Fatigue crack growth of composite patch repaired Al-alloy plates under variable amplitude loading

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ABSTRACT. In this paper, fatigue crack growth of edge crack in 2024 T351 aluminum alloy plate repaired with bonded composite patch under constant and variable amplitude loading (VAL) was predicted. The Al-alloy plate was repaired with symmetric patch "Boron/Epoxy". Additionally to constant amplitude loading (CAL), the effects of single overload and band overload were investigated for repaired and unrepaired Al-alloy plates.

The obtained results confirm the improvement in repair performances by composite patch on fatigue lives and crack growth rates for all applied cycles (CAL and VAL) compared to unrepaired plates. A retardation effect was observed in application of single overload compared to band overload with the same stress ratio (R=0.2) for unpatched plate and characterized by instantaneous delays. However, this retardation effect is increased by the presence of the patch repair which leads to the higher fatigue life. Retardation effect was neglected for lower overload ratio (ORL>1.8). Comparison in fatigue life and crack growth rates under the same overload ratio (ORL=2.4) between repaired and unrepaired plates show the supplementary beneficial effects in combination of overloading and patch repair

KEYWORDS. Fatigue crack; Composite patch repair; Variable amplitude loading; Retardation; Al-alloy.

INTRODUCTION

As a consequence, fatigue problems become an important topic for the maintenance of damaged aircrafts structures. Bonded composite patch repair presents an advanced technology to reinforce damaged metallic of theses [1-3]. The crack patching bonding technique goes back to the 1980s, when it was successfully applied to

the fracture-critical D6AC steel wing pivot fitting of the F-lli bomber [4]. This technique offers significant advantages over traditional repair methods "riveting, fastening, and welding" [5].

In literature, several studies have investigated this technique, both numerical and experimentally, for different specimen's dimensions, and which have the following advantages: provides a high structural efficiency and extends the life of cracked structural components at an economical cost, and reduce the stress field near the crack, leads to retardation or complete arrest of the crack growth [3, 5-7], hence, as a result, crack growth is delayed and the service life of the repaired structures is extended. Fortunately, the presence of humidity presents harmful effect on fatigue behavior of adhesive joints bonded with aluminium [8]. In patched zone, the composite patch and aluminum alloy present a bi-material, also the interface assembly presents shielding in repaired material [9] and increase the fatigue and decrease the fatigue crack growth rate.

Sabelkin et al. [10] performed an experimental fatigue investigation on 2024 T3 aluminium alloy reinforced with composite patch Boron/Epoxy under constant amplitude loading. The results shown that bonded composite patch repair increase fatigue life about fivefold in the case of stiffened panels while it increased about ten fold in the case of un-stiffened panels. In absence of patch repair Negru and al. [11] have predicted the fatigue life in 2024 T3 aluminium alloy notched specimens (v-notch and semi circular notch) using three method based on finite element method.

In the fatigue investigation conducted by Salehi-Khojin et al. [12] on 6.125 mm cracked 7075 Al-alloy without patch repair and repaired with 4-ply boron/epoxy composite patches of several geometries under constant amplitude loading (R=0.1). The results show that the propagation lifetime of the four kinds of patched (repaired) specimens is improved greatly compared to the unpatched Al-alloy plate and the fatigue crack growth depends on dimension of patch along the path of crack. The study of fatigue crack growth under constant amplitude loading in repaired aluminium alloy plate 6061 T6 by carbon-epoxy composite patch has shown that the fatigue life has significantly increased. The applied patch has provided about a 100-110% improvement in the fatigue life and a 30-35% decrease in the stress intensity factor [13]. Performance for a composite repair patch to prolong the service life of pre-cracked of some aluminium alloys plates under fatigue conditions in a corrosive environment and constant amplitude loading was conducted by Schubbe et al. [14]. The Boron-Epoxy used for repairs of all Al-alloys plates, showed a positive life improvement comparatively to Graphite-Epoxy.

Several experimental research on fatigue crack growth of patched structures were oriented to the fatigue tests under constant amplitude loading including effect of stress ratio, amplitude of loading, number of ply', geometry of patches…i.e. [15-19]. It is recognized that aeronautics structures were subjected to variable amplitude loading, essentially characterized by overload and underload [20]. Research on variable applied loading (VAL), determined that appreciable crack growth retardation can occur following tensile overloading for unpatched specimens [21, 22]. In recent study, Wang et al. [23] have investigated the effect of single and block loading on fatigue crack growth rates of 2024 T4 Al-alloy unpatched CT specimen. It is concluded that single tensile overload introduced in the constant amplitude loading causes a significant retardation of crack growth and similar retardation of crack growth occurs for three-step sequence loading.

In repaired of crack specimens, few investigations were conducted to shown the simultaneous effects of patch repair and variable amplitude loading (single overload, stepped sequence loading, underload…). Albedah et al. [24] have investigated experimentally the associated effects of patch repair of cracks of plate in aluminum alloy 7075 and stepped variable amplitude loading on fatigue life. A small improvement on fatigue life is noticed in decreasing of loading amplitude blocks. Recent study on fatigue crack growth of patched 2024 T3 V-notch specimen was conducted by the same authors [25] under applied of two stepped block (increase/decrease and decrease/increase). A retardation effect was observed for decreasing blocks loading in unrepaired (unpatched) specimens. However, this retardation effect is attenuated by the presence of the patch which leads to lower fatigue life for repaired (patched) specimens.

