



## Continuum damage modeling through theoretical and experimental pressure limit formulas

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**ABSTRACT.** In this paper, we developed a mathematical modeling to represent the damage of thermoplastic pipes. On the one hand, we adapted the theories of the rupture pressure to fit the High Density Polyethylene (HDPE) case. Indeed, the theories for calculating the rupture pressure are multiple, designed originally for steels and alloys. For polymer materials, we have found that these theories can be adapted using a coefficient related to the nature of the studied material. The HDPE is characterized by two important values of pressure, deduced from the ductile form of the internal pressure's evolution until burst. For this reason, we have designed an alpha coefficient taking into account these two pressures and giving a good approximation of the evolution of the experimental burst pressures through the theoretically corrected ones, using Faupel's pressure formula. Then, we can deduce the evolution of the theoretical damage using the calculated pressures. On the other hand, two other mathematical models were undertaken. The first one has given rise to an adaptive model referring to an expression of the pressure as a function of the life fraction, the characteristic pressures and the critical life fraction. The second model represents a continuum damage model incorporating the pressure equations as a function of the life fraction and based on the burst pressure's static damage model. These models represent important tools for industrials to assess the failure of thermoplastic pipes and proceed quick checks.

**KEYWORDS.** Continuum Damage; HDPE; Burst pressure; Life fraction.



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### INTRODUCTION

Polymers have changed all facets of the industrial life and are almost a part of all fields, including the medical and food industries. Universities, organizations and industrial companies are getting deeply concerned about these materials. They are contributing to the advance and the prosperity of several industries such as petrochemicals, gas, water networks and slurries' transport. This multitude of applications explains the huge number of product ranges, which have been produced. Many researches were interested to the extreme environmental conditions, the manufacturing methods

and the technological advances of polymer's applications [1–5]. In this context, several discoveries have been recorded in the history of plastic materials starting with PVC in 1913, Plexiglas in 1924, polystyrene in 1933, polyethylene in 1935, Teflon in 1938, ABS in 1946 and polypropylene in 1954. The polymers are generally classified into three main categories, which are thermosetting, thermoplastic and elastomer. For our case, the High Density Polyethylene (HDPE) materials have gained a huge importance in the industrial field because of their durability and high performances.

To contribute to these advances, we led many simplifying approaches of failure assessment and prediction of HDPE pipes [14-16]. All the developed concepts have been based on experimental tensile and burst tests. Indeed, our aim is to assess the degradations through the damage modelling by referring only to static tests and models instead of tedious and very costly dynamic ones. Therefore, we are using the results of the burst tests of HDPE pipes for a mathematical modeling of burst pressure evolution and damage evaluation. For that reason, we evaluated the characteristic pressures of these pipes, obtained from the internal pressure curve, which have been integrated in theoretical equations leading to the damage assessment. In this paper, new concepts based on the limit pressure formulas such as Faupel one [6] and continuum equation of pressure have been introduced. These formulas led to three ways of damage modeling of HDPE pipes:

- A theoretical model based on the Faupel formula applying a corrective coefficient  $\alpha$ ;
- An adaptive model using the pressure formulation,  $P(\beta)$ , as a function of the life fraction and the critical life fraction and assuming that the applied pressure corresponds to the calculated pressure  $P(\beta)$  corresponding to the last experimental life fraction (86%);
- A continuum damage model  $D(\beta)$ , which takes values as a function of the life fraction  $\beta$  and the constant  $\alpha$  and  $\eta$  and assumes an applied pressure equivalent to the experimental one (11.9 bar).

This modeling is considering the theoretical calculations of the HDPE pipe's burst pressure, for different notches' depths, by either the Faupel or  $P(\beta)$  formulas. Therefore, we consider the burst pressure of a neat pipe as the ultimate rupture pressure, maximum pressure, while the other pressures, related to the notched pipes, are considered as the residual ones for both the theoretical and the adaptive damage models. For the continuum damage, it is taking into consideration the different pressures of the HDPE pipes as a function of the intervals of  $\beta$ . These approaches are validated and compared to the model of static damage obtained through the experimental burst pressures in order to evaluate their accuracy.

