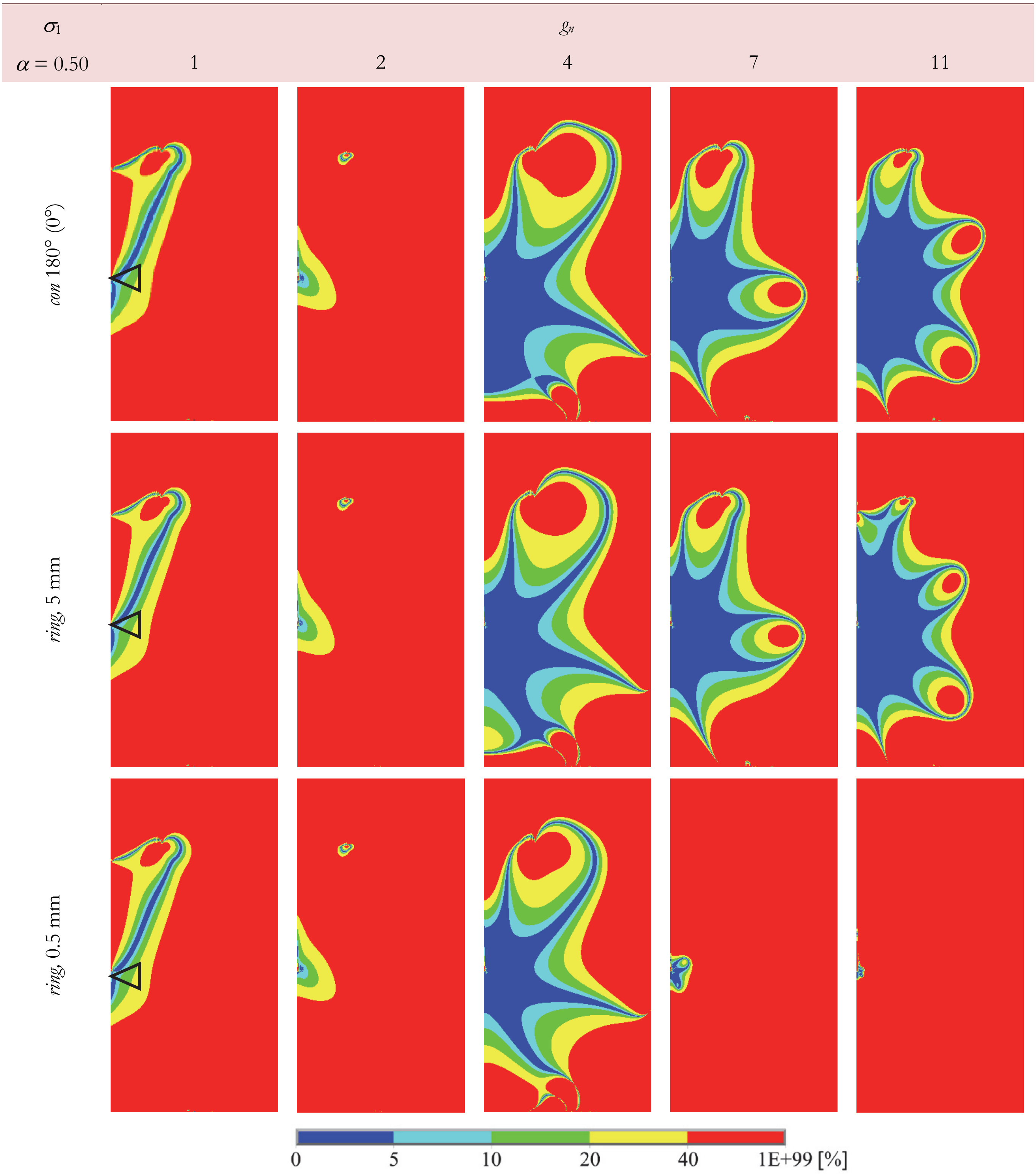

Figure 6: Relative error of principal tensile stress σ_1 field approximation for chosen variants of nodal selection.

