Statistical analysis of fatigue crack propagation data of materials from ancient portuguese metallic bridges

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ABSTRACT. In Portugal there is a number of old metallic riveted railway and highway bridges that were erected by the end of the 19th century and beginning of the 20th century, and are still in operation, requiring inspections and remediation measures to overcome fatigue damage. Residual fatigue life predictions should be based on actual fatigue data from bridge materials which is scarce due to the material specificities. Fatigue crack propagation data of materials from representative Portuguese riveted bridges, namely the Pinhão and Luiz I road bridges, the Viana road/railway bridge, the Fão road bridge and the Trezói railway bridge were considered in this study. The fatigue crack growth rates were correlated using the Paris's law. Also, a statistical analysis of the pure mode I fatigue crack growth (FCG) data available for the materials from the ancient riveted metallic bridges is presented. Based on this analysis, design FCG curves are proposed and compared with BS7910 standard proposal, for the Paris region, which is one important fatigue regime concerning the application of the Fracture Mechanics approaches, to predict the remnant fatigue life of structural details.

KEYWORDS. Fracture Mechanics; Fatigue Crack Growth; Statistical Analysis; Old Steels; Ancient Bridges; BS7910.

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INTRODUCTION

In Portugal, the major concern of governmental agencies is related with the maintenance and safety of centenaries riveted steel bridges. These old riveted road and railway bridges were fabricated and placed into service during the 19th century and beginning of 20th century. The traffic conditions both in terms of vehicle gross weight and frequency, are today completely different from those considered in the design phase. Additionally, the original design procedures of these bridges did not account for fatigue, since fatigue understanding only achieved the maturity after the design of those structures. In the 19th century the designer engineers were not aware of some important damage phenomena including fatigue. Fatigue was only intensively studied in the 20th century. In order to maintain the high safety levels of old riveted steel bridges, road and railway authorities have to invest heavily in inspection, maintenance and retrofitting, with those activities supported by fatigue assessment studies, including remnant life calculations [1].

The crack propagation data is essential to perform fatigue life predictions according to the Linear Elastic Fracture Mechanics (LEFM), which is an important alternative to the usual code-based S-N curve procedures, mainly in what concerns residual life estimations. In this perspective, the knowledge about design fatigue crack growth curves for materials from ancient Portuguese steel bridges is extremely appropriate [1].

The present paper reports research work carried out to determine the design FCG curves of materials from ancient Portuguese riveted bridges, namely the Pinhão, Fão and Luiz I road bridges, built in 1906, 1891 and 1886, respectively, the Eiffel road/railway bridge (inaugurated in 1878) and the Trezói railway bridge (inaugurated in 1956). The design FCG curves were obtained using the procedure proposed by Gallegos Mayorga et al. [2] and a comparison with design crack propagation curves proposed by BS7910 standard [3] is made. The authors have investigated the mechanical behaviour of those bridge materials, such as, metallographic, chemical composition, monotonic and fatigue behaviours of the materials of the old riveted steel bridges have been characterized [4-7]. Others authors have performed similar work for other centenary bridges [8,9]. Correia et al. [6,10,11] and Bogdanov et al. [12] have proposed probabilistic curves for the fatigue crack growth propagation curves for several materials. Correia et al. [6,10,11] proposed the probabilistic fields for fatigue crack growth using the probabilistic fatigue local approaches using old materials from riveted steel bridges and current steels. Meanwhile, Bogdanov et al. [12] proposed a probabilistic analysis of the fatigue crack growth rates based on the Monte-Carlo method applied to the *Unigrow* model.

FATIGUE LIFE EVALUATION BASED ON FRACTURE MECHANICS

Typically, the fatigue life predictions of structural details based on Fracture Mechanics are used for residual fatigue life assessments containing initially known defects acting as cracks and are typically used as evaluation criteria for planning in-service inspections [13]. De Jesus et al. [13] evaluated the residual fatigue life of an ancient riveted steel road bridge using Fracture Mechanics approach based on experimental fatigue crack propagation data obtained for the old material from the Pinhão bridge. Thus, fatigue crack propagation data are fundamental to be used in fatigue life prediction approaches using Fracture Mechanics. In this sense, design fatigue crack growth curves are extremely important to establish and maintain the safety levels of the structural details and consequently of the structures.