In the case of a bonded repair, different mechanisms may be involved in prediction of fatigue crack growth under variable amplitude loading (VAL) [26, 27]. It is recognized that peak loads might cause debonds under the patch, resulting in an increased crack growth rate. It was concluded that the effects of peak loads on bonded repairs presents a similar effects of peak loads on unrepaired structures; the crack shows retardation, and no debonds after application of the peak loads were found [27]. Fatigue crack growth of fibre reinforced metal laminates (Glare) under constant amplitude loading following a single overload was studied experimentally by Wu and Guo [28]. In this investigation, the mechanisms for the effect of a single overload on the crack growth rates and the delamination growth rates were identified. Fatigue tests on bonded Glare repairs were performed at room temperature in C-5A Galaxy fuselage fatigue spectrum [29]. As can be concluded from obtained result with fatigue spectrum, small load cycles are less important for repaired specimens than for unpatched specimens.

The present work is aiming at performing a combined experimental fatigue crack growth data of 2024 T351 Al-alloy, analytical integrated models of variable amplitude loading (Generalized Willenborg model) and composite patch repair (Ratwani model) for investigation of fatigue behaviour of repaired edge cracked plate. The study of the fatigue behaviour is conducted on Al-alloy edge-cracked plate reinforced by a one-side composite patch. Applied spectrum characterized by constant amplitude loading, single or band overloading were investigated.

THEORETICAL BACKGROUND

Stress intensity factor

Calculation of stress intensity factor at the repaired crack tip is need in order to predict the fatigue life or the fatigue life enhancement of bonded patch repaired of structures or tests specimens. The most commonly materials used in bonded patch repair are boron, graphite, and Glare. Each of these materials has an application in composite patch repair depending on the metallic material thickness, type of load spectrum, and the stress level in the spectrum [30]. It recognized that repair to a cracked structure involves an upper bound on stress intensity [31]. The stress intensity factor after repair of a cracked structure is lower, by an order of magnitude than for other repair. The thickness of the patch can be chosen based on the following equivalent stiffness criterion [32]:

$$S = \frac{E_r . t_r}{E_p . t_p} \tag{1}$$

where Ep is the cracked structure's Young modulus, tp is the cracked structure's thickness, E_R is the equivalent Young modulus of the patch perpendicular to the crack and t_R is the patch thickness.
The stress intensity factor for a cracked structure with no repair can is written as:

$$K_I = \sigma \sqrt{\pi a} . f\left(\frac{a}{w}\right) \tag{2}$$

where "σ" is the remote uniform tensile stress, "a" is half crack length and f(a/W) is finite geometrical correction function.
In patched specimen, the stress intensity factor depends on presence of composite patch and width of the patch and numbers of plies. Evaluation of boundary correction function for evaluation of stress intensity factor is detailed in research of Ratwani [33], Boyd et al. [30] and Ricci et al. [34]. This analysis is based on a Green's function method. Currently, this developed formulation implemented in Afgrow code [35] is only valid for the following conditions: through the thickness cracks, thin structure (<3.17 mm), non-stiffened panels and crack remains under the patch.
The stress intensity factor in the cracked structure with repair patch and applied stress of σ is K_IP. The equivalent stress σe acting on a sheet with no repair patch and giving stress intensity factor of K_IP is given by:

$$K_{IP} = \sigma_e \left(\sqrt{\pi a}\right) . f\left(\frac{a}{w}\right) \tag{3}$$

K_IP may be expressed as: $K_{IP} = \sigma_e . K_I / \sigma$
The stress transferred to the repair patch is the difference between the remotely applied stress in the repaired structure and the uniform in-plane stress that produces the same stress intensity factor in a single sheet as in the repaired structure. The stress transferred, σt, may be written as:

$$\sigma_t = \sigma - \sigma_e \tag{4}$$

then, $\sigma_t = \sigma - K_{IP}\sigma / K_I$
In the case of one sided repairs, Ratwani [33] provided a bending correction factor "BC" and the stress intensity factor is expressed by:

$$K_{IP} = (1 + BC)K_I \tag{5}$$

The bending correction factor is given by:

$$BC = a.y_{\max} \left(1 - \frac{K_I}{K_{IP}}\right)^{\frac{t_p\left(t_p + t_r\right)}{I}} \tag{6}$$

where I, tp, tr and y_{max} are respectively the I is the total moment of inertia of the plate repair, thickness of specimen, thickness of patch and y_{max} is the distance of the outer sheet edge from neutral axis.

Fatigue crack growth model

The Forman/Mettu equation [36] also called Nasgro equation, used by several researchers [37-40] for prediction and investigation of fatigue crack growth of metals and alloys, is given by:

$$\frac{da}{dN} = C\left[\left(\frac{1-f}{1-R}\right)\Delta K_I\right]^n \frac{\left(1 - \frac{\Delta K_{th}}{\Delta K_I}\right)^{p'}}{\left(1 - \frac{K_{\mathrm{Im}\,ax}}{K_{crit}}\right)^{q'}} \tag{7}$$

where C, n, p', and q' are empirically derived from experimental fatigue crack growth rate for specified material. The others parameters in equation 7 are detailed in Afgrow technical manual [37].