## MATERIAL AND METHOD

To determine the experimental burst pressures of the studied pipes, we used a hydrostatic tester allowing the control of the internal pressure until burst, Fig. 1. This test allowed us to determine the HDPE pipes' resistance through their burst pressure. Thus, undamaged HDPE pipes and notched pipes, with a groove of 100 mm length, 5 mm width, and a variable depth from 1 to 5 mm, have been exposed to an increasing pressure until burst for the ultimate and residuals burst pressures determination. The specimens have been chosen according the ASTM code D1599 that requires a specimen with a length that should not exceed five times the diameter of the pipe. The specimens are prepared and conditioned at the room temperature (23 °C) prior to pressurization.

The burst pressures were evaluated, according to the notches' depths experimentally and theoretically, as explained by the methodology shown in the Fig. 2. Then, the experimental burst pressures have been obtained and recorded, Tab. 1. It is having a proportional drop according to the increase of the notch depth.

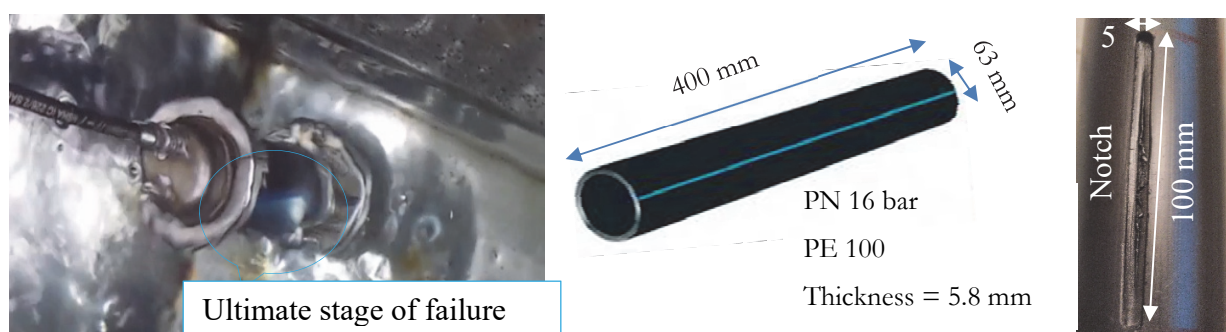


Figure 1: Bursting of HDPE pipes and the specimen and notch dimensions.



This paper highlight a new and simplified approach for the industrials to get through the fastidious checks required by the different codes. On the one hand, we propose two ways to get fast to this purpose. The first approach is an accelerated damage creation through notches in HDPE pipes. From the latter, ultimate and residual pressures have been measured and used in a static damage model obtained through the Eq. (3). The establishment of the corresponding graph give rise to the prediction of the damage behavior of HDPE pipes.

On the other hand, the same thing, which has been done through experimental burst tests, can be done only by theoretical calculations through rupture pressure formulas or the newly developed formulas presented in this paper. From then, we get to the theoretical static damage evaluation and to the plot of the damage curves, which tells about the material behavior.

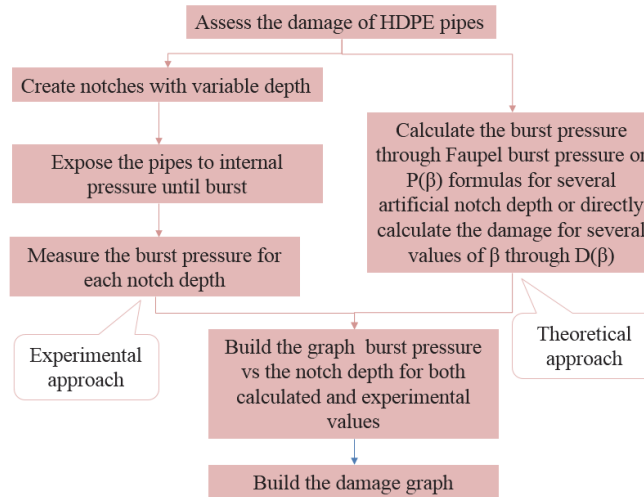


Figure 2: Damage building methodology.