In Fig. 1, a schematic bi-logarithm representation of the three fatigue crack growth regimes, between da/dN (crack growth rate) and ΔK (stress intensity factor range), is showed. This behaviour shows two vertical asymptotes: one on the left, at $\Delta K = \Delta K_{th}$, indicates that ΔK values below this threshold level are too low to cause macro crack growth; and the right asymptote occurs for a ΔK cycle with $K_{max} = K_c$, which leads to complete failure of the specimen. These three regimes can be denominated as: I – the threshold region; II – the Paris region; III - the unstable tearing crack growth region [14].

Paris and Erdogan [15] established a power function (Eq. (1)) to describe the relation between da/dN and ΔK:

$$\frac{da}{dN} = C \cdot \left(\Delta K\right)^m \tag{1}$$

where C and m are material parameters. This law is used to describe the so-called Paris region and the experimental data follows a linear relation when using bi-logarithmic scales are used, as shown in Fig. 1.

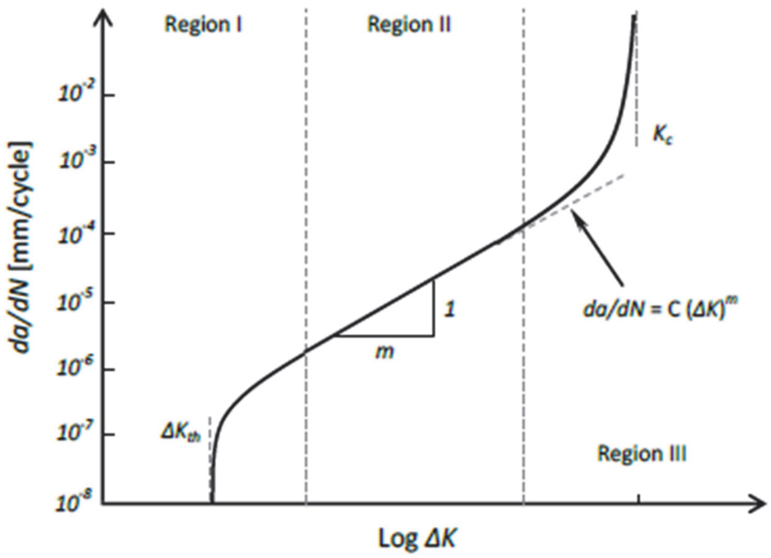

Figure 1: Three crack growth regimes for (da/dN) versus ΔK [11].

The integration of the fatigue crack propagation laws should be made between the initial crack size, a_i, and the final crack size, a_f, as indicated by Eq. (2):

$$N_f = \int_{a_i}^{a_f} \frac{1}{C \cdot \Delta K^m} da \tag{2}$$

The final crack size, a_f, is established by unstable crack propagation, dictated by material toughness, or plastic failure at the net section. The initial crack size, a_i, is assumed around 0.25 to 1 mm for metals underestimating fatigue life of the component [16-18]. Furthermore, a crack depth of 0.5 mm can be assumed in Fracture Mechanics analysis if it is not indicated by the available standards [3].

The stress intensity factor, K, can be evaluated using the weight functions [19] and finite element analysis or using finite element analysis [20] to calculate the stresses and displacements on the crack front followed by the implementation of the virtual crack closure technique (VCCT) [21]. Alternatively, this parameter can be obtained by analytical relations that were established by several authors depending of the geometry.

The definition of a design fatigue crack propagation curve for current steels and old materials is of great importance for obtaining safe residual fatigue lives of structural details from old metallic bridges.

STATISTICAL EVALUATION OF EXPERIMENTAL FATIGUE CRACK PROPAGATION DATA

Statistical procedure

In this paper, the statistical procedure to determine the design curves for the experimental fatigue crack propagation data used was proposed by Gallegos Mayorga et al. [2]. This procedure follows the same recommendations that were proposed by ASTM E739-91 standard [22]. This statistical procedure is based on the linear Paris law that is described by Eq. (3), where $\frac{da}{dN}$ is the fatigue crack growth (FCG) rates, ΔK is the applied stress intensity factor range, C and m are material constants, $\left(\frac{da}{dN}\right)^* = \log\left(\frac{da}{dN}\right)$, $C^* = \log(C)$, $m^* = m$ and $\Delta K^* = \log(\Delta K)$.

