Focused on Crack Tip Fields

Visual Abstract

Multi-parameter fracture mechanics: crack path in a mixed-mode specimen

L. Malíková, H. Šimonová, B. Kucharczyková
Brno University of Technology, Faculty of Civil Engineering, Institute of Structural Mechanics, Veveří 331/95, 602 00 Brno, Czech Republic
malikova.l@fce.vutbr.cz, http://orcid.org/0000-0001-5868-5717
simonova.h@vutbr.cz, http://orcid.org/0000-0003-1537-6388
Barbara.Kucarczykova@vutbr.cz, http://orcid.org/0000-0002-7123-5099

P. Miarka
Academy of Sciences of the Czech Republic, Institute of Physics of Materials, v. v. i., Žižkova 22, 616 62 Brno, Czech Republic and Brno University of Technology, Faculty of Civil Engineering, Institute of Structural Mechanics, Veveří 331/95, 602 00 Brno, Czech Republic
Petr.miarka@vut.cz, http://orcid.org/0000-0002-4953-4324

ABSTRACT. A mixed-mode geometry has been chosen to investigate a crack propagation using the multi-parameter fracture mechanics concept. The so-called Williams' series expansion is used for the crack-tip stress field approximation. It has been shown that application of the generalized fracture mechanics concept can be crucial for materials with specific fracture behaviour, such as elastic-plastic or quasi-brittle one, when fracture occurs not only in the very vicinity of the crack tip, but also in a more distant surrounding. Then, considering the higher-order terms of the Williams' expansion in fracture criteria (describing the crack stability and/or crack propagation direction) can bring more precise results. The coefficients of the Williams' expansion must be calculated numerically (for instance by means of the over-deterministic method in this work) for each cracked configuration, which is very time-consuming, and the analysis is very extensive even for a few basic cracked specimen configurations. On the other hand, a suitable choice of the geometrical configuration of the cracked disc enables performing experiments only on the specimens that could prove the theory about the importance of using the higher-order terms.

KEYWORDS. Crack-tip field; Multi-parameter fracture mechanics; Finite element analysis; Mixed-mode; Williams' expansion.

Citation: Malíková, L., Šimonová, H., Kucharczyková, B., Miarka, P., Multi-parameter fracture mechanics: crack path in a mixed-mode specimen, Frattura ed Integrità Strutturale, 49 (2019) 65-73.

Received: 21.04.2019
Accepted: 25.04.2019
Published: 01.07.2019

INTRODUCTION

Although fracture mechanics has been studied for approximately one century, proper assessment of crack behaviour is still under investigation of many researchers. One of the current tasks is more precise description of the crack-tip stress/displacement field that is fundamental information necessary for additional more complex fracture analyses. In the last years, the multi-parameter concept seems to be helpful, when some of the fracture mechanics issues are solved [1-3]. Contrary to the most common single-parameter linear elastic fracture mechanics approach working mainly only with the stress intensity factor (first term of the Williams' expansion [4]), the generalized fracture mechanics takes into account also the second (non-singular) term of the WE, the so-called *T*-stress, as well as the terms of higher orders. Significance of the truncated form of the Williams' series has been investigated by the authors' collective for instance in [5-8]. Moreover, generalized fracture criteria are taking into account more terms of the Williams' expansion start to appear in scientific works. For example in [9, 10] the generalized maximum tangential stress criterion is introduced, which considers in the fracture criterion not only the stress intensity factor but also the *T*-stress.

In this work, a mixed-mode cracked configuration was chosen and the crack propagation angle was investigated numerically by means of finite elements. The study presented represents a pilot analysis that should help to select a suitable configuration of the semi-circular disc, particularly the initial crack length and the crack inclination angle. This specimen will be considered in planned bending fracture experiments on specimens made of fine-grained composites based on the alkali-activated matrix (AAM). Several configurations are searched, where it can be expected that the description of the crack-tip stress field via multi-parameter fracture mechanics brings much better results of the crack path than the classical one-parameter fracture mechanics concept.