Beretta and Carboni [38] have used Nasgro equation in the investigation of the effect constant amplitude loading (R ratio) and spectrum loading on fatigue crack growth of 30NiCrMoV12 steel. In order to investigate the effect of variable amplitude loading on fatigue crack growth of patched specimens is necessary to account of the interaction of levels of applied load. Generalized Willenborg model [41] presents the most common load interaction models used in crack growth life prediction. The model is based on early fracture mechanics work performed at Wright-Patterson AFB, OH. The model uses an "effective" stress intensity factor based on the size of the yield zone in front of the crack tip. The formulation of the generalized Willenborg retardation model used in fatigue code and integrated in crack growth equation (3) is given below:

$$\begin{cases} K_{\max(eff)} = K_{\mathrm{Im}\,ax} - K_r \\ K_{\min(eff)} = K_{\mathrm{Im}\,in} - K_r \\ R_{eff} = K_{\min(eff)} / K_{\max(eff)} \end{cases} \tag{8}$$

K_r is the residual stress intensity factor due to overload, it is given by the following equation and R_{eff} is the effective stress ratio.

$$K_r = \varphi\left(K_{\mathrm{Im}\,ax(ol)}\right)\sqrt{1 - \frac{\left(a - a_{ol}\right)}{R_{y(ol)}}} - K_{\mathrm{Im}\,ax} \tag{9}$$

Factor ϕ expressed by equation 10, define the level of residual stress induced by application of overload.

$$\varphi = \left(1 - \Delta K_{th} / K_{\mathrm{Im}\,ax}\right) / \left(SORL - 1\right) \tag{10}$$

and the yield zone created by overload $R_{y(ol)}$ is expressed by the following equation:

$$R_{y(ol)} = \frac{1}{2\pi} \cdot \left(\frac{K_{\mathrm{Im}\,ax(ol)}}{\sigma_e}\right)^2 \tag{11}$$

where "a" is crack length, "aol" crack length at overload, ΔK_{th} threshold value of stress intensity factor at R=0.
In patch repair stress intensity factor K_{IP}, given by equation 5, is integrated in equation 8.

SPECIMEN, MATERIAL & PATCH REPAIR

The geometry and dimensions of the single edge through crack specimen aluminium specimens investigated in cyclic loading are presented in Fig. 1. The nominal thickness of the aluminium alloy plate, t_p, is 3 mm. Specimen have length L_p=320 mm and width W_p=160 mm. At the edge of each specimen there is a through thickness initial crack, having length "a=3 mm". A composite patch was laminated on one of the sides of the specimen, having thickness t_r=2 mm, effective length L_r=80 mm and width W_r=80 mm. The material, used in this study, was 2024-T351 sheet aluminium alloy. The mechanical properties of are presented in Table 1. The composite patch used for repair is boron/epoxy. The mechanical properties of this composite patch (Boron/Epoxy) are presented in Table 2 and adhesive film used for patching is FM-73 where shear's modulus is G_{xy}= 413.68 MPa and thickness t=0.15 mm. The composite patch was composed by four symmetric ply oriented to the direction ($\pm$45.02). The maximal applied remote stress on edge-cracked plate with adhesively bonded composite patch for constant amplitude loading is σ_{aMax}=100 MPa. The spectrum forms of applied cyclic stress for variable amplitude loading are given in Fig. 2 for single overload and band overload. The main parameters of fatigue crack growth Nasgro equation for investigated material at R=0 are given in Table 3.
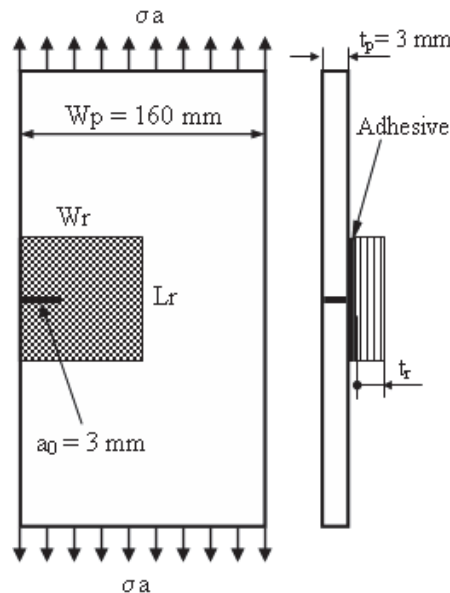

Figure 1: Edge-cracked plate with adhesively bonded composite patch

$\sigma_{0.2}$	E (GPa)	UTS	K_{IC} (MPa.m$^{1/2}$)	K_C (MPa.m$^{1/2}$)	ν
372.31	73.08	469	37.36	74.72	0.33

Table 1: Mechanical properties of 2024 T351 Al-alloy.

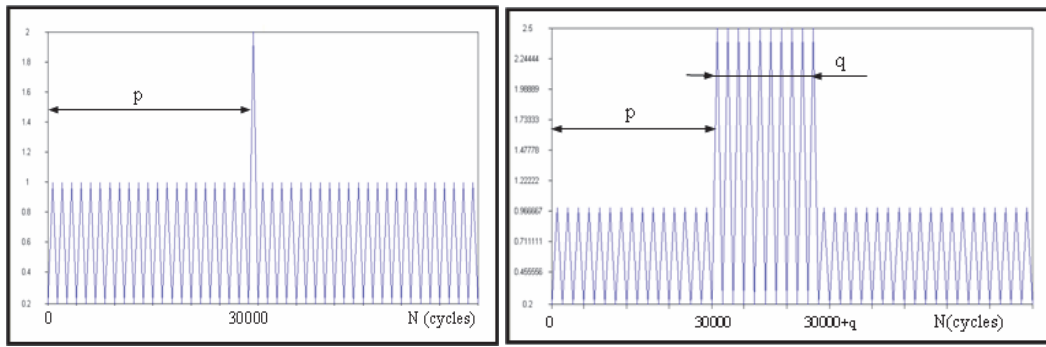| E_L(GPa)	E_T (GPa)	G_{LT}	ν
206.84	193.05	5171	0.33

Table 2: Mechanical properties of Boron/Epoxy.