$$\left(\frac{da}{dN}\right)^* = C^* + m^* \cdot (\Delta K)^* \tag{3}$$

This procedure considers that the fatigue crack growth (FCG) data pertain to a random sample where all $\left(\frac{da}{dN}\right)^*_i$ are independent and there are no run-outs or suspended tests for the entire range of $\left(\Delta K\right)^*$. Furthermore, the linear model for the Paris relation can be rewritten as Eq. (4) shows, where $\widehat{\left(\frac{da}{dN}\right)^*_i}$ is the estimative (estimator) for the values of FCG rates.

$$\widehat{\left(\frac{da}{dN}\right)^*_i} = C^* + m^* \cdot \left(\Delta K\right)^*_i \tag{4}$$

The characterization parameters of the statistical analysis, such as the variance and the standard deviation must be defined. Regarding the variance of the log-normal distribution, it is constant and maximum likelihood estimators of C^* and m^* are, respectively, defined by Eqs. (5) and (6).

$$C^* = \sum_{i=1}^{k} \frac{\left(\frac{da}{dN}\right)^*_i}{k} - m^* \cdot \sum_{i=1}^{k} \frac{\left(\Delta K\right)^*_i}{k} = \overline{\left(\frac{da}{dN}\right)^*} - m^* \cdot \overline{\left(\Delta K\right)^*} \tag{5}$$

$$m^* = \frac{\sum_{i=1}^{k}\left(\left(\Delta K\right)^*_i - \overline{\left(\Delta K\right)^*}\right)\left(\left(\frac{da}{dN}\right)^*_i - \overline{\left(\frac{da}{dN}\right)^*}\right)}{\sum_{i=1}^{k}\left(\left(\Delta K\right)^*_i - \overline{\left(\Delta K\right)^*}\right)^2} \tag{6}$$

where $\overline{\left(\frac{da}{dN}\right)^*}$ is the average values of $\left(\frac{da}{dN}\right)^*_i$, $\overline{\left(\Delta K\right)^*}$ is the average value of $\left(\Delta K\right)^*_i$ and k is the total number of readings during the test by specimen. The average values $\overline{\left(\frac{da}{dN}\right)^*}$ and $\overline{\left(\Delta K\right)^*}$ are determined as shown in Eq. (7).

$$\overline{\left(\Delta K\right)^*} = \sum_{i}^{k} \frac{\left(\Delta K\right)^*_i}{k}; \overline{\left(\frac{da}{dN}\right)^*} = \sum_{i}^{k} \frac{\left(\frac{da}{dN}\right)^*_i}{k} \tag{7}$$

where $\left(\Delta K\right)^*_i$ represents the computed during the test of the stress intensity factor ranges and $\left(\frac{da}{dN}\right)^*_i$ represents the readings during the test of the FCG rates.
Concerning the standard deviation of the normal distribution for $\log\left(\Delta K\right)$, it is defined through the Eq. (8).

$$S = \sqrt{\frac{\sum_{i=1}^{k}\left(\left(\frac{da}{dN}\right)^*_i - \widehat{\left(\frac{da}{dN}\right)^*_i}\right)}{k-2}} \tag{8}$$

Aiming the definition of a design FCG curve, rectilinear confident bands were defined as Eq. (9) shows, where α is an integer.

$$\left(\frac{da}{dN}\right)^* = C^* + m^* \cdot (\Delta K)^* \pm \alpha \cdot S = \left(C^* \pm \alpha \cdot S\right) + m^* \cdot (\Delta K)^* \tag{9}$$

In this analysis, it is assumed that $\alpha = 2$ which means that the confident band will cover approximately 95%. Design FCG curve is defined through the upper boundary of this confident band. The material parameters C and m are defined by Eqs. (10) and (11).

$$C = 10^{\left(C^* + 2S\right)} \tag{10}$$

$$m = m^* \tag{11}$$

This statistical procedure can be completed for several slopes in the FCG law using the notes proposed by Bogdanov et al. [12]. In these notes, the FCG law may have three or four slopes identified [12,23,24], hence three or four pairs of $\left\{C_i^*, m_i^*\right\}$ coefficients that are needed to fit the experimental fatigue crack propagation data. Each pair $\{C_i^*, m_i^*\}$ corresponds to a segment with linear behaviour between $\log(da/dN)$ and $\log(\Delta K)$ values. Slopes $m_i^* (= m_i)$ are obtained using Eq. (6). The materials constants of fatigue crack growth $\{C_i^*, m_i^*\}$ and standard deviations can be obtained using Eqs. (5), (6) and (8). Several authors have discussed the evaluation of the fatigue crack propagation rates using statistical assumptions [6,7,10, 12,23,24], demonstrating the importance that the subject raises in the scientific community and engineers.

