Effect of spectral cross-correlation on multiaxial fatigue damage: simulations using the critical plane approach

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ABSTRACT. The present paper aims to discuss a frequency-domain multiaxial fatigue criterion based on the critical plane approach, suitable for fatigue life estimations in the presence of proportional and non-proportional random loading. The criterion consists of the following three steps: definition of the critical plane, Power Spectral Density (PSD) evaluation of an equivalent normal stress, and estimation of fatigue damage. Such a frequency-domain criterion has recently been validated by using experimental data available in the literature, related to combined proportional and non-proportional bending and torsion random loading. The comparison with such experimental data has been quite satisfactory. In order to further validate the above criterion, numerical simulations are herein performed by employing a wide group of combined bending and torsion signals. Each of such signals is described by an ergodic, stationary and Gaussian stochastic process, with zero mean value. The spectrum of each signal is assumed to be represented by a PSD function with rectangular shape. Different values of correlation degree, variance and spectral content are examined.

KEYWORDS. Critical plane-based criterion; Frequency-domain criterion; Power Spectral Density.

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INTRODUCTION

Engineering structures suffer from failure associated with fatigue, both at the stage of manufacturing and under service conditions [1]. Examples in the civil engineering field are jack-up platforms, bridges, towers and masts, for which their structural integrity is of paramount importance both to avoid huge material losses and ecological disaster and to ensure the life and health of people.

Fatigue damage in structures is caused by progressive crack growth under time-varying loading [2]. A method of analysis, named time-domain analysis, is that to consider a loading time history on the structure to find the output response caused

by such an input. However, most structures experience a variety of loadings. Therefore, since many load cases are necessary for fatigue analysis, the computational time can become extremely high or even unacceptable [2].
A much more efficient method of analysis, named frequency-domain or spectral analysis, is that to consider the Power Spectral Density (PSD) function of the loading on the structure, which represents the frequency content of the loading time history. Generally, such methods employ an equivalent uniaxial loading to represent the actual multiaxial stress state, opening the possibility to use time- and frequency-domain methods originally proposed for fatigue analysis under uniaxial variable or random amplitude loading.
The spectral method employed in this paper was proposed by the authors to determine fatigue damage in structures under multiaxial stationary random Gaussian loading [3]. In the recent past, the method was validated by employing experimental data available in the literature [3-7]. Numerical simulations are here developed, by considering random biaxial loading characterized by different values of the correlation coefficient, zero order moments ratio and central frequencies ratio.

FREQUENCY-DOMAIN CRITICAL PLANE (F-D/CP) CRITERION

The input data for the fatigue damage calculation is the PSD matrix of the stress tensor (Step 1 in Fig.1). The determination of the expected critical plane orientation constitutes an important part of the algorithm (Step from 2 to 4 in Fig.1). The PSD function of an equivalent stress is defined as a linear combination of the PSD functions of suitable stress components acting on the critical plane. Finally, the expected fatigue damage per unit time is obtained (Step 5 in Fig.1). Details of each step of the F-D/CP criterion are reported in Refs [3-7].

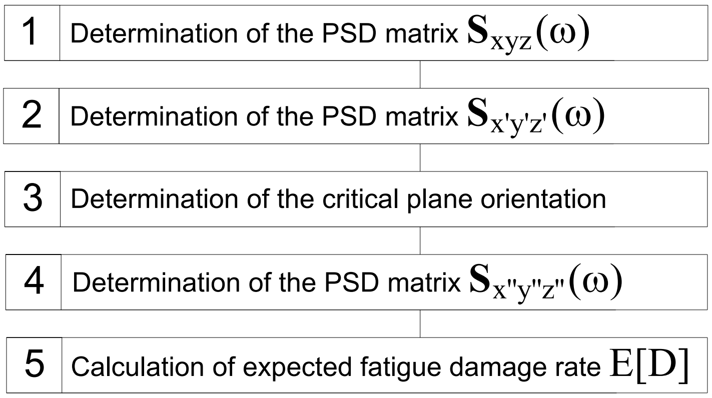

Figure 1: Algorithm for damage determination using the F-D/CP criterion.