Figure 2: Applied spectrum form: (a) constant amplitude loading with single overload (b) with band overload.

C	n	p'	q'	ΔK_{th} at R=0
1.71×10^{-10}	3.353	0.5	1	2.857

Table 3: Parameters of fatigue crack growth according to Nasgro equation.

RESULTS & DISCUSSION

This section provides and discusses the obtained results including the effects of variable amplitude loading characterized by single overload ratio and band overload on fatigue life and fatigue crack growth rates respectively in cracked unpatched plate and cracked patched composite plate. Also, we investigated the effect of patch repair under constant and variable amplitude loading and presented the result in the following subsections.

Combined effect of overload ratio and patch repair on the crack growth

Fig. 3 presents the fatigue life response (crack length vs. cycle number) for unpatched 2024 T351 Al-alloy specimen, pre-cracked to a length of 3 mm, under spectrum block (Fig. 2a) characterized by the presence of single overload and stress ratio R=0.2. From the presented, we notice a no effect for OLR = 1.5 [42]. But from ORL=1.8 to 2.0, we notice a little effect on fatigue life. At high overload ratio (ORL=2.4), we show significant effect on fatigue life compared to the others overloads ratios and constant amplitude loading. The retardation cycles N_d between applied spectrums with constant amplitude loading and with single overload ratio is about 1.4×10^4 cycles.
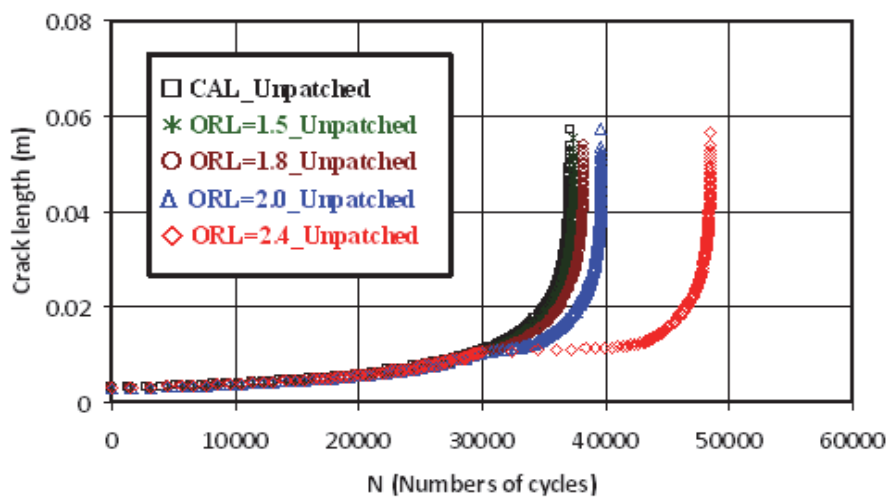

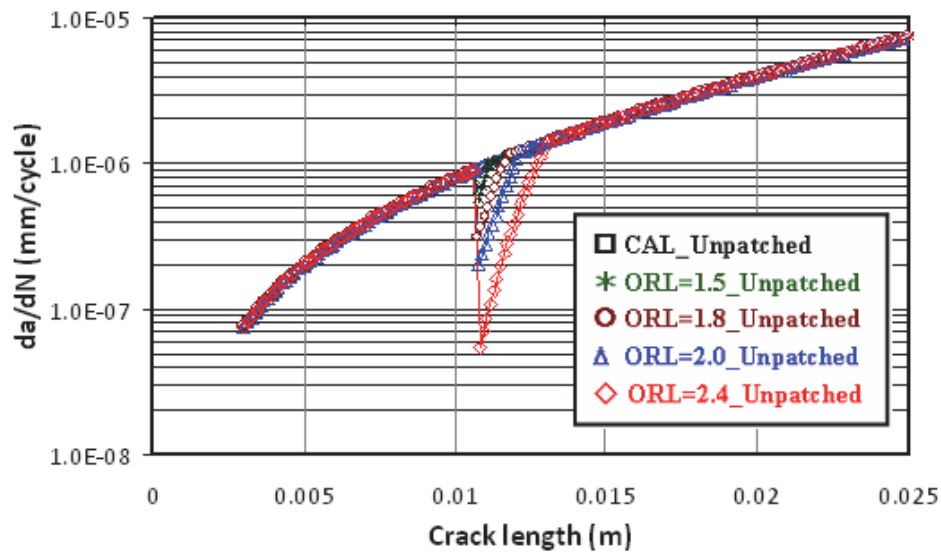Figure 3: Overload ratio effect on fatigue life for unpatched 2024 T351 Al-alloy

Figure 4: Overload ratio effect on fatigue crack growth rate for unpatched 2024 T351 Al-alloy

The results shown in Fig. 5 indicate that the fatigue life of repaired specimens was improved for all overload ratios. It noticed clearly that the fatigue lives for all overloads ratios were affected highly by composite patch repair (Fig. 5) compared to the unpatched plate (Fig. 3). Also, at low overload ratio (ORL=1.5) the same effect of overload ratio comparatively to the unpatched plate. At high overload ratio (ORL=2.4) with the presence of patch, the fatigue life is increased from 1.18×10^5 cycles for constant amplitude loading to 2.27×10^5 cycles for ORL=2.4 witch represents an increasing ratio in fatigue life of 2 times. Fig. 6 presents the evolution of the crack growth rate (da/dN) as a function of the crack length "a" for repaired samples under block loading with single overload for the stress ratio R=0.2.