Experimental data

The experimental fatigue crack growth data from the old riveted metallic bridges are collected for the statistical analysis proposed by Gallegos Mayorca et al. [2] aiming at obtaining the design curves for these materials. The experimental fatigue crack propagation data was derived accordingly ASTM E647 standard procedures [25]. This standard establishes the geometry of Compact Tension specimens – CT specimens and Middle Tension specimens – MT specimens. CT specimens were used for materials from Eiffel (W = 40 mm; B = 4.35 mm), Fão (W = 50 mm; B = 8 mm), Pinhão (W = 40 mm; B = 4.35 mm) and Trezói (W = 50 mm; B = 8 mm) bridges. Specimens from Luiz I bridge were manufactured as MT specimens (W = 40 mm; B = 10 mm). All tests were carried out under a sinusoidal waveform with a frequency of 20 Hz except for Luiz I bridge specimens that were tested at a frequency of 10 Hz. Two travelling microscopes with accuracy of 0.001 mm were used to measure the crack growth on both faces of the specimens by direct visual inspection. Regarding the number of tested specimens, five were manufactured from the Eiffel bridge (four according to the transverse direction and one according to the longitudinal direction), twelve from the Fão bridge, thirteen from the Pinhão bridge (six from a diagonal and seven from a bracing), eight from the Trezói bridge and four specimens from the Luiz I bridge [1]. The following stress ratios were investigated for each material:

- Eiffel bridge: $R_\sigma = 0.1$ and $R_\sigma = 0.5$;
- Luiz I bridge: $R_\sigma = 0.1$;
- Fão bridge: $R_\sigma = 0.1$;
- Pinhão bridge: $R_\sigma = 0.0$, $R_\sigma = 0.1$ and $R_\sigma = 0.5$;
- Trezói bridge: $R_\sigma = 0.0$, $R_\sigma = 0.25$ and $R_\sigma = 0.5$.

Experimental results from all tested specimens are presented in Fig. 2. In each case, fatigue crack growth data is correlated using the previously referred power law developed by Paris and Erdogan [15] (see Eq. (1)).

Application and discussion

The statistical analysis described in the research work proposed by Gallegos Mayorca et al. [2] was used to estimate the probabilistic field of the FCG data for all old materials from the ancient riveted metallic bridges.
The C and m parameters for the materials from the Eiffel and Fão bridges were estimated by Gallegos Mayorca et al. [2]. The FCG constants of the material from Eiffel bridge using the statistical procedure are the following: $C^* = -17.614$

($C = 1.199 \times 10^{-17}$), $m^* = 4.69$ (m=4.69) and $S = 0.3463$. For the material from the Fão bridge, the FCG constants are: $C^* = -15.126$ ($C = 1.237 \times 10^{-15}$), $m^* = 4.03$ ($m = 4.03$) and $S = 0.2378$. All FCG constants were obtained with da / dN in mm/cycle and ΔK in N.mm$^{-1.5}$.