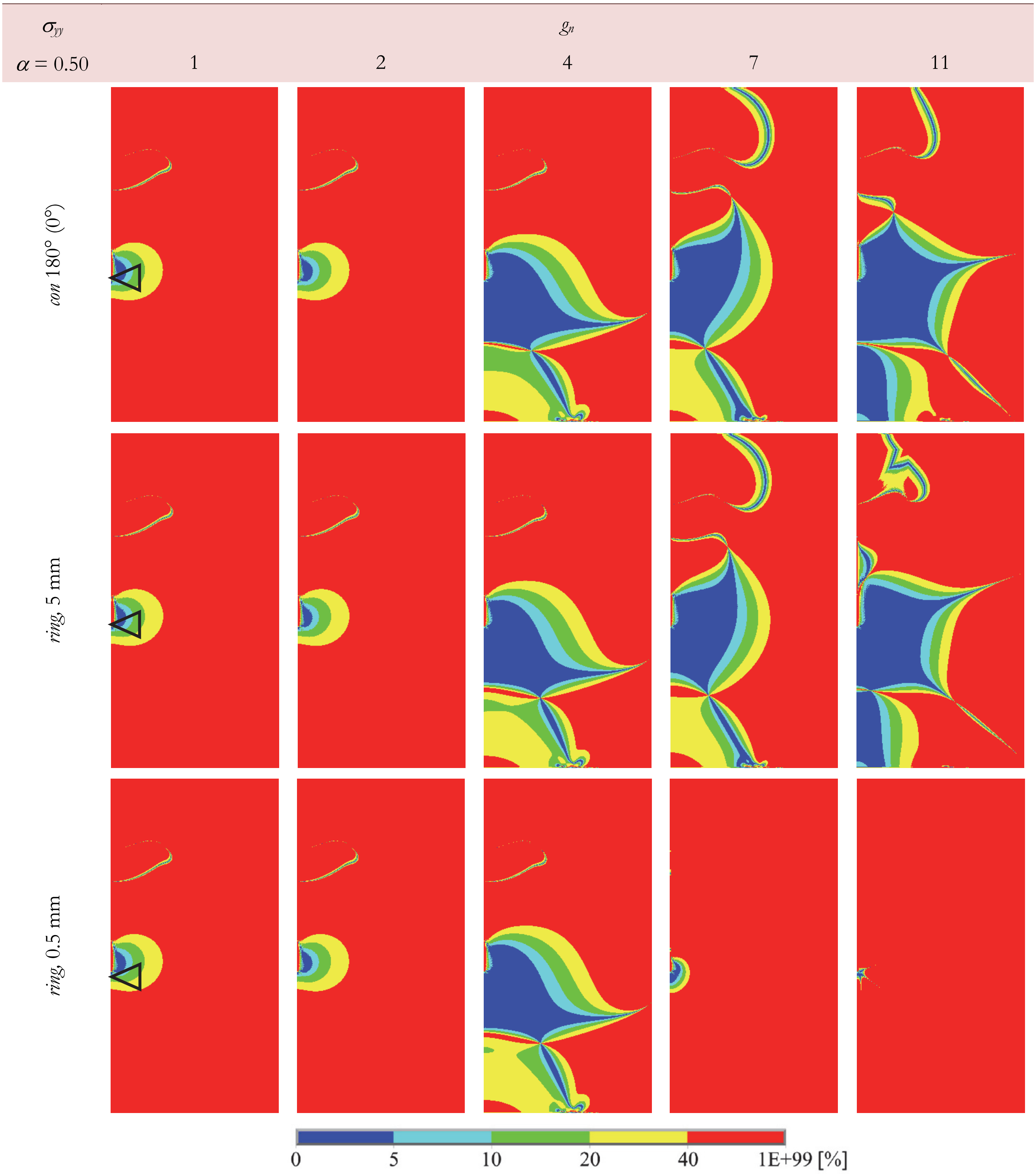

Figure 7: Relative error of crack opening stress σ_{yy} field approximation for chosen variants of nodal selection.

Results of variant *con* 180° (0°) and *ring* 5 mm seem to be very close which can be investigated in detail also on the comparison of the relative error of crack opening stress field σ_{yy} progress for all considered variants in Fig. 7 with α = 0.5. Again, the variant *ring* 0.5 mm shows very inaccurate progress of the stress field reconstruction and should be avoided without usage of more than four terms of the Williams expansion. This fact is described in detail in Fig. 8, which illustrates the relative errors just for *ring* 0.5 mm variant and its progress with the usage of 4, 5, 6 and 7 terms of expansion for reconstruction of principal and crack opening stress field with α = 0.5. From the fourth term the error grows rapidly.