At low crack length (a < 7 mm), we show instantaneous delay for ORL=2.0 and 2.4. The decrease in fatigue crack growth rate (da/dN) occurs at a length of 5 mm and varies from 8.95×10^{-8} m/cycle in CAL to 4.8×10^{-8} m/cycle and 1.90×10^{-8} m/cycle respectively for overload ratios "2" and "2.4". At this stage, patch gives significant rigidity.

For ORL=2.0 and at crack length "a=7.7 mm", deferred delay was shown. But for ORL=2.4, the presence of retardation is characterized by deferred delay in various crack length (a= 8.66; 11.10; 14.1; 18.0 and 28.37). At crack length "a=28.37", the maximum crack growth rate does not exceed the value of 1.12×10^{-4} m/cycle at application of overload ratio and decrease to 6.7×10^{-8} m/cycle. For ORL=1.8, instantaneous delay is predominant until crack length "a=34 mm" with low effect crack growth rate. For all overload ratios, fatigue crack growth rate keep to constant amplitude fatigue crack growth rate from crack length "a=37 mm" and patch repair effect of disappeared.

In detail, Figs. 7 and 8 present respectively comparison in fatigue life and crack growth rates between repaired and unrepaired 2024 T351 Al-alloy plates for overload ratio ORL=2.4. In unrepaired plate under application of the spectrum with single overload, the number of delay cycles "N_D" due to overload is about 1.25×10^4 cycles and retarded crack length, "a_d", is 2.97 mm. But in repaired Al-alloy plate by Boron/Epoxy, the total number of delayed cycles, "N_D", is about 19.3×10^4 cycles. This result represents a ratio of improvement in fatigue life about 15.5 times in presence of patch repair comparatively to unrepaired plate. Also, the retarded crack length, "a_d", is about 26.5 mm influenced conjointly by applied spectrum with single overload and patch repair.

Additionally to the combined effect of overload ratio and patch repair given in Fig. 6, Fig. 8 gives comparisons between fatigue crack growth rates in repaired and unrepaired alloy plates under spectrum with single overload (ORL=2.4). In unrepaired plate, is noticed a presence of single instantaneous delay at crack length "a=10.9 mm". Fatigue crack growth rate is increased from 9.02×10^{-7} to 5.52×10^{-8} m/cycle. At same crack length "10.9 mm" in repaired plate, the curves in Fig. 8 demonstrate that the crack growth rate decrease with differed delay case and leads to 2.26×10^{-8} m/cycle. The ratio in delayed crack growth rate is 2.44 times. In general, a significant reduction in crack growth rate after 5 mm of crack length is shown. Crack growth rates ratio between unrepaired and repaired plates in constant amplitude phases varies from 2.5 to 16.6 times. The high percentage of the beneficial effect in fatigue crack growth rate is related to the effect of patch repair characterized by the reduction in stress intensity factor [43] compared to the effect of variable amplitude loading with single overload.
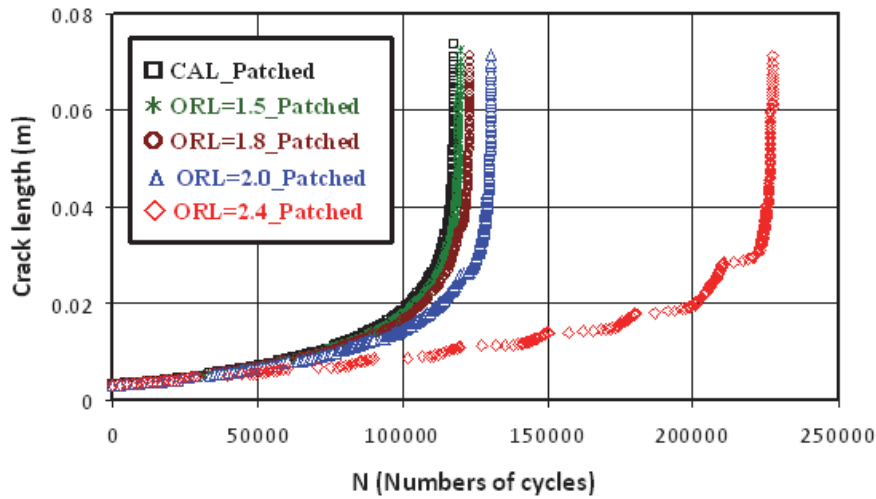
Figure 5: Overload ratio effect on fatigue life for repaired 2024 T351 Al-alloy plate.
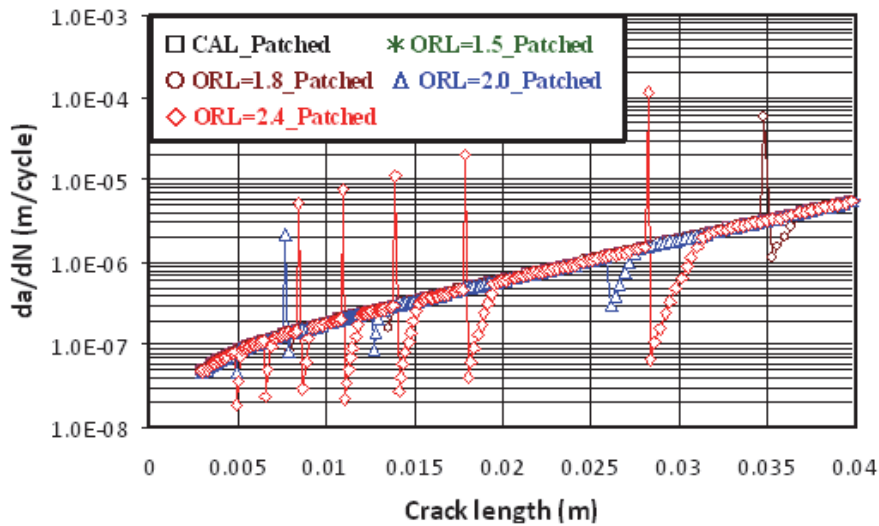