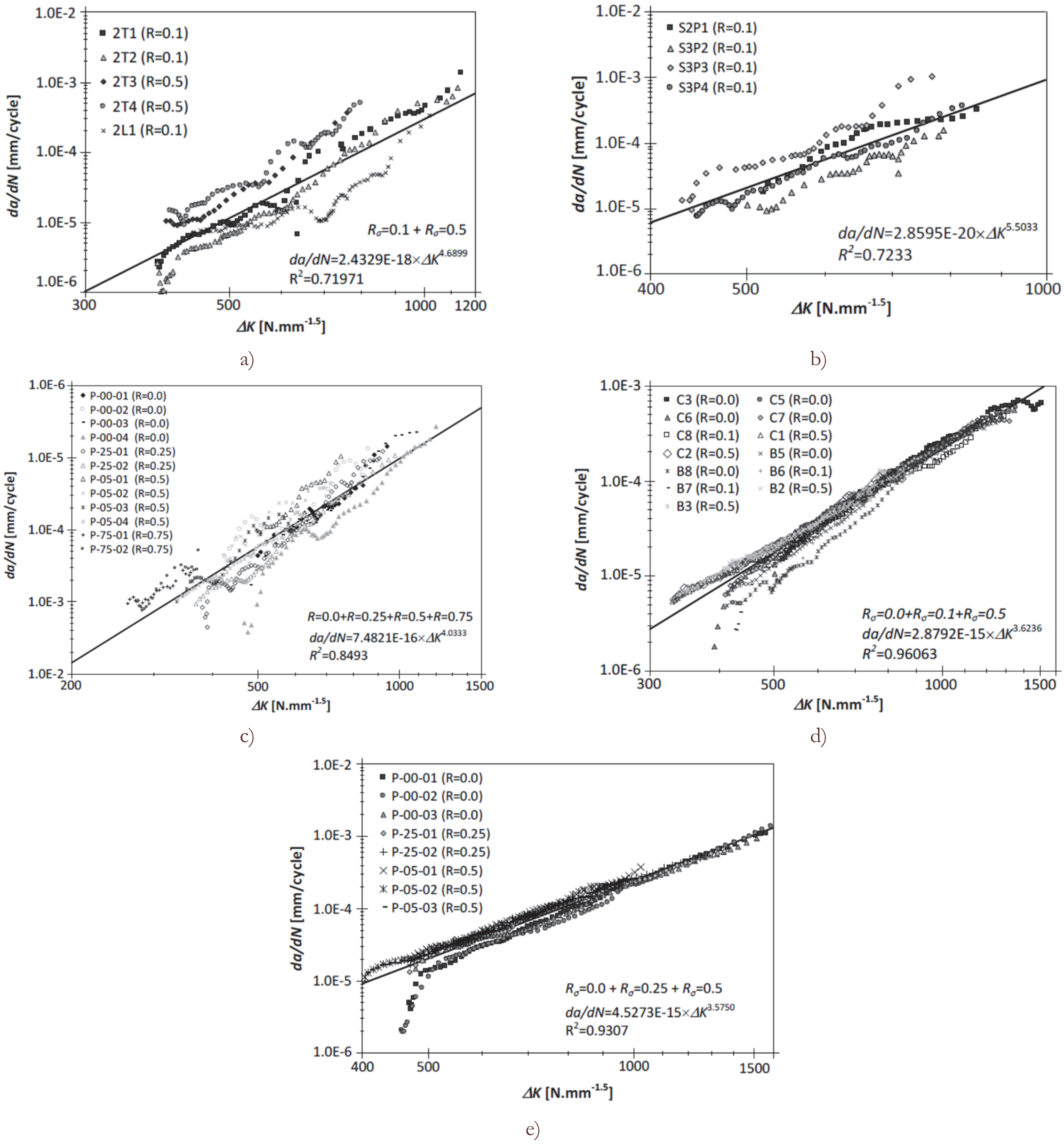

Figure 2. Crack propagation data correlated with the Paris law: (a) Eiffel; (b) Luiz I; (c) Fão; (d) Pinhão; (e) Trezói.

Figs. 3 to 8 show the experimental FCG data, mean curve and the mean curve $\pm 2S$ (5% and 95% of probability of failure). All experimental results and probabilistic fields for the fatigue crack propagation data were compared with the design FCG curve of the Stage B (mean curve $+2S$, with $m = 2.88$ and $C = 6.77 \times 10^{-13}$ where da / dN is in $mm / cycle$ and ΔK in $N.mm^{-1.5}$) proposed by the BS7910 standard [3]. It should be noted that the design FCG curve proposed in

the BS7910 standard [3] is indicated for current structural steels. This standard use the mean curve $+2S$ to describe the design curve for the fatigue crack growth rates.
In Tab. 1 the C and m parameters are presented for all materials from Portuguese metallic bridges and also the standard deviation, S, which is used to define the mean FCG curve $+2S$. This design curve is important to be used in rehabilitation studies of historical bridges (to analyse the fatigue residual life of structural details).
The design FCG curve proposed by BS7910 standard when compared with each material from the Eiffel, Luiz I and Fão bridges revealed not be representative of these materials. However, when compared to the experimental FCG data of the Pinhão and Trezói bridges, the design FCG curve proposed by BS7910 standard, seems to be more representative of these old materials. It should be noted that the materials of the Pinhão and Trezói bridges are more recent when compared to the other materials.
In the Fig. 8, the statistical analysis using the mean FCG curve $+2S$ applied to the experimental data for all materials from ancient riveted bridges proved to be reasonably satisfactory, however the use of the mean FCG curve $+3S$ to cover all experimental data is suggested.
The slopes of the FCG curves of the materials from the Eiffel, Luiz I and Fão bridges revealed to be similar, indicating the tendency exhibited for old materials. However, the slopes of the FCG curves for the materials from Pinhão and Trezói bridges exhibited a consistent behaviour with the slope of the design FCG curve proposed by BS 7910 standard [3]. The statistical analysis applied to FCG data proved to be efficient.