Notch depth (mm)	Life fraction ( $\beta$ )	Burst pressure (bar)
0	0	63.2
1	0.17	57.8
1.5	0.26	49.7
2	0.34	47.5
2.5	0.43	42.3
3	0.52	37.5
3.5	0.60	23.5
4	0.69	20.4
4.5	0.78	18.2
5	0.86	11.9

Table 1: Burst pressure evolution in function of the life fraction.

## THEORY

### *Rupture pressure theories*

Several theories have been developed to predict the rupture of a cylinder under pressure by determining the limit loads. Hill, in 1950, developed a theory predicting the pressure at boundary load conditions. It gives the evolution of the internal pressure at break as a function of the value of the stress  $\sigma_y$ , the internal diameter  $D_0$  and the external diameter  $D_i$  [7]. For the same purpose, Nadai proposed calculating the burst pressure using the ultimate stress instead of the yield



stress [8]. In 1953, Faupel developed a formula that takes into account the ultimate stresses and the internal and external diameters of a pipe, Eq. (1) [6]. Lately in 2006, Klever developed a relationship based on thickness, mean diameter and ultimate stress [9]. More recently, Brabin has presented in his works a new formula using the same data as Faupel in addition to a parameter  $\lambda$ , which depends on the characteristics of the studied material [10]. Finally, in 2010, the DNV standards simplified the formulas proposed in the literature by introducing a new formula based only on the yield stress, the thickness and the mean diameter [11].

$$P = \frac{2}{\sqrt{3}} \sigma_y \left( 2 - \frac{\sigma_y}{\sigma_{UTS}} \right) \ln \left( \frac{D_0}{D_i} \right) \quad (1)$$

- $D_0$  and  $D_i$  are the external and internal diameters of the cylinder ;
- $\sigma_y$  is the yield stress ;
- $\sigma_{UTS}$  is the ultimate stress.

### Static damage

Kachanov formulated the approach of continuum damage mechanics in 1958 [12]. From a physical point of view, the author considered the initiation of damage as an internal phenomenon of progressive deterioration of the material reflected by the presence of cavities and micro-cracks under the effect of repetitive loadings generating the reduction of the area of the cross section [13]. Chaboche who proposed a law of fatigue damage with nonlinear evolution took up this approach to quantify the level of damage. The damage value  $D$  takes imposed values varying from zero, for the undamaged material, to a value equal to one corresponding to the appearance of a detectable crack or rupture.

These damage theories, developed in the literature, are elaborated based either on the concepts of Palmgren, Langer and Kommers or on Kachanov's continuum damage approach. The most common and used model, adopted by the international codes like ASME and ISO, is the linear damage model which is directly proportional to the life fraction. Miner has presented the damage as equal to the life fraction.

This law is expressed as below:

$$D = \beta = n / N_f \quad (2)$$

The life fraction presented by Miner is referring to the ratio between the instantaneous number of cycles under an applied stress ( $n$ ) and the total number of cycles at the rupture ( $N_f$ ) in the fatigue case. In this paper, as per the fatigue phenomenon, the notch level is considered as the preloading for fatigue because it is consuming a number of cycle of the material that corresponds to the level of weakening of it. Therefore, the ratio between the thickness fluctuation, which is measuring the notch impact and the thickness, which represents the full strength of the HDPE pipe, can be considered as the life fraction for the material.

In this paper, we evaluated the damage through a combined theory using the static damage of the unified theory [14–16] and burst pressure equations herein developed. Indeed, we exposed neat and notched HDPE pipes to an increasing internal pressure until rupture (Burst). Each notch level is corresponding to a life fraction calculated through the thickness fluctuation. This fluctuation is influencing the burst pressure of the pipes. Therefore, each ( $\beta$ ) is corresponding to a burst pressure. For the neat pipe, this pressure is considered as the ultimate one ( $P_u$ ), while they are considered as the ultimate residual pressures ( $P_{ur1}, P_{ur2}, \dots, P_a$ ) for the others.

$$D_s = \frac{1 - \frac{P_{ur}}{P_u}}{1 - \frac{P_a}{P_u}} \quad (3)$$

where,

- $P_{ur}$  is the burst pressure for damaged pipes ;
- $P_u$  is the burst pressure for a neat pipe ;
- $P_a$  is the pressure just before rupture

## RESULTS AND DISCUSSIONS

### *Theoretical damage based on corrected Faupel formula*

The theoretical burst pressures obtained by the formula (1), and those obtained experimentally, Tab. 1, are shown in Fig. 3. They have a decreasing trend according to the life fraction ( $\beta$ ).