Figure 6: Overload ratio effect on crack growth rates for repaired 2024 T351 Al-alloy plate.

The relationships between retard ratio " N_D/N_{CAL} " and overload ratio "ORL" for repaired and unrepaired plate are plotted on a log-log graph in Fig. 9. It is seen from the plot that all the points seem to lie on a straight line on the log-log scale for unrepaired plate given by power function [42]. On the others hand, the plot in repaired plate case is given by logarithmic function. The developed equations for the both cases (repaired and unrepaired) are given below:

$$
\begin{cases}
\dfrac{N_D}{N_{CAL}} = 16.9 \times 10^{-4} \left(ORL\right)^{3.483} & \textit{Unrepaired} \\[4mm]
\dfrac{N_D}{N_{CAL}} = 3.305 \times Ln\left(ORL\right) - 1.31 & \textit{Re paired}
\end{cases}
\tag{12}
$$

Influence of band overloading parameter on patch repair

A few investigators have studied the effect of multi overload and band overloading cycles on crack growth retardation. However no attempt has been made to predict fatigue life and fatigue crack growth in multi band overloading case. In the present attempt interaction of multi band overload is investigated. The spectrum of band overloading given by figure 2b is applied and the overload band is characterized by the parameter "q".
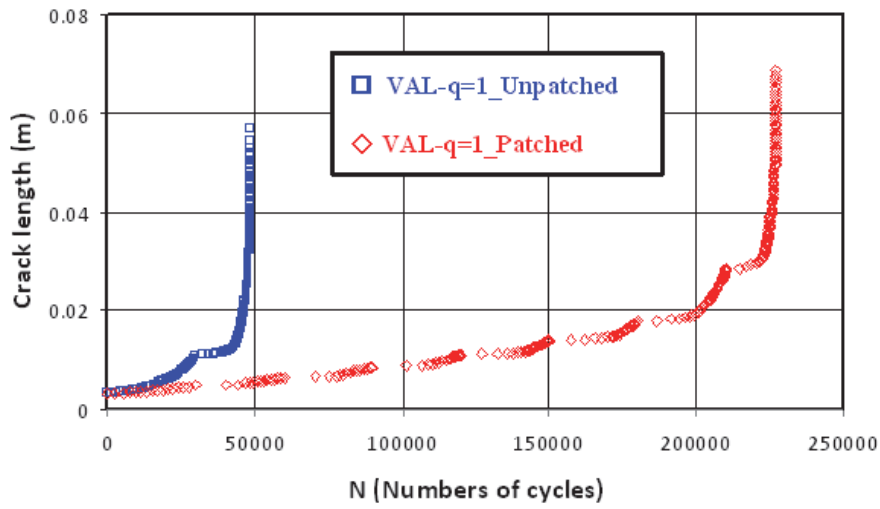
Figure 7: Influence of composite patch repair on fatigue life under variable amplitude loading (ORL=2.4)
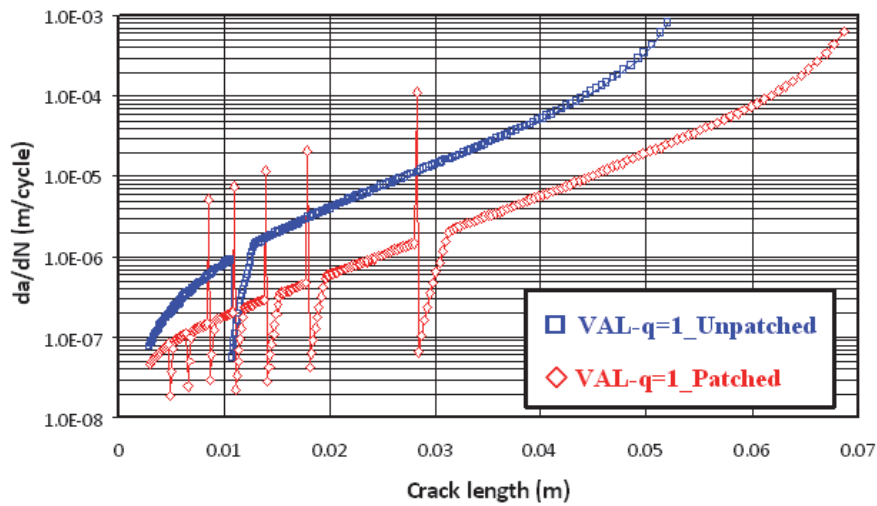