Material	C^*	S	C	$m^* = m$
Eiffel	−17.614	0.3463	1.199×10^{-17}	4.69
Luiz I	−19.543	0.2548	9.243×10^{-20}	5.50
Fão	−15.126	0.2378	2.237×10^{-15}	4.03
Pinhão	−14.539	0.1082	4.757×10^{-15}	3.62
Trezói	−14.278	0.1317	9.660×10^{-15}	3.54
All materials	−13.998	0.2967	3.940×10^{-14}	3.47

Table 1: Constants of the mean FCG curve for all materials from the Portuguese old metallic bridges, with da/dN in $mm/cycle$ and ΔK in $N.mm^{-1.5}$.

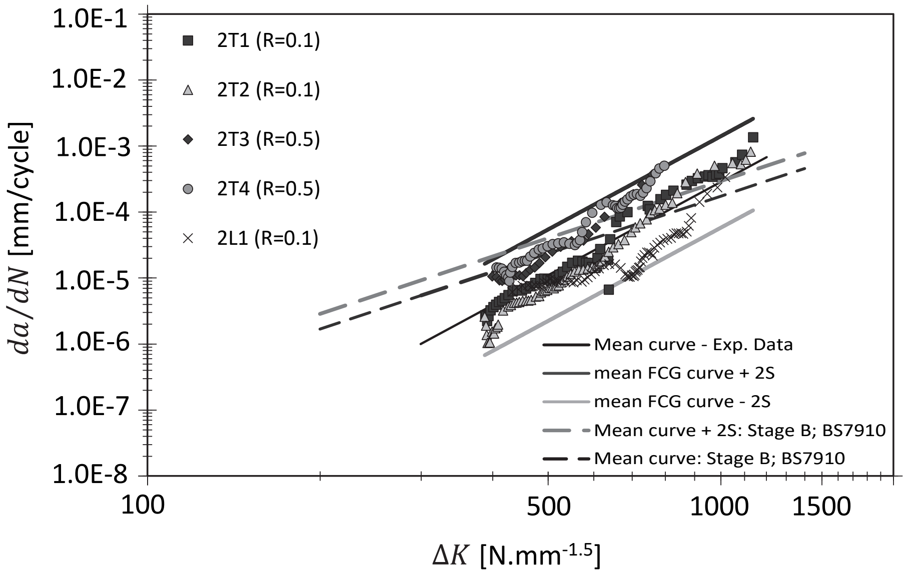

Figure 3: Statistical analysis of the FCG data for the material from the Eiffel bridge.