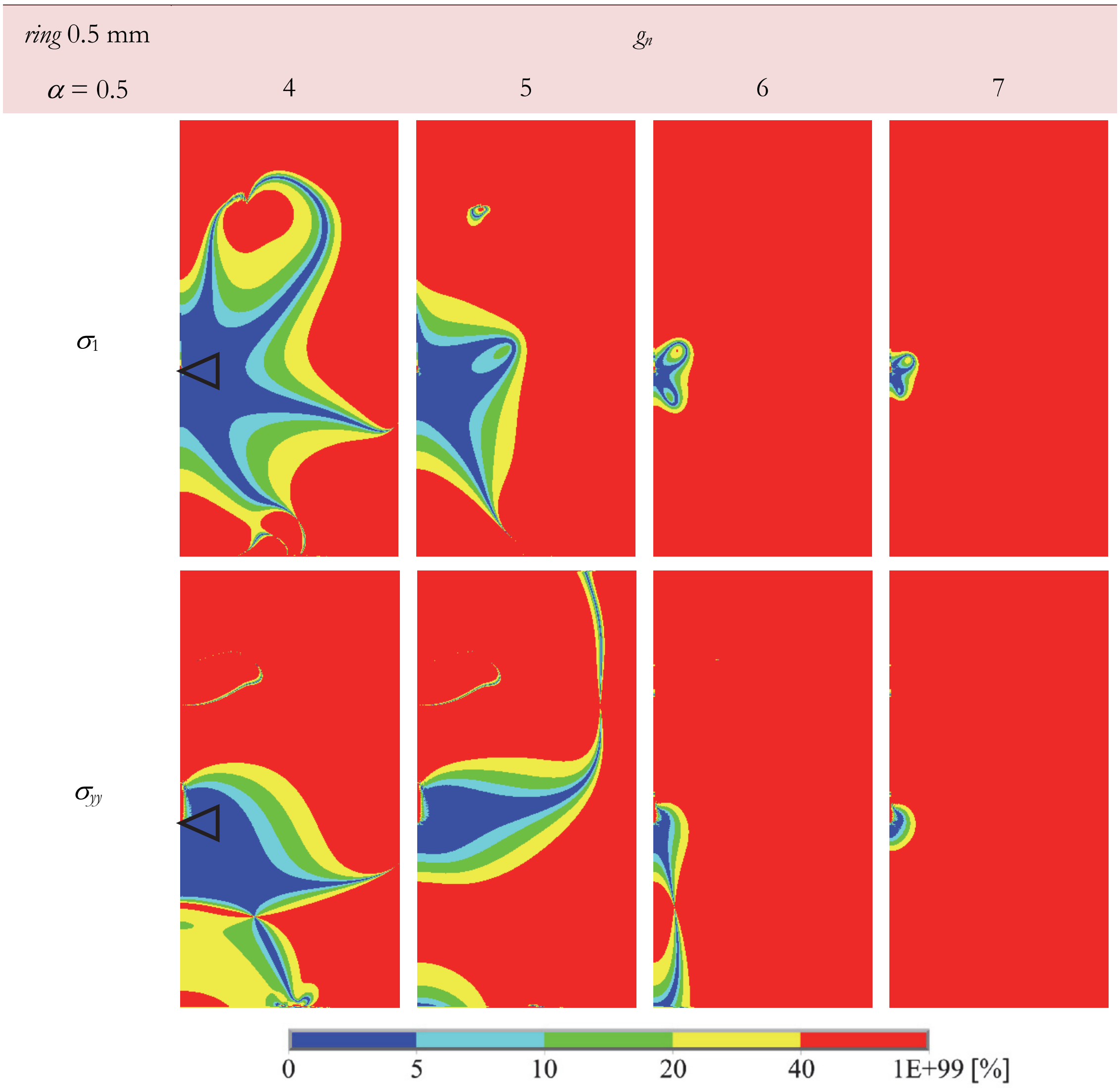

Figure 8: Progress of relative error of the σ_1 and σ_{yy} reconstruction with increasing number of Williams series terms used.

Results depicted in Figs. 9 and 10 show the percent differences of σ_1 and σ_{yy}, respectively, stress field reconstruction. Fig. 9 represents result with α = 0.25 for selected terms of Williams series (g_4, g_7 and g_{11}). These images confirm conclusion that variant *ring* 0.5 mm is very inaccurate for reconstruction of stress field far from the vicinity of the crack tip. Comparison between *ring* 0.5 mm and *con* 180° (0°) variant of nodal selection provides a conclusion that a usage of constant variant is better in case of α = 0.25 (Fig. 9) and also α = 0.75 (Fig. 10). With one exception in Fig. 10 – progress of the g_4 for α = 0.75 in σ_{yy} reconstruction. So, in some cases, it is slightly better to use *ring* 5 mm variant instead of constant distribution function from the whole body of the test specimen.