Figure 8: Influence of composite patch repair on fatigue crack growth rate under variable amplitude loading (ORL=2.4)
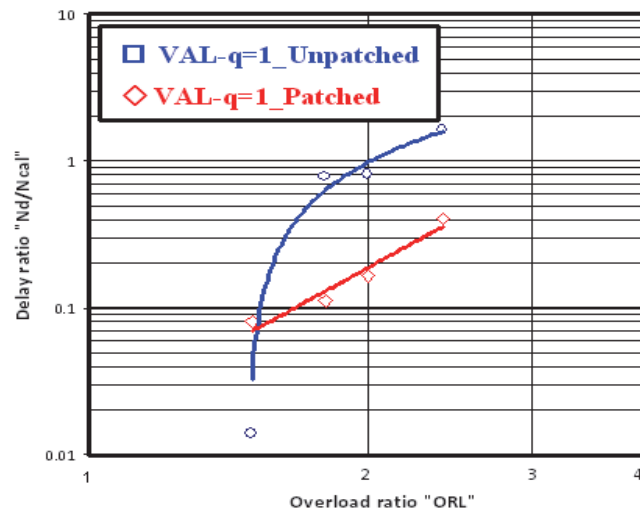

Figure 9: Retard ratio " N_D/N_{CAL} " vs overload ratio

The results of predicted fatigue life and crack growth rates of repaired plate are presented respectively in Figs. 10 and 11 at ORL=2.4. It has shown clearly that applied spectrum causes noticeable crack growth delay for "q" equal or less then 100 cycles after constant amplitude compared to constant amplitude loading.
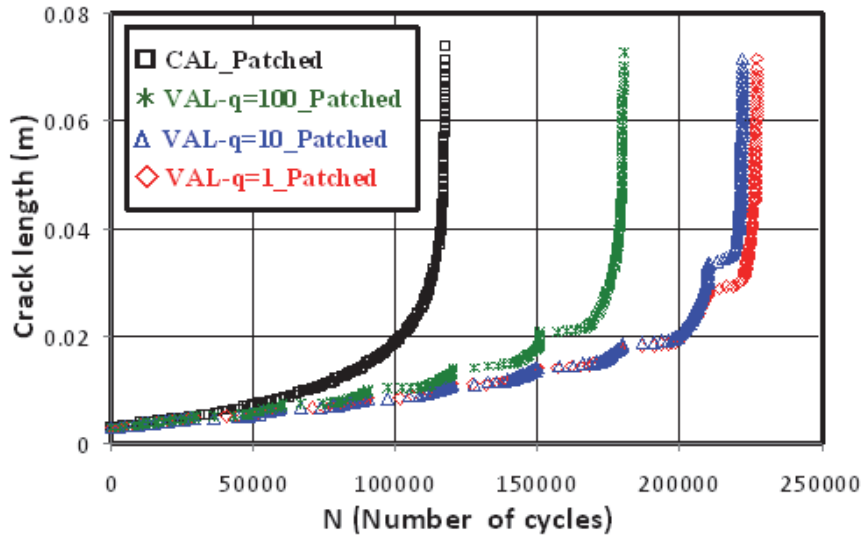

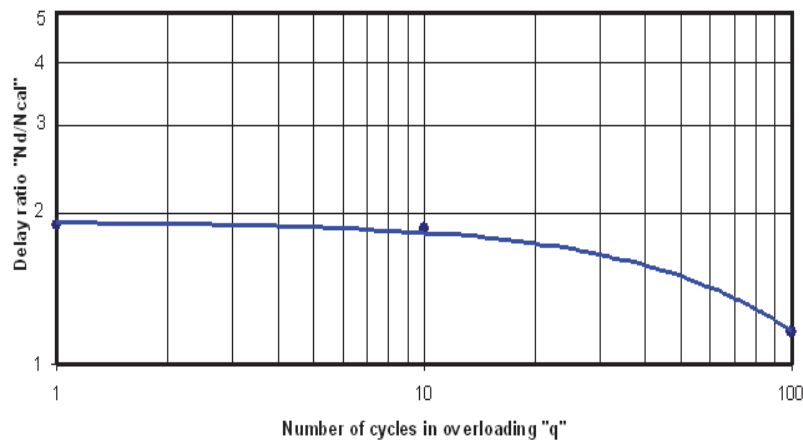Figure 10: Influence of band overloading parameter "q" on fatigue life for patched crack repair at ORL=2.4.

Figure 11. Influence of band overloading parameter "q" on delay ratio N_D/N_{CAL} in repaired plate.

Delay in fatigue life decrease with increasing band overloading parameter "q" from 1 to 100. No effect of band overloading at "q=10" is noticed comparatively to the single overload on fatigue life, expected in delayed crack length "a_D". At "q=1 cycle", the total delayed crack length "a_D" is about of 26.82 mm; but for "q=10 cycles", total delayed crack length is about of 31.51 mm witch presents approximately 4.69 mm in difference.