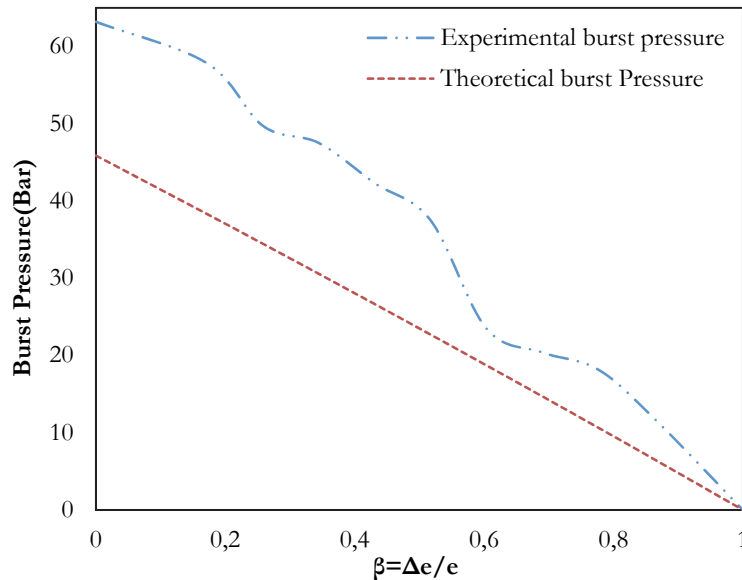


Figure 3: Theoretical and experimental burst pressures as a function of the life fraction.

The observed difference at the small notch depths can be explained by the ductile behavior of the studied material, which is not taken into account by the theoretical burst pressure formulas. In order to correct this discrepancy, the formula (1) is corrected by a new coefficient  $\alpha$  that represents the ratio between the maximum pressure  $P_{\max}$  and the pressure at break  $P_r$ . Then, the approximate corrected equation of Faupel is given by:

$$P = \frac{2}{\sqrt{3}} \sigma_y \left( 2 - \frac{\sigma_y}{\sigma_{UTS}} \right) \ln \left( \frac{D_0}{D_i} \right) \cdot \alpha \quad (4)$$

where:

$$\alpha = \frac{P_{\max}}{P_r} \quad (5)$$

$\alpha$  is a parameter that depends on the studied material.

$P_{\max}$  and  $P_r$  represent the maximum pressure and rupture pressure of an HDPE pipe as shown in Fig. 4.

Afterwards, the corrected pressures obtained by the Eq. (4) and (5) is represented as shown in Fig. 5.

The corrected theoretical damage model is obtained by combining the static damage, based on the rupture pressures, to the corrected burst pressures formula of Faupel for different life fractions. Therefore, we obtain the ultimate pressure ( $P_u$ ), the ultimate residual pressures ( $P_{ur}$ ) and the applied pressure, which corresponds to the life fraction just before rupture ( $P_a$ ). Then, these pressures were integrated in the static damage Eq. (3), to obtain the corrected model. Finally, we represent the theoretical damage and the experimental damage in comparison as shown in Fig. 6.

We note from this figure that these damages have the same tendencies. Therefore, we deduce that the corrected theoretical damage can be adopted as a simple and an efficient tool to represent the damage of HDPE pipes by bursting a neat pipe only.

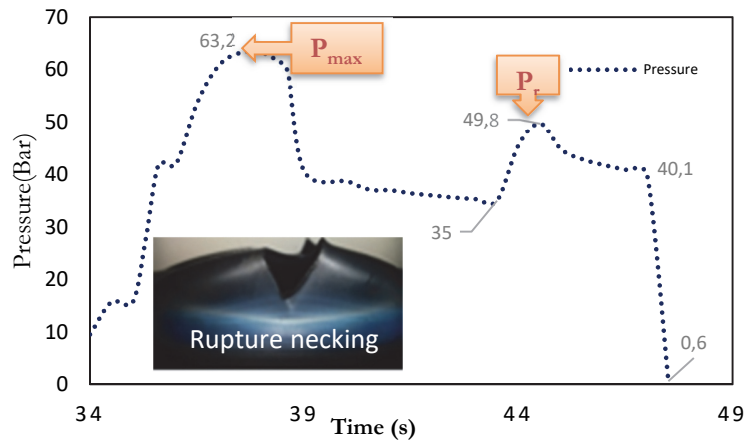


Figure 4: Internal pressure evolution as a function of the time.