General quantification of the accuracy of the performed variants of the multi-parameter approximation is not easy to obtain, because the large amount of the nodal selection variants, as were used in the study, lead to combinations, where recommendations cannot be simply stated. An attempt of at least some aspects of such quantification is introduced in this work through the relative error contour plots. The progress of the relative error in the contour maps shows a distance from the crack tip, where the approximation of the stress field is of sufficient accuracy – the blue field may be considered as such a region, because the relative error of approximation here is up to 5 %.

Particular example of such quantification can be explained with the help of results from Fig. 7: with usage (for all variants of nodal selection the trend is the same) of only the first term (g_1) and then also with the first two terms (g_2) the blue region (of the accurate enough approximation) is within a radius of about 5 mm. For the variant *con* 180° (0°) and with usage of 11 terms of the WE the radius increases to about 23 mm. On the other hand, the variant *ring* 0.5 mm with the number of WE terms up to 11 shows a decreasing trend where the utilizable area is of a radius of only about 1 mm.

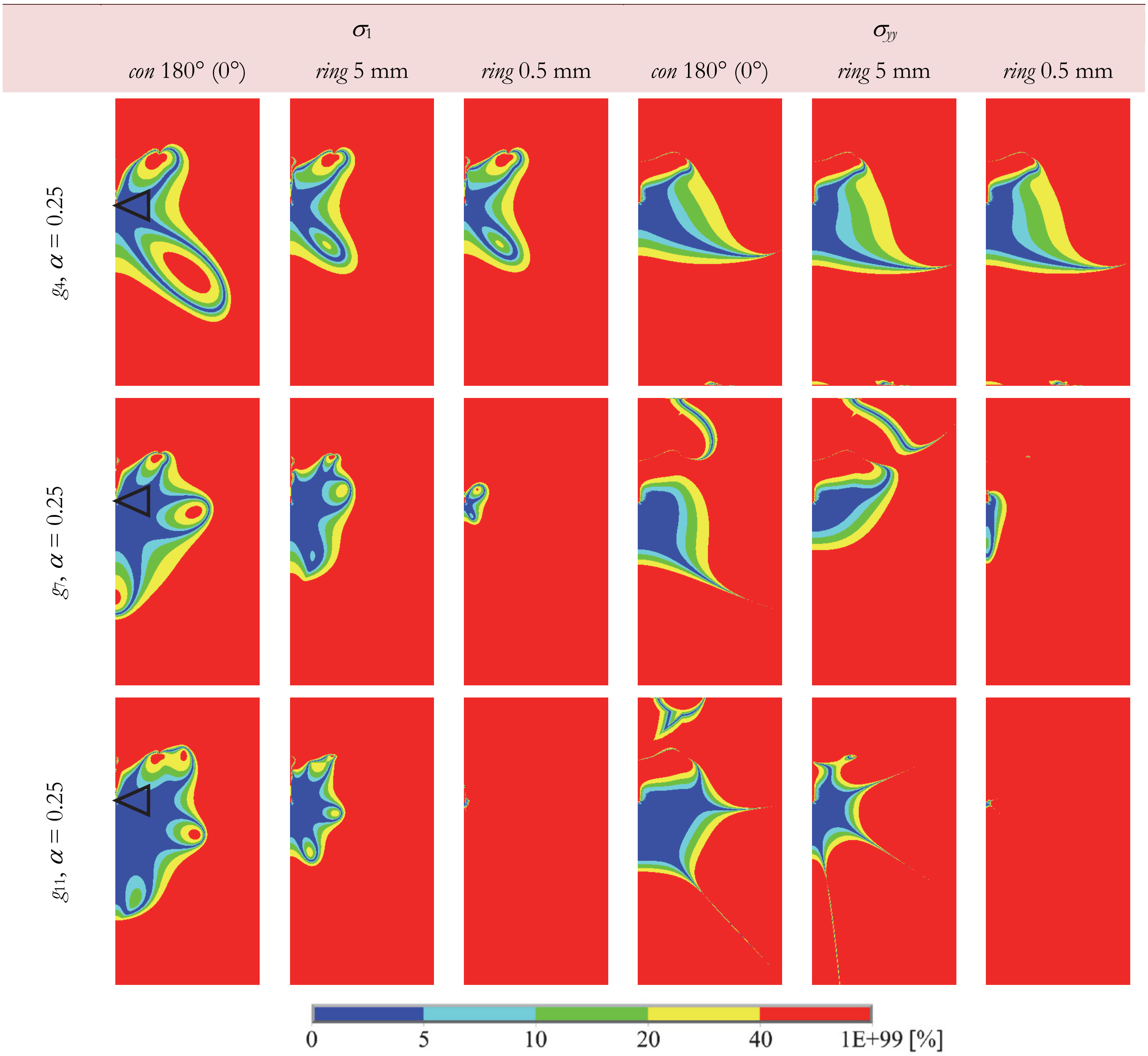

Figure 9: Progress of relative error of the σ_1 and σ_{yy} reconstruction for selected terms of Williams series used.

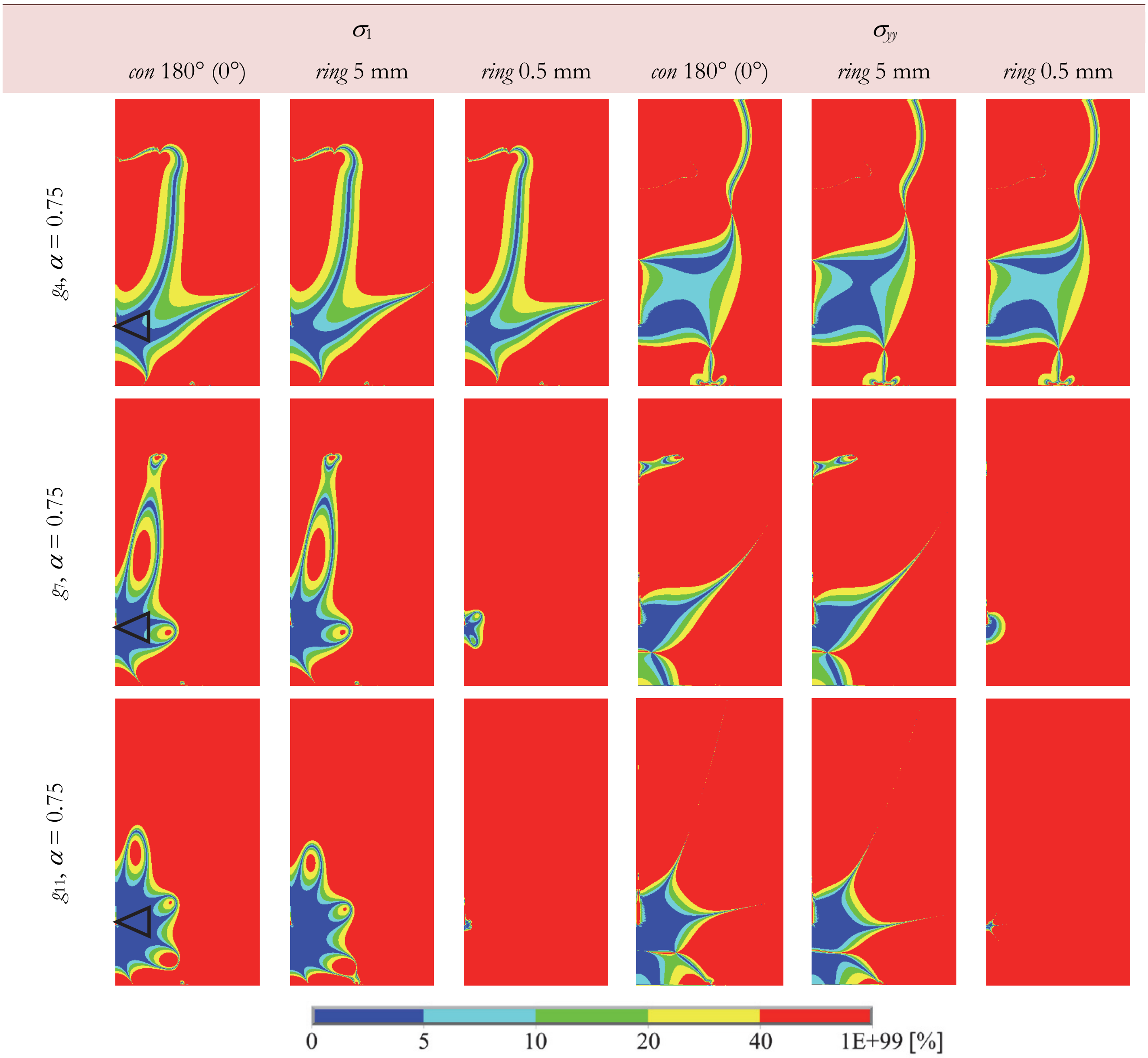

Figure 10: Progress of relative error of the σ_1 and σ_{yy} reconstruction for selected terms of Williams series used.