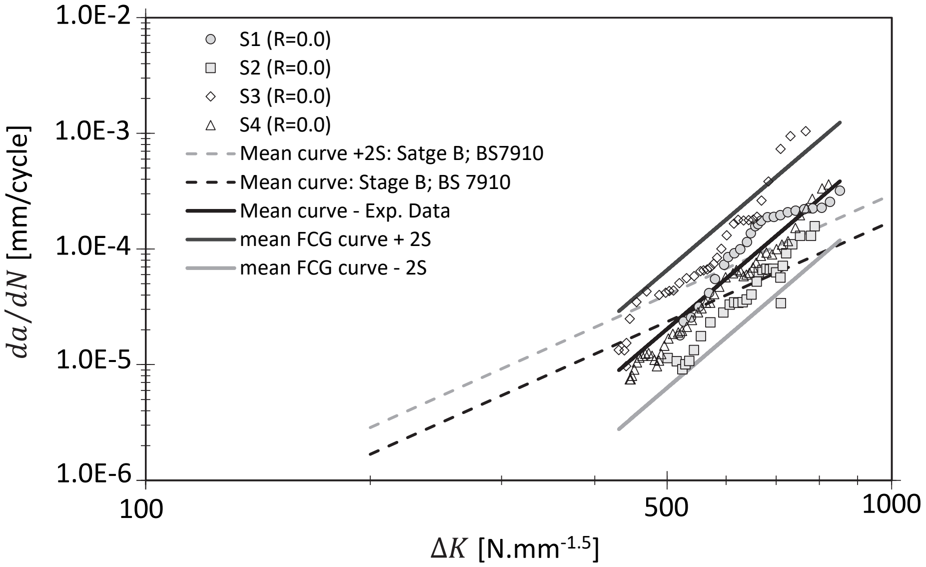

Figure 4: Statistical analysis of the FCG data for the material from the Luiz I Bridge.

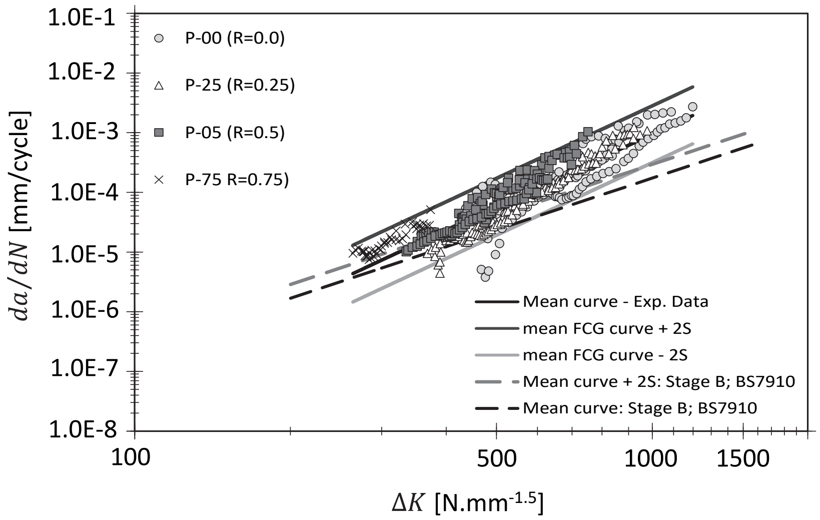

Figure 5: Statistical analysis of the FCG data for the material from the Fão Bridge.

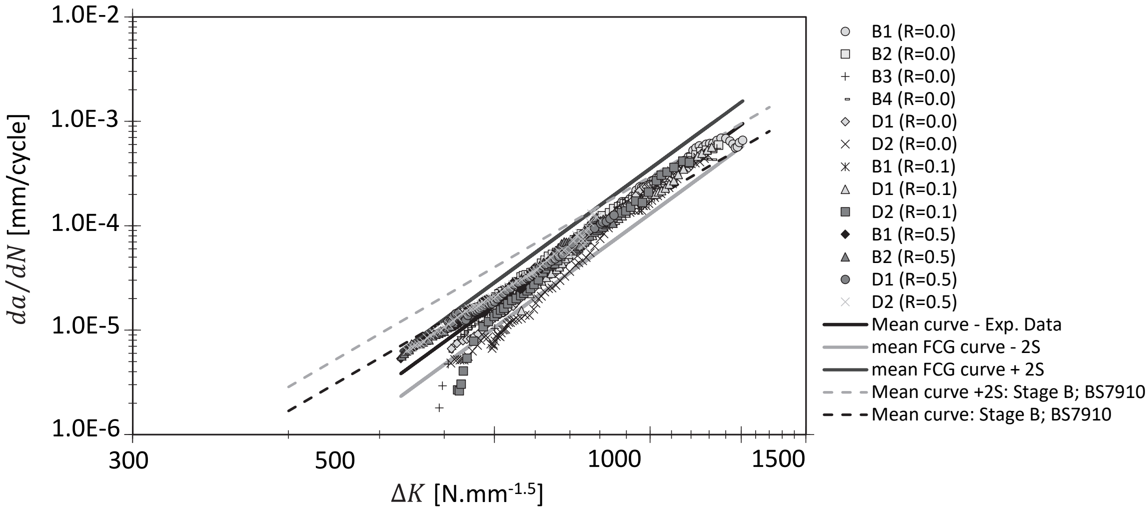

Figure 6: Statistical analysis of the FCG data for the material from the Pinhão Bridge.