The delay cycles after application of specified spectrum with the presence of patch repair for band overloading cycles "q=100 cycles" is about 1.37×10^5 cycles compared to constant amplitude loading. These delays cycles increases for single overload "q=1" and band overload "q=10" and are respectively, 2.22×10^5 and 2.19×10^5 cycles. Also, these delays is attributed to the crack closure effect [44-46] under application of multiple variable spectrums with band overload from "q=1" to "q=100" cycles. The relationship between the retard ratio N_D/N_{CLA} and band overload "q" is plotted in Fig. 12. The developed equation has exponential form and is given below:

$$\frac{N_D}{N_{CAL}} = 1.925 \times Exp\left(0.005 \times q\right) \tag{13}$$

In term of crack growth rate, fatigue behavior under application of band overloading is characterized by differed delay for "q" greater than one cycle (single overloading) (Fig. 11). Crack growth rates are stabilisation at high value after application of band overloading in high stress intensity factor i.e. at high of crack length values with loss of patch rigidity.
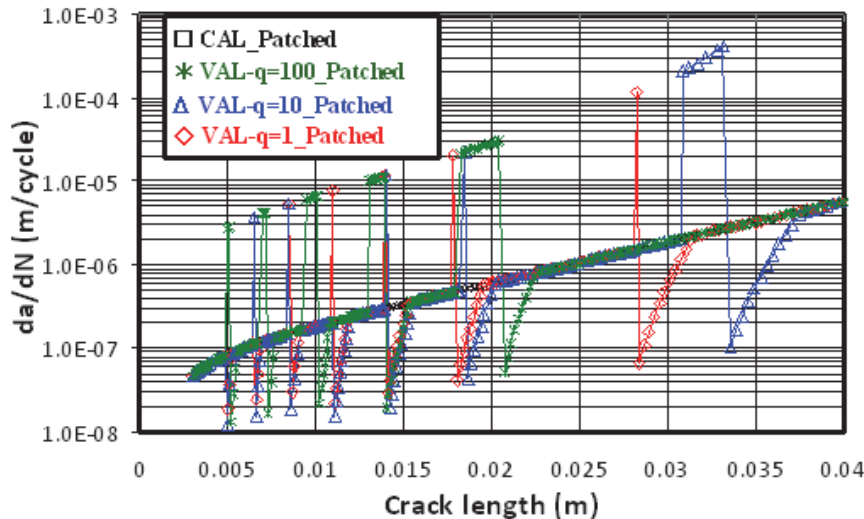

Figure 12: Influence of band overloading parameter "q" on crack growth rates of crack patch repair at ORL=2.4.

CONCLUSION

Fatigue crack propagation predictions were carried out on patched and unpatched 2024 T351 Al-alloy plates. Performance of composite patch repair was evaluated under constant amplitude and variable amplitude loading conditions. Combination of retardation model (modified Willenborg model) and patch repair accounting of Ratwani model have given good prediction of composite patch repair performances. From the results of the present investigation, we can conclude that:

- After application of overload, instantaneous delay is noticed in unrepaired specimens witch theirs levels depends upon the amount of overload.
- Also, in composite patch repair, differed delays have affected the total fatigue life and crack growth rates
- High efficiency is given in fatigue life for repaired specimens.
- As the overload ratio (ORL) increases, the fatigue life increases for both repaired and unrepaired specimens
- The fatigue life is found to be larger if the applied overload was important (ORL=2.4) compared to constant amplitude loading in unrepaired and repaired specimens.
- The fatigue life increases in each periodic band of overload "q" as compared to constant amplitude loading. Also the fatigue life decreases in increasing of band of overload "q".
- Finally, experimental studies are needed in order to understanding of the key mechanisms that affect the efficiency of the patch repair under variable amplitude loading effects.

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NOMENCLATURE

a	Crack length
a_{ol}	Crack length at applied overload
a_0	Initial crack length
BC	Bending correction
C, n	Material constants of fatigue crack growth
da/dN	Fatigue crack growth rate
Ep	Cracked structure's Young modulus
E_L	Young modulus of composite patch in longitudinal direction
E_R	Equivalent Young modulus of the patch perpendicular to the crack
E_T	Young modulus of composite patch in transversal direction
f	Crack closure factor
f(a/W)	Finite geometrical correction function
Gxy	Shear's modulus of adhesive film
G_{LT}	Shear's modulus of composite patch
K_I	Stress intensity factor for cracked structures
K_{IP}	Stress intensity factor for cracked structures with patch
K_{IC}	Critical stress intensity factor for plane strain
K_C	Critical stress intensity factor for plane stress
ΔK_I	Amplitude of stress intensity factor
ΔK_{th}	threshold of stress intensity factor
K_{crit}	Critical stress intensity factor
K_{Imax}	Maximal stress intensity factor
K_{maxeff}, K_{mineff}	effective stress intensity factor
Kr	Residual stress intensity factor
Lr	Length of composite patch
R	Stress ratio
SORL	Overload zone factor
ORL	Overload ratio
CAL	Constant amplitude loading
VAL	Variable amplitude loading
R_{eff}	Effective stress ratio
$R_{y(ol)}$	Yield zone created by overload
N_D/N_{CAL}	Retard ratio
Wp	Width of plate
Wr	Width of composite patch
I	Total moment of inertia
tr	thickness of patch
tp	Cracked structure's thickness,
y_{max}	Distance of the outer sheet edge from neutral axis
p' , q'	Constant empirically derived from experimental fatigue crack growth rate
p, q	Length of applied spectrum at constant and variable amplitude
σ_{aMax}	Maximum applied stress
σ	Remote uniform tensile stress
σ_e	Equivalent stress
σ_t	Transferred stress
$\sigma_{0.2}$	Limit of elasticity at 0.2%
ν	Poisson's coefficient
UTS	Ultimate tensile stress