Random biaxial loading: numerical simulations

Let us consider the following stress tensor

$$\mathbf{s}_{\mathrm{xyz}}(t)=\{s_1,s_2,s_3,s_4,s_5,s_6\}^T=\{\sigma_{\mathrm{x}},\sigma_{\mathrm{y}},\sigma_{\mathrm{z}},\tau_{\mathrm{yz}},\tau_{\mathrm{xz}},\tau_{xy}\}^T=\{\sigma_{\mathrm{x}},0,0,0,0,\tau_{xy}\}^T$$

with respect to the fixed frame XYZ. By assuming that the random features can be described by a two-dimensional ergodic stationary Gaussian stochastic process with zero mean value, the PSD matrix with respect to XYZ is here displayed:

$$\mathbf{S}_{\mathrm{xyz}}(\omega)=\begin{bmatrix} S_{11} & 0 & 0 & 0 & 0 & S_{16} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ S_{61} & 0 & 0 & 0 & 0 & S_{66} \end{bmatrix} \tag{1}$$

where ω is the pulsation. The symbols S_{11} and S_{66} represent the auto-spectral density function of the stress components s_1 and s_6 (i.e. σ_x and τ_{xy}), respectively, whereas S_{16} and S_{61} represent the cross-spectral density functions of the above stress components. It is here assumed that the imaginary part of S_{16} is equal to zero and, consequently, even the imaginary part of S_{61} is equal to zero: that is, $S_{16} = S_{61}$ [8]. In such a case, the PSD matrix is symmetric.
Now a rectangular spectrum is assumed for both S_{11} and S_{66}. Let us introduce the correlation coefficient given by:

$$r_{16} = \frac{\lambda_{0,16}}{\sqrt{\lambda_{0,11} \cdot \lambda_{0,66}}} \tag{2}$$

where $\lambda_{0,16}$, $\lambda_{0,11}$, and $\lambda_{0,66}$ represent the zero order moment of S_{16}, S_{11} and S_{66}, respectively (more precisely, $\lambda_{0,16}$ is the co-variance of S_{16}, whereas $\lambda_{0,11}$, and $\lambda_{0,66}$ are the variances of S_{11} and S_{66}, respectively). Note that, for proportional stress components, r_{16} tends to the unity, while r_{16} tends to zero for highly uncorrelated loading.
The central frequency, $\omega_{c,11}$, and the height, h_{11}, related to S_{11} are assumed to be equal to 10Hz and 11MPa²/Hz, respectively. Different values of the ratio between the central frequencies, $\omega_{c,r} = \omega_{c,66} / \omega_{c,11}$ are examined: $\omega_{c,r} =$ 0.1, 1.0, 1.1, 5.0, and 10.0. The maximum to minimum frequency ratio is equal to 1.1/0.9. The variance of the stress component σ_x turns out to be equal to 22 MPa².
Different values of the ratio between the zero order moments, $\lambda_{0,r} = \lambda_{0,66} / \lambda_{0,11}$ are examined: $\lambda_{0,r} =$ 0, 1, 100, and ∞.
Three different types of spectral cross-correlation for S_{11} and S_{66} are examined, that is:
(i) totally separated spectra (Fig.2(a)). In such a case: $\omega_{c,r} =$ 0.1, 5.0 and 10.0, and r_{16} is equal to zero;
(ii) completely overlapped spectra (Fig.2(b)). In such a case: $\omega_{c,r} = 1.0$, and different values of r_{16} are assumed, i.e. $r_{16} =$ 0.00, 0.25, 0.50, 0.75, and 1.00;
(iii) partially overlapped spectra (Fig.2(c)). In such a case: $\omega_{c,r} = 1.1$, and different values of r_{16} are assumed, i.e. $r_{16} =$ 0.00, 0.25, 0.50, 0.75, and 1.00.
By exploiting both Eq.(1) and the Schwartz inequality [8], which states that S_{16} is non-negative only where S_{11} and S_{66} are overlapped and zero elsewhere, the height h_{16} of the rectangular cross-spectrum is computed for each considered biaxial loading state.