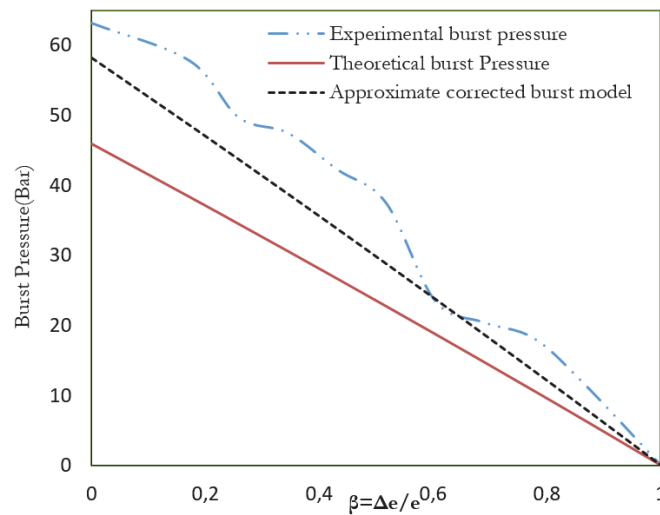


Figure 5: The theoretically corrected pressure versus the experimental one as a function of the life fraction.

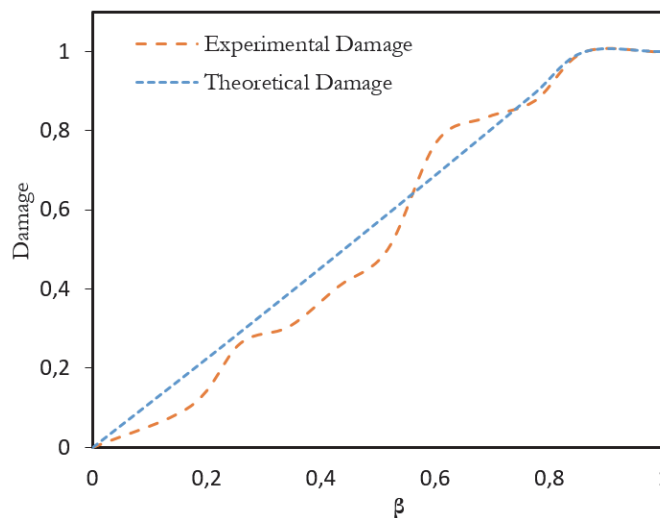


Figure 6: Theoretical and experimental damage as a function of the life fraction.



*Adaptive and continuum damage based on burst pressure formula P (β)*

In this part, we developed a new formula for burst pressure calculations, which takes into account the intervals of the life fraction and the characteristic pressures shown in the first part of this paper. This equation is independent of the limit pressure's equations such as Faupel one. It is conditioned by the critical life fraction as shown by the Eq. (6) and (7):

$$\begin{cases} P(\beta) = -P_r * \beta + P_{\max}; [0, \beta_c] \\ P(\beta) = (-P_r * \beta + P_{\max}) * \frac{\beta_c}{\beta}; [\beta_c, 1] \end{cases} \quad (6)$$

Taking into account the parameter  $\alpha$ , we obtain:

$$\begin{cases} P(\beta) = (-\beta + \alpha)P_r; [0, \beta_c] \\ P(\beta) = \left(-1 + \frac{\alpha}{\beta}\right) * \beta_c * P_r; [\beta_c, 1] \end{cases} \quad (7)$$

The model presented above is an adaptive tool for estimating the burst pressure through a burst test of a neat HDPE pipe only as shown in Fig. 7:

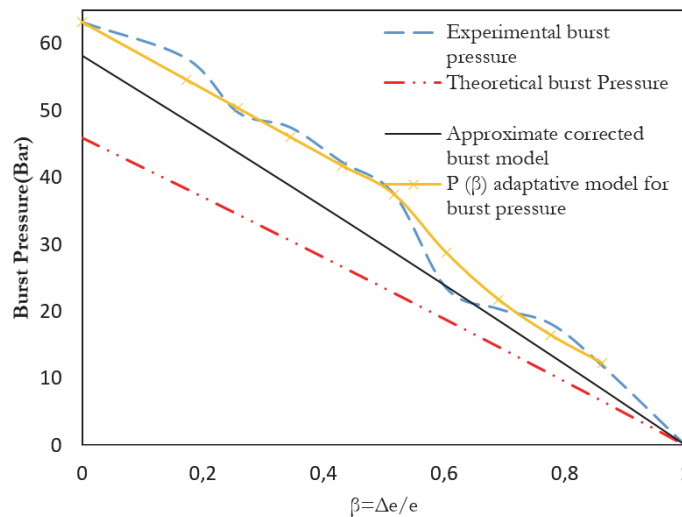


Figure 7: Theoretically corrected and adaptive estimation of burst pressure.

Starting from the expression of the pressure  $P(\beta)$ , we have developed two ways to calculate the damage. The first consists of a model of damage inspired from the experimental static damage. In fact, we calculate the adaptive pressures through the  $P(\beta)$  equation for the same life fractions as the experimental case. Consequently, we consider that the pressure is ultimate ( $P_u$ ) for a neat pipe ( $\beta = 0$ ). For the other intermediate life fractions, we obtain the residual ultimate pressures ( $P_{ur}$ ). The applied pressure ( $P_a$ ) corresponds to the last pressure before the rupture corresponding to the last possible loading level ( $\beta = 0.86$ ). Furthermore, the integration of these pressures into the burst pressure static damage model allows us to obtain the adaptive damage that is shown in Fig. 8.

The second consists of the creation of a continuum damage model that supports the characteristic pressures of HDPE,  $P_r$  and  $P_{\max}$ , represented by the coefficient  $\alpha$  and a non-dimensional coefficient representing the ratio of the applied pressure ( $P_a$ ) and the rupture pressure ( $P_r$ ),  $\eta$ . In fact, this model presents a continuum function represented according to three intervals of  $\beta$ . Calculations of the damage are possible by choosing the iteration of the desired life fraction and by knowing the various experimental parameters such as the applied pressure ( $P_a$ ) corresponding to the last experimental pressure before the rupture of 11.9 bar for the studied HDPE.

The continuum static damage is expressed by:



$$\left\{ \begin{array}{l} D(\beta) = \frac{1 - \frac{(-P_r * \beta + P_{\max})}{P_u}}{1 - \frac{P_a}{P_u}}; [0, \beta_c]; P_{\max} = P_u \\ D(\beta) = \frac{(-P_r * \beta + P_{\max}) * \frac{\beta_c}{\beta}}{1 - \frac{P_a}{P_u}}; [\beta_c, \beta_{P_a}] \\ D(\beta) = 1; [\beta_{P_a}, 1] \end{array} \right. \quad (8)$$

So:

$$\left\{ \begin{array}{l} D(\beta) = \frac{P_{\max} - (-P_r * \beta + P_{\max})}{P_{\max} - P_a}; [0, \beta_c]; u \\ D(\beta) = \frac{P_{\max} - (-P_r * \beta + P_{\max}) * \frac{\beta_c}{\beta}}{P_{\max} - P_a}; [\beta_c, \beta_{P_a}] \\ D(\beta) = 1; [\beta_{P_a}, 1] \end{array} \right. \quad (9)$$

So:

$$\left\{ \begin{array}{l} D(\beta) = \frac{P_r * \beta}{P_{\max} - P_a}; [0, \beta_c]; u \\ D(\beta) = \frac{P_{\max} + (P_r * \beta - P_{\max}) * \frac{\beta_c}{\beta}}{P_{\max} - P_a}; [\beta_c, \beta_{P_a}] \\ D(\beta) = 1; [\beta(P_a), 1] \end{array} \right. \quad (10)$$