CONCLUSIONS

A detailed analysis of the accuracy of the Williams series approximation of the stress tensor components (principal stress and crack opening stress) in the cracked WST specimen was shown. The area of interest around the crack tip was investigated from two stand points within this analysis:

– The area from where the nodes are taken into account for the regression technique should be somehow easily defined (easiest way is the ring of nodes around the crack tip), moreover it is convenient to use the ring of small radius to be able to create reasonably good FE mesh also for very short and very long cracks.

In total, five variants of nodal selection were introduced (which followed on at previous published studies) and recommendations are as follows:

– Variant *con* 180° (0°), which represents the nodal selection governed by a uniform distribution from the whole body of the test specimen, seems to be the best choice (and was used as the reference for the next analysis step).
– Nodal selection of variant *ring* 5 mm is still comparable with the uniform nodal distribution variant. This is true up to the highest number of the used Williams series terms that was tested in this study, i.e. $N = 11$.
– Using of variant *ring* 0.5 mm provides a sufficiently accurate results only up to the number of Williams expansion terms $N = 4$. This is not valid for the (very) close vicinity of the crack tip, where the results are still accurate enough.

Future work will provide analysis of variant *ring* 5 mm (or other convenient value of the ring radius) with a particular focus on how many terms N of the Williams series are optimal for the crack-tip field reconstruction at a given distance from the crack tip.

Physical significance of the higher order terms of the Williams series should be better explained and will be analysed via a real experiment (with utilization of digital image correlation technique, similarly to the case of [28]), which is under preparation by authors of this paper. However, for the purpose of the analysis shown in this paper this knowledge is not relevant – the coefficients of terms of the regression are not used here as fracture parameters as it is within the classical or the two-parameter fracture mechanics (K or/and T). Here, they are used only as coefficients of a regression function for the stress field approximation. The aim of this approach is to use the approximation of the field in further fracture-mechanical application (i.e. the plastic zone or the fracture process zone size and shape estimation, etc.).

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REFERENCES

[1] Williams, M.L., On the stress distribution at the base of a stationary crack, Journal of Applied Mechanics (ASME), 24 (1957) 109–114.
[2] Anderson, T.L., Fracture mechanics. Fundamentals and Applications, Boca Raton: CRC Press, (2005).
[3] Berto, F., Lazzarin, P., On higher order terms in the crack tip stress field, International Journal of Fracture, 161 (2010) 221–226.
[4] Pook, L.P., The linear elastic analysis of cracked bodies and crack paths, Theoretical and Applied Fracture Mechanics, 79 (2015) 34–50.
[5] Ayatollahi, M.R., Rashidi Moghaddam, M., Razavi, S.M.J., Berto, F., Geometry effects on fracture trajectory of PMMA samples under pure mode-I loading, Engineering Fracture Mechanics, (2016), in press.
[6] Pook, L.P., The linear elastic analysis of cracked bodies, crack paths and some practical crack path examples, Engineering Fracture Mechanics, (2016), in press.
[7] Berto, F., Lazzarin, P., Kotousov, A., On higher order terms and out-of-plane singular mode, Mechanics of Materials, 43 (2011) 332–341.
[8] Berto, F., Lazzarin, P., Multiparametric full-field representations of the in-plane stress fields ahead of cracked components under mixed mode loading, International Journal of Fatigue, 46 (2013) 16–26.
[9] Pook, L.P., Berto, F., Campagnolo, A., Lazzarin, P., Coupled fracture mode of a cracked disc under anti-plane loading, Engineering Fracture Mechanics, 128 (2014) 22–36.
[10] Ayatollahi, M.R., Rashidi Moghaddam, M., Berto, F., A generalized strain energy density criterion for mixed mode fracture analysis in brittle and quasi-brittle materials, Theoretical and Applied Fracture Mechanics, 79 (2015) 70–76.
[11] Pook, L.P., Campagnolo, A., Berto, F., Lazzarin, P., Coupled fracture mode of a cracked plate under anti-plane loading, Engineering Fracture Mechanics, 134 (2015) 391–403.
[12] Saboori, B., Ayatollahi, M.R., Torabi, A.R., Berto, F., Mixed mode I/III brittle fracture in round-tip V-notches, Theoretical and Applied Fracture Mechanics, 83 (2016) 135–151.
[13] Berto, F., Lazzarin, P., Recent developments in brittle and quasi-brittle failure assessment of engineering materials by means of local approaches, Material Science and Engineering R, 75 (2014) 1–48.