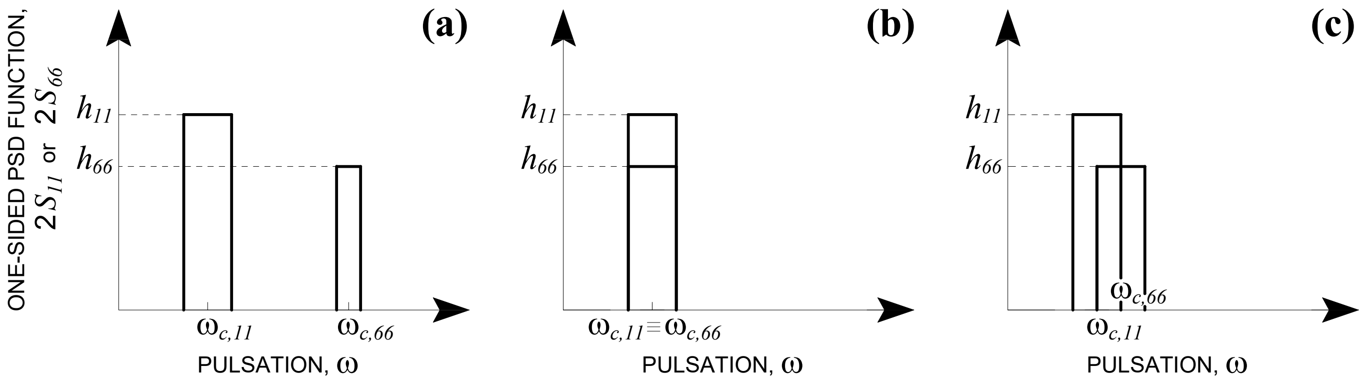

Figure 2: One-sided PSD functions: (a) totally separated spectra (i), (b) completely overlapped spectra (ii), and (c) partially overlapped spectra (iii).

The reference fatigue parameters used in the analysis are: $\sigma_{af,-1} = 79.37(10)^{-4}$ MPa, $N_0 = 2(10)^6$ cycles, $C = 1.0$ and $k = 3.0$ for fully reversed tension or bending, whereas $\tau_{af,-1} = \sigma_{af,-1} / \sqrt{3}$ for fully reversed torsion [8].

Figures 3-5 present overall damage comparisons, by plotting the ratio of expected fatigue damage per unit time, $E[D]$, to the reference damage per unit time, $E_{ref}[D]$. Such a reference value $E_{ref}[D]$ depends on the spectra type:

- for type (i), it represents the damage produced by two sinusoidal signals characterized by pulsations equal to $\omega_{c,11}$ and $\omega_{c,66}$, respectively, and variance equal to $\lambda_{0,11}$ and $\lambda_{0,66}$, respectively;
- for types (ii) and (iii), it represents the damage produced by three sinusoidal signals characterized by pulsations equal to $\omega_{c,11}$, $\omega_{c,66}$, and $\omega_{c,16}$, respectively, and variance equal to $\lambda_{0,11}$, $\lambda_{0,66}$ and $\lambda_{0,16}$, respectively.

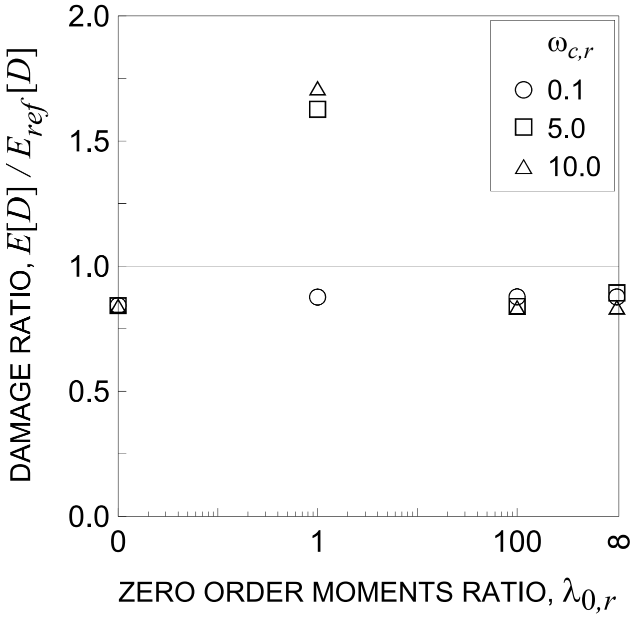

Figure 3: Comparison between $E[D]$ and $E_{ref}[D]$: totally separated spectra.