In addition:

$$\left\{ \begin{array}{l} D(\beta) = \frac{\beta}{\alpha - \eta}; [0, \beta_c]; \eta = \frac{P_a}{P_r} : \\ D(\beta) = \frac{\alpha + (\beta - \alpha) * \frac{\beta_c}{\beta}}{\alpha - \eta}; [\beta_c, \beta_{P_a}] \\ D(\beta) = 1; [\beta_{P_a}, 1] \end{array} \right. \quad (11)$$





The coefficient  $\alpha$ ,  $\beta$  and  $\eta$  represent respectively the ratio of the ductile rupture pressures, the life fraction and the non-dimensional ratio of the applied pressure and the rupture one.  $\beta_{Pa}$  is the life fraction corresponding to the pressure before rupture.

The representation of the adaptive model is given in Fig. 8:

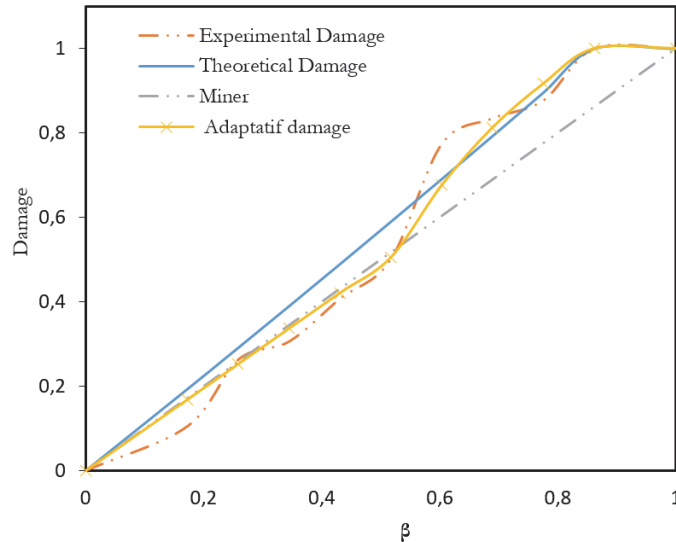


Figure 8: Approximate theoretical damage, through corrected Faupel, adaptive and experimental damages according to the life fraction.

The representation of the continuum damage model is almost the same as the one given by the adaptive damage model as illustrated in Fig. 8. The difference is explained by taking into account the applied experimental pressure ( $P_a = 11.9$  bar) for the continuum damage model independently of the theoretical equations.

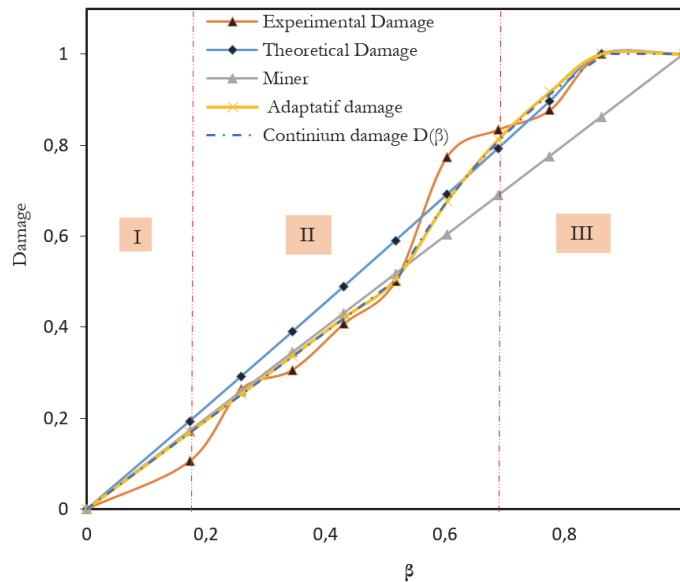


Figure 9: Approximate theoretical damage, through corrected Faupel, adaptive, continuum  $D(\beta)$  and experimental damages as a function of the life fraction.

We observe from the damage curves of Figs. 8 and 9 that the theoretical damage is linear as the damage of Miner, while the experimental damage clearly shows three phases of evolution. The first phase is characterized by slow evolution under the theoretical damage up to the life fraction of 17%. In the second phase, we observe a steady increase of the latter up to the 65% of life fraction. In the third phase, a significant acceleration of the damage to reach the unit was registered. Concerning the adaptive damage, we found that the developed model traces perfectly the model of static damage obtained



experimentally. The same evolution was observed for continuum damage model, which has the same tendency as the adaptive one, Fig. 9.