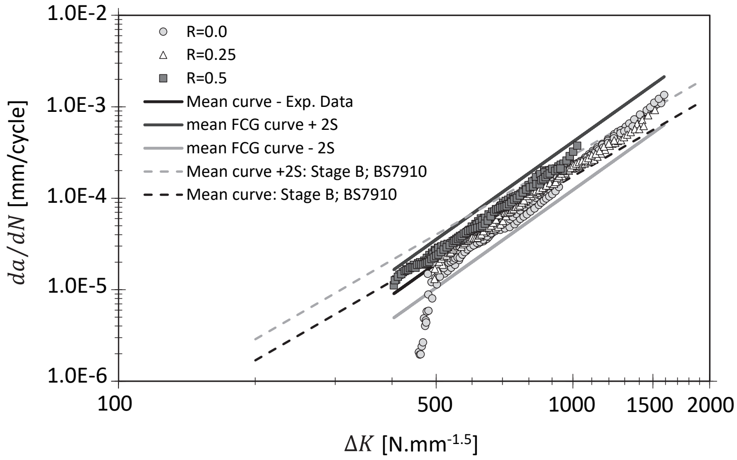

Figure 7: Statistical analysis of the FCG data for the material from the Trezói Bridge.

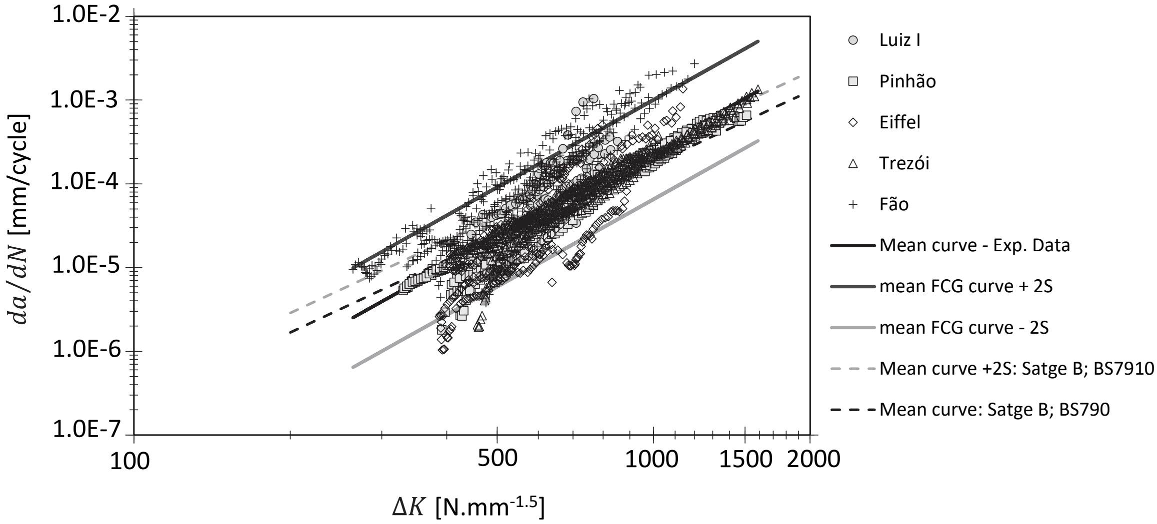

Figure 8: Statistical analysis of the FCG data for all materials from ancient riveted bridges.

CONCLUSIONS

The statistical procedure used to analyse the experimental FCG data of the materials from the Portuguese ancient metallic bridges proved to be efficient. The design FCG curves proposed by BS7910 standard are not representative of all materials from old metallic bridges. The slopes of the design curves for the fatigue crack growth data of the materials from Eiffel, Luiz I and Fão bridges revealed to be similar. However, the slopes of the design curves for the FCG data of the materials from Pinhão and Trezói bridges exhibited a consistent behaviour and similar when compared with the slope of the design FCG curve proposed by BS 7910 standard. It should be noted that the latter materials are more recent and have mechanical properties similar to current constructional steels. Further statistical analysis of the experimental FCG data from old materials is therefore necessary to better represent their behaviour. A comparison between this statistical analysis for FCG curve using several pairs $\{C_i^*, m_i^*\}$ and the probabilistic approaches for evaluating the FCG rates using local approaches should also be performed.

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