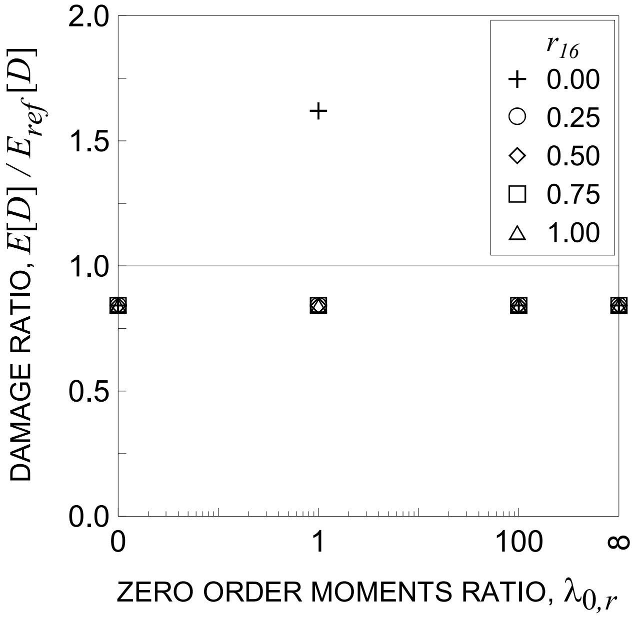

Figure 4: Comparison between $E[D]$ and $E_{ref}[D]$: completely overlapped spectra.

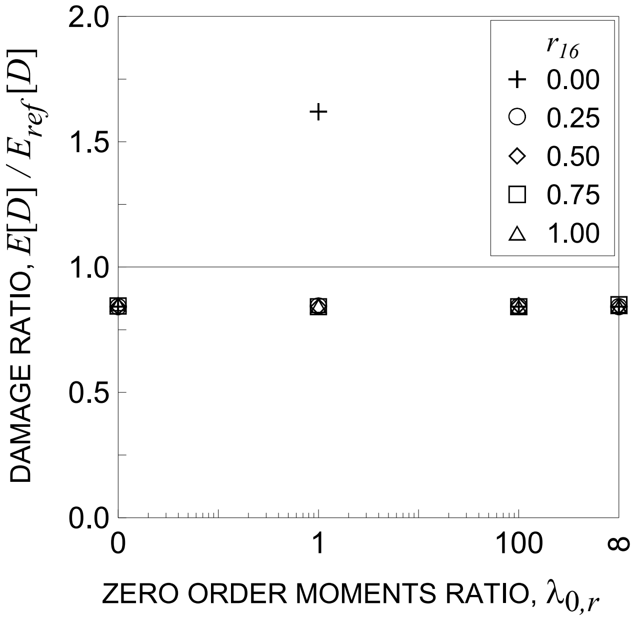

Figure 5: Comparison between $E[D]$ and $E_{ref}[D]$: partially overlapped spectra.

It can be observed that, in general, fatigue damage rate is slightly lower than that of the reference sinusoidal loading. Only in the case of bending and torsion loads of the same variance, a fatigue damage rate up to about 60% higher than that of the reference loading case is recorded for completely uncorrelated signals ($r_{16} = 0$).

CONCLUSIONS

In the present paper a frequency-domain multiaxial fatigue criterion based on the critical plane approach, suitable for fatigue life estimations in the presence of proportional and non-proportional random loading, has been discussed.

In order to validate the above criterion, numerical simulations have herein been performed by employing a wide group of combined bending and torsion signals. The narrow-band spectrum of each signal has been assumed to be represented by a PSD function with rectangular shape. Different values of correlation degree, variance and spectral content have been examined. Among the cases being simulated, the most damaging one is identified in the loading combination of bending and torsion having same variance but being completely uncorrelated (correlation coefficient equal to zero).

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