## CONCLUSION

The assessment and establishment of the damage of HDPE pipes can't be considered as a simple task. On the one hand, we propose a new approach based on the calculated pressures through the Faupel formula. This theoretical pressure can substitute the experimental burst pressures of the HDPE pipes, which are exposed to an increasing internal pressure until burst. Nevertheless, the calculations obtained by the Faupel formula must take into account the specificities of the used material since they were originally proposed for metals (steels). In this perspective, we have developed a modified equation based on a factor  $\alpha$  that depends strongly on the behavior of the internal pressure used for the bursting of the neat studied pipes. Indeed, we have highlighted the effect of elongation of the pipe during the elastic phase to reach a maximum pressure. After that, the pipe reaches a phase of large strains before getting to another pressure peak representing the pressure of rupture.

In the used static damage models presented in this paper, we substitute the experimental pressures by the calculated and modified ones. Then, we obtain a close approximate model more representative of the experimental model. The latter represents a powerful and rapid tool that can be used by manufacturers to launch audits and fast checks by knowing just the value of the coefficient  $\alpha$  deduced from the evolution of the internal pressure of a neat pipe. Moreover, we have shown another powerful tool, adaptive damage model, perfectly reproducing the experimental static damage through a single burst test of a neat HDPE pipe. Therefore, if we want to calculate the burst pressure and subsequently the corresponding damage, we simply replace the value of the life fraction in the set of Eq. (6) or (7) knowing that the parameters  $P_r$ ,  $P_{max}$  and  $\beta_c$  are constants that depend on the thermoplastic material and on the class of the pipeline. For the HDPE PE100 PN16 63 mm x 5.8 mm, we found that the values of these parameters are respectively 49.8 bar, 63.2 bar and 0.52.

A theoretical modeling of the damage giving rise to a continuum damage model, Eqs. (8) to (11), showed that the latter could be evaluated by knowing the values of the coefficients  $\alpha$  and  $\eta$ . These coefficients are constants of the studied material depending respectively on the pressures  $P_r$  and  $P_{max}$  for the first and on the applied experimental pressure  $P_a$  and the rupture pressure  $P_r$  for the second as well as the critical life fraction. The continuum model is represented as a function of the theoretical life fraction by choosing an iteration for its evolution according to the wanted accuracy.

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## NOMENCLATURE

$\Delta e$	Thickness fluctuation of each created notch (groove).
$e$	Thickness of the pipe.
$n$	Instantaneous number of cycles under an applied stress.
$N_f$	Total number of cycles at the rupture.
$\beta$	Life fraction representing the thickness fluctuation over the pipe's thickness ( $\beta = \Delta e/e$ ).
$\beta_c$	Critical life fraction.
$\beta_{pa}$	Life fraction corresponding to the pressure before rupture.
$P$	Burst or limit pressure.
$P_u$	Ultimate pressure corresponding to a neat HDPE specimen.
$P_{max}$	Maximum pressure of the internal pressure curve of neat HDPE pipes
$P_r$	Rupture pressure of HDPE pipes.
$P_{ur}$	Ultimate residual burst pressure of notched pipes.
$P_a$	Pressure just before the rupture corresponding to the last possible notch.
$P_{ur1}, P_{ur2}, P_a$	Ultimate residual burst pressure of notched pipes according to the notch level.
$\alpha$	Parameter depending on the studied material ( $\alpha = P_{max}/P_r$ ).
$\eta$	Parameter depending on the applied pressure corresponding to the notch just before rupture ( $\eta = P_a/P_r$ ).
$D_o, D_i$	External and internal diameters of a cylinder.
$\sigma_y$	Yield stress.
$\sigma_{UTS}$	Ultimate stress
$P(\beta)$	Continuum burst pressure as a function of the life fraction $\beta$ ,
$D(\beta)$	Continuum damage as a function of the life fraction $\beta$ .
$D_s$	Static damage based on burst pressures of HDPE pipes.
$D$	Linear damage of Miner.