Currently, identification of the basic material characteristics of the specially selected material are taking place. Alkali activated aluminosilicate materials represent an alternative to ordinary Portland-cement-based materials, reducing the environmental impact of building industry and offering new improved properties. These binders are made through the mixing of some non- or poorly-crystalline aluminosilicate-based material, such as blast furnace slag or fly ash, with an alkaline activator (hydroxides, carbonates or the most preferably silicates) and water [11, 12]. Type and dosage of the activator as well as the way of curing have a significant effect on the hydration course and final mechanical properties [13]. The choice of the composite with AAM for the research corresponds to the lack of information about its fracture behaviour.

THEORETICAL BACKGROUND

Multi-Parameter Fracture Mechanics (MPFM)

The multi-parameter fracture mechanics concept assumes that the crack-tip stress/displacement field is described by means of the Williams' expansion [4] that was derived for a homogeneous elastic isotropic cracked body with an arbitrary remote loading. The infinite power series for the crack-tip stress field can be written in the form:

$$\sigma_{ij} = \sum_{n=1}^{\infty} A_n \frac{n}{2} r^{\frac{n}{2}-1} f_{ij}(n,\theta) + \sum_{n=1}^{\infty} B_m \frac{m}{2} r^{\frac{m}{2}-1} g_{ij}(m,\theta) \text{ , where } \quad i,j \in \{x,y\} \text{ ;} \tag{1}$$

similarly, the infinite power series for the crack-tip displacement field can be written in the form:

$$u_i = \sum_{n=0}^{\infty} A_n r^{\frac{n}{2}} f_i(n,\theta,E,\nu) + \sum_{n=0}^{\infty} B_m r^{\frac{m}{2}} g_i(m,\theta,E,\nu) \text{ , where } \quad i \in \{x,y\} \text{ .} \tag{2}$$

These are the two basic equations and their truncated form is used in the research introduced in this paper. Regarding the symbols used, it holds following: σ_{ij} and u_i represent the stress tensor and displacement vector components, respectively; (r, θ) symbolize the polar coordinates (considering the centre of the coordinate system at the crack tip and the crack faces lying on the negative x-axis); f_{ij} ,g_{ij} and f_i , g_i stand for the known functions corresponding to loading mode I (f) and loading mode II (g) and their expressions can be found in classical textbooks on fracture mechanics; E and ν represent Young's modulus and Poisson's ratio, respectively. The only unknown symbols are the coefficients A_n and B_m of the individual terms of the series – their values depend on the relative crack length, load, geometry or generally, on the cracked specimen configuration. Therefore, it is necessary to calculate these parameters numerically and the method used within this work is described in the following text.

Over-Deterministic Method (ODM)

Over-deterministic method (ODM) was chosen as the procedure for calculation of the coefficients of the individual terms of the Williams' expansion. These terms are usually neglected and only the first (singular) term corresponding to the well-known stress intensity factor is taken into account. ODM enables determination of an arbitrary number of the coefficients of the series introduced in Eqns. 1 and 2 when particular conditions are satisfied.

The principle of the method consists in direct application of Eqn. 2 for displacement vector components. When k nodes are selected around the crack tip, a system of $2k$ algebraic equations for the variables A_n and B_m can be defined, when displacements u and v as well as the nodes coordinates r and θ are known (from a finite element model or from an experiment). In order to fulfil the basic idea of the method (solving of an over-determined system of equations), it must hold that $2k$ is larger than $N+M+2$, where N and M are the numbers of the mode I and mode II Williams' expansion terms in the its truncated form, respectively. More details, recommendations and/or restrictions regarding the application of the method can be found for instance in [14, 15]. Note that the ODM was chosen with regard to its low requirements on the software and mathematical definitions/procedures in contrast to other method derived in literature, such as Hybrid Crack Element (HCE) method, Boundary Collocation Method (BCM) etc. [16-18].

Maximum Tangential Stress criterion (MTS)

When a crack is located arbitrarily to the loading direction, the mixed-mode loading is spoken about. In order to describe the crack behaviour, particularly its trajectory through the specimen, several fracture criteria have been derived, see for instance [19]. In this work, the Maximum Tangential Stress (MTS) criterion and minimum Strain Energy Density (SED) criterion were chosen. Both are common and often used when the crack path shall be investigated. The MTS criterion is a stress-based criterion and is independent on the plane stress or plane strain conditions. Its basic idea consists in the assumption that a crack will propagate in the direction where the tangential stress $\sigma_{\theta\theta}$ reaches its maximum [20]. Written mathematically:

$$\frac{\partial \sigma_{\theta\theta}}{\partial \theta} = 0 \quad \text{and} \quad \frac{\partial^2 \sigma_{\theta\theta}}{\partial \theta^2} < 0 \,. \tag{3}$$

When the criterion is used in this research, the tangential stress component is expressed via Williams' expansion similarly to Eqn. 1 and the maximum of that function is searched numerically in Wolfram Mathematica code [21].

Minimum Strain Energy Density criterion (SED)

Similarly, the kink angle of the crack can be found by means of the minimum Strain Energy Density (SED) criterion [22][23], but the minimum of the function S is searched. This condition can be again written by means of the derivatives:

$$\frac{\partial S}{\partial \theta} = 0 \quad \text{and} \quad \frac{\partial^2 S}{\partial \theta^2} > 0 \,, \text{ where } \; S = \frac{1}{2\mu}\left[\frac{\kappa+1}{8}(\sigma_{rr} + \sigma_{\theta\theta})^2 - \sigma_{rr}\sigma_{\theta\theta} + \sigma_{r\theta}^2\right] \,. \tag{4}$$

The stress tensor components are approximated by means of the Williams' expansion considering various numbers of the initial terms and a procedure for finding the minimum of the strain energy density factor S is programmed.

The basic difference between the multi-parameter (generalized) criteria and the classical (one-parameter) ones is the existence of a length parameter where the multi-parameter criterion is applied. Because there does not exist any unambiguous recommendation how to choose this radial distance (only some theories that it should be related to material characteristics [24-26]), several various values were tested within this study.

NUMERICAL MODEL

The analysis on the crack behaviour and importance of the MPFM has been performed on a semi-circular disc loaded in bending (SCB), see e.g. [27]. One of the advantages of this kind of specimens is the initiation of the tension crack even under the compressive loading, see Fig. 1. Also, the inclination of the crack enables to test specimens under various mixed mode (I+II) conditions.

The parameters of the numerical model were as follows: radius of the disc R = 50 mm, span between supports S = 80 mm, the crack length varied between a = 10 ÷ 35 mm and the crack inclination angle varied between β = 10° ÷ 50°. Thus, thirty various configurations were investigated with mixed-mode level between K_I/K_{II} = 1.5 ÷ 8.

In order to get basic fracture mechanical properties of the alkali-activated material under study, several tests have been performed on prisms with dimensions 40 × 40 × 160 mm. Young's modulus of 35 GPa and Poisson's ratio of 0.23 were then applied in the numerical simulations based on the results of 3-point bending test and resonance method, respectively.

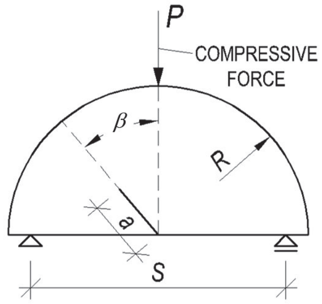
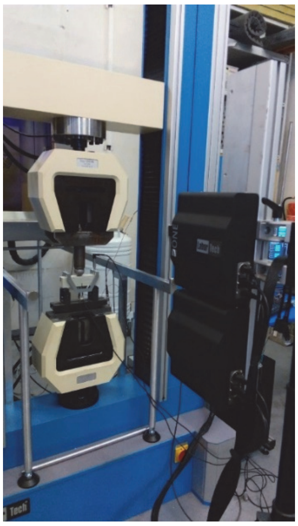


Figure 1: Schema and geometry of the semi-circular disc loaded in bending used for the fracture analysis in this research.



a)

b)

Figure 2: Experimental campaign on 40 × 40 × 160 mm cracked prisms under 3-point-bending: a) experimental setup; b) detail of the crack path through the specimen.

The numerical model has been created as a two-dimensional (2D), commercial finite element software ANSYS [28] has been used. The crack deflection has been studied under the compressive force $P = 1$ kN considering plane strain boundary conditions. A quadrilateral element type (PLANE183) with quadratic basis functions have been used for modelling of the semi-circular disc. The FE mesh at the crack tip has been refined in order to take into account crack tip singularity. For application of the ODM, nodes at the radial distance of 4 mm have been selected and their displacements used for evaluation of the coefficients of higher-order terms. Stress components necessary for application of mentioned fracture criteria have been expressed via Williams' expansion taking into account up to 10 initial terms of the power series. The criteria have been applied at critical distances r_C of 0.1, 0.5 and 1 mm. Results obtained are presented and discussed in the following section.

RESULTS & DISCUSSION

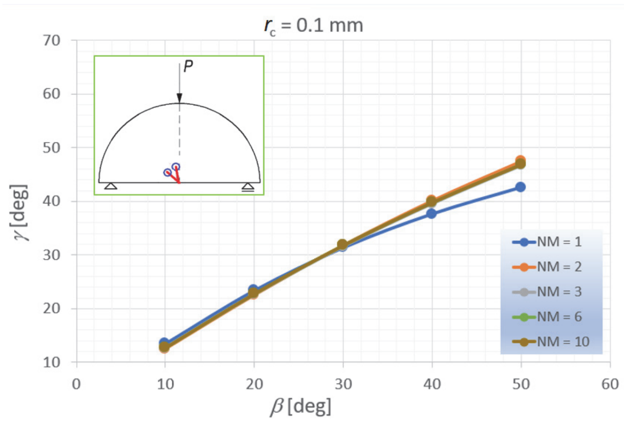
In the following figures, the dependences of the crack deflection angle obtained for various cracked configurations and considering various parameters are presented. Let's summarize what parameters have been varied:

- Initial relative crack length, $\alpha = a/R$;
- Crack inclination angle, β;

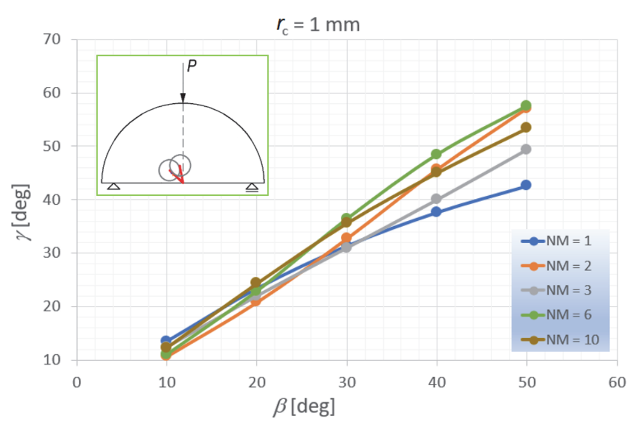
- Critical distance, where the multi-parameter fracture criteria are applied, r_c;
- Number of initial terms of the Williams' expansion considered for crack-tip stress field approximation.

In Figs. 3 to 6 only selected results are introduced, because the extent of the analysis is too large. Influence of the above-mentioned parameters on the crack deflection angles obtained has been investigated in order to find the appropriate configurations for subsequent experimental campaign that should prove the significance of considering the higher-order terms when the crack-tip stress field is reconstructed.

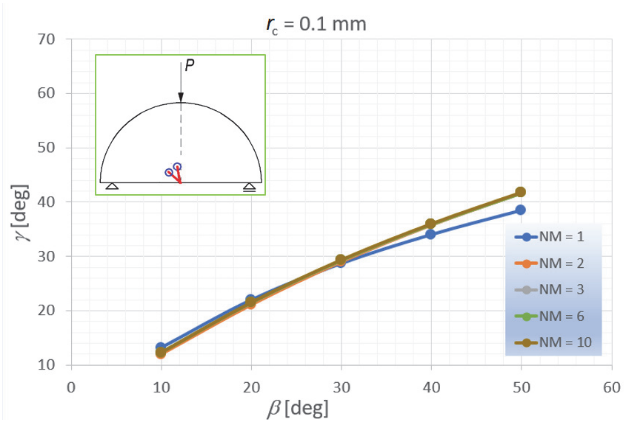
Comparison between multi-parameter MTS and SED criterion and their application at various distances can be observed for short cracks with α = 0.2 in Fig. 3 and for longer cracks with α = 0.5 in Fig. 4, respectively.



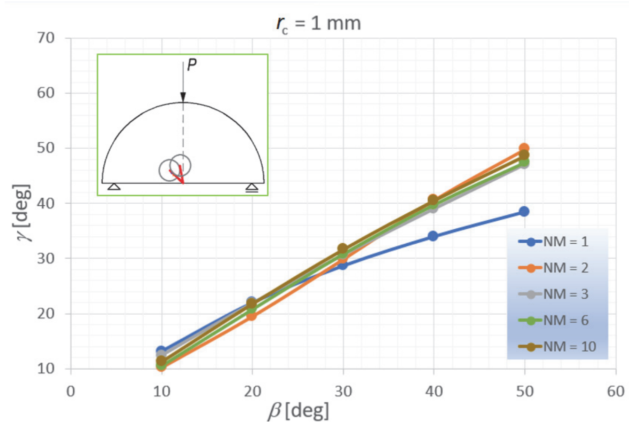
a) MTS, $\alpha = a/R = 0.2$, r_c = 0.1 mm



b) MTS, $\alpha = a/R = 0.2$, r_c = 1.0 mm



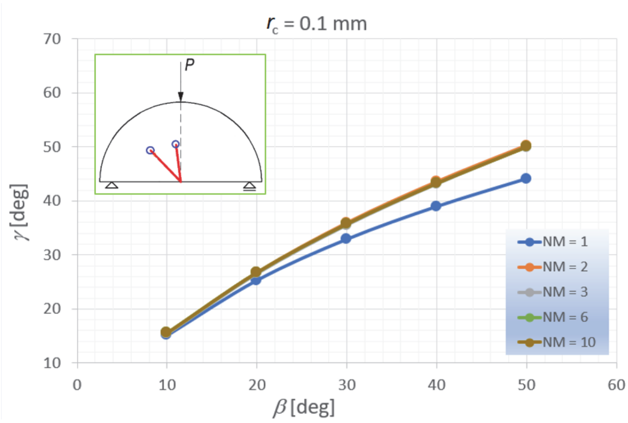
c) SED, $\alpha = a/R = 0.2$, r_c = 0.1 mm



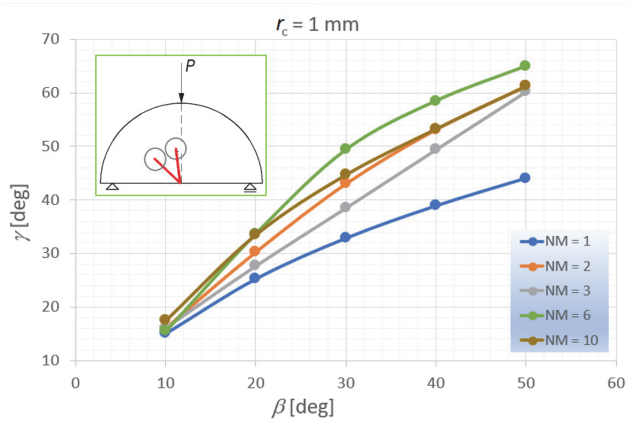
d) SED, $\alpha = a/R = 0.2$, r_c = 1.0 mm

Figure 3: Crack deflection angles γ obtained on the SCB specimens for short cracks (a/R = 0.2) via multi-parameter MTS and SED criterion applied at critical distances of 0.1 mm and 1.0 mm for various crack inclination angles (β = 10, 20, 30, 40 and 50°) and considering various numbers of initial terms of Williams's expansion (N, M = 1, 2, 3, 6 and 10).

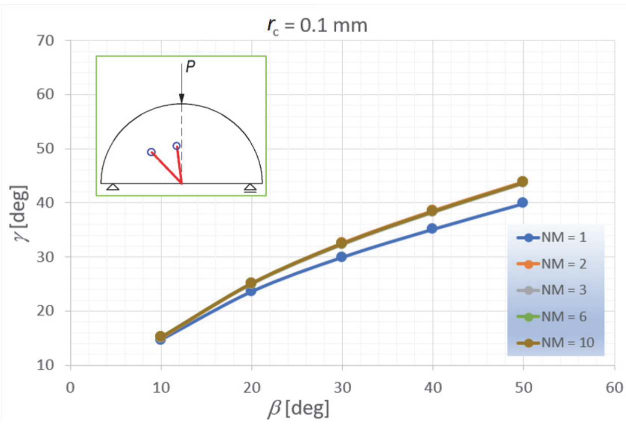
Results presented in Figs. 5 and 6 represents comparison of the crack deflection angles obtained via multi-parameter MTS and SED criterion at various critical distances in dependence on the choice of the number of initial terms of the Williams' expansion for two inclination angles (β = 10° and β = 50°) for short cracks with α = 0.2 (Fig. 5) and for longer cracks with α = 0.5 (Fig. 6), respectively.



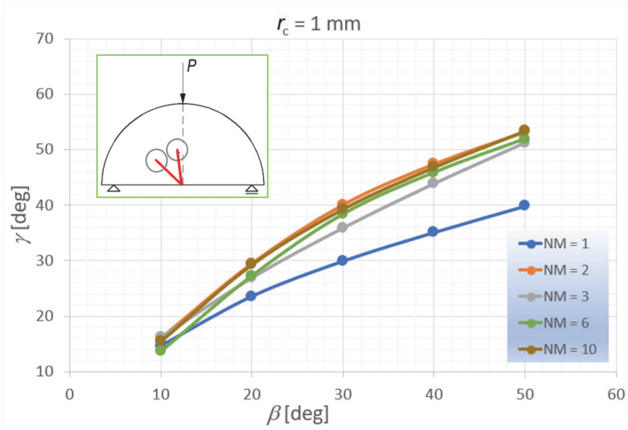
a) MTS, $\alpha = a/R = 0.5$, $r_c = 0.1$ mm



b) MTS, $\alpha = a/R = 0.5$, $r_c = 1.0$ mm



c) SED, $\alpha = a/R = 0.5$, $r_c = 0.1$ mm



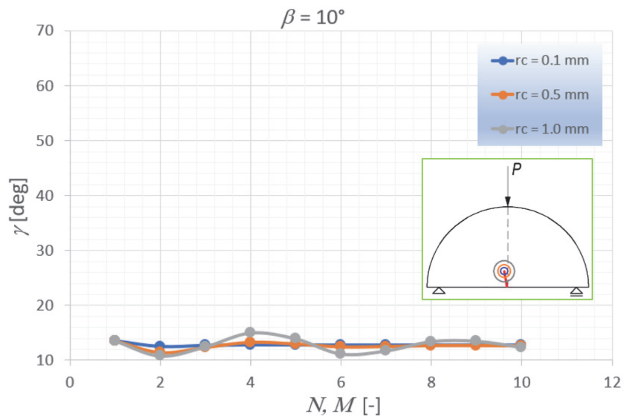
d) SED, $\alpha = a/R = 0.5$, $r_c = 1.0$ mm

Figure 4: Crack deflection angles γ obtained on the SCB specimens for longer cracks ($a/R = 0.5$) via multi-parameter MTS and SED criterion applied at critical distances of 0.1 mm and 1.0 mm for various crack inclination angles ($\beta = 10$, 20, 30, 40 and 50°) and considering various numbers of initial terms of Williams's expansion (N, M = 1, 2, 3, 6 and 10).

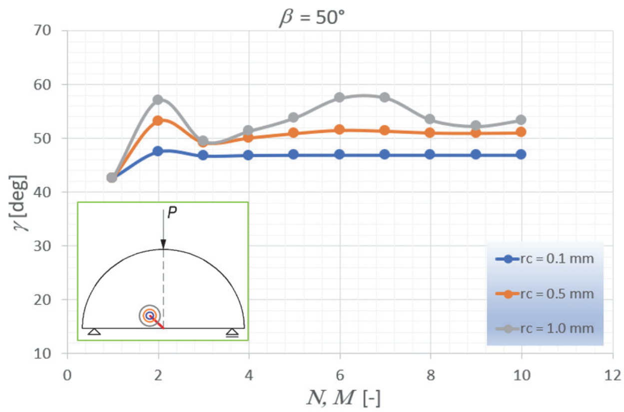
Based on the graphs presented above, following conclusions can be stated:

- Even at small critical distances from the crack tip (r_c = 0.1 mm) it has been proved that higher-order terms of the Williams' expansion could improve the accuracy of the estimated crack deflection angle.
- This effect is stronger for longer cracks and when larger distances from the crack tip are used for application of the generalized fracture criteria.
- The SED criterion is less sensitive to the choice of the number of the higher-order terms than the MTS criterion.
- Generalized fracture criteria are more important for cracks under mixed mode I+II, than for cases when loading mode I prevails (i.e. in our case for larger β angles).

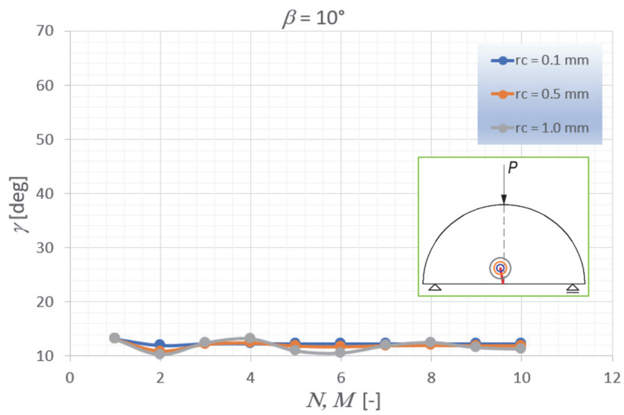
In connection with the experimental research intended by the authors, following recommendations can be concluded: The experimental campaign should be performed (as much as possible) on SCB specimens with larger cracks and larger initial deflection angles β in order to be able to investigate the effect of the higher-order terms considered in the generalized fracture criteria on estimation of further crack propagation angles.



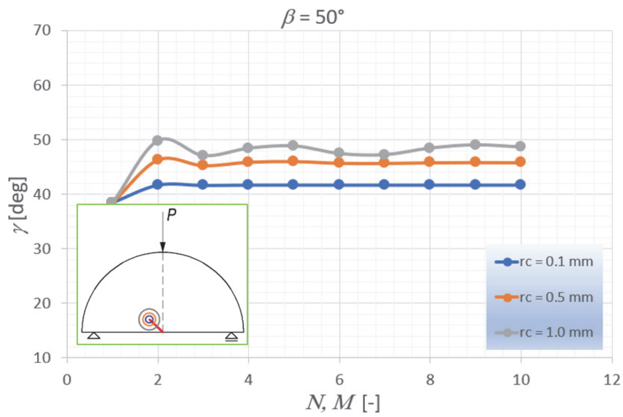
a) MTS, $\alpha = a/R = 0.2$, $\beta = 10°$



b) MTS, $\alpha = a/R = 0.2$, $\beta = 50°$



c) SED, $\alpha = a/R = 0.2$, $\beta = 10°$