[14] Veselý, V., Sobek, J., Malíková, L., Frantík, P., Seitl, S., Multi-parameter crack tip stress state description for estimation of fracture process zone extent in silicate composite WST specimens, Frattura ed Integrità Strutturale, 25 (2013) 69–78.
[15] Veselý, V., Frantík, P., Sobek, J., Malíková, L., Seitl, S., Multi-parameter crack tip stress state description for evaluation of nonlinear zone width in silicate composite specimens in component splitting/bending test geometry, Fatigue & Fracture of Engineering Materials & Structures, 38-2 (2014) 200–214. DOI: 10.1111/ffe.12170.
[16] Karihaloo, B.L., Abdalla, H., Xiao, Q.Z., Coefficients of the crack tip asymptotic field for wedge splitting specimens, Engineering Fracture Mechanics, 70 (2003) 2407–2420.
[17] Ayatollahi, M.R., Nejati, M., An over-deterministic method for calculation of coefficients of crack tip asymptotic field from finite element analysis, Fatigue & Fracture of Engineering Materials & Structures, 34 (2010) 159–176.
[18] Sobek, J., Veselý, V., Šestáková, L., Accuracy of approximation of stress and displacement fields in cracked body for estimation of failure zone extent, Transactions of the VŠB–Technical University of Ostrava, Civil Engineering Series, XII-2 (2012) 1–10. DOI: 10.2478/v10160-012-0031-5.
[19] Malíková, L., Multi-parameter fracture criteria for the estimation of crack propagation direction applied to a mixed-mode geometry, Engineering Fracture Mechanics, 143 (2015) 32–46.
[20] Sobek, J., Pail, T., Veselý, V., Evaluation of crack tip Stress field using combination of advanced implementation of over-deterministic method and FE analysis, Key Engineering Materials, 627 (2015) 273–276. DOI: 10.4028/www.scientific.net/KEM.627.273.
[21] Veselý, V., Sobek, J., Tesař, D., Frantík, P., Pail, T., Seitl, S., Multi-parameter approximation of stress field in a cracked specimen using purpose-built Java applications, Frattura ed Integrità Strutturale, 33 (2015) 120–133.
[22] Veselý, V., Sobek, J., Frantík, P., Seitl, S., Multi-parameter approximation of the stress field in a cracked body in the more distant surroundings of the crack tip, International Journal of Fatigue, 89 (2016) 20–35. DOI: 10.1016/j.ijfatigue.2016.02.016
[23] Malíková, L., Veselý, V., Seitl, S., Estimation of the crack propagation direction in a mixed-mode geometry via multi-parameter fracture criteria, Frattura ed Integrità Strutturale, 33 (2015) 25–32.
[24] Malíková, L., Veselý, V., Significance of Higher-order Terms of the Williams Expansion for Plastic Zone Extent Estimation Demonstrated on a Mixed-mode Geometry, Procedia Materials Science, 3 (2014) 1383–1388.
[25] Veselý, V., Frantík, P., An application for the fracture characterisation of quasi-brittle materials taking into account fracture process zone influence, Advances in Engineering Software, 72 (2014) 66–76. DOI: 10.1016/j.advengsoft.2013.06.004.
[26] Linsbauer, H.N., Tscheg, E.K., Fracture energy determination of concrete with cube-shaped specimens, Zement und Beton, 31 (1986) 38–40.
[27] ANSYS Documentation, Version 11.0, Swanson Analysis System, Inc., Houston, Pennsylvania, (2007).
[28] Vargas, R., Neggers, J., Canto, R.B., Rodrigues, J.A., Hild, F., Analysis of wedge splitting test on refractory castable via integrated DIC, Journal of the European Ceramics Society, 36 (2016) 4309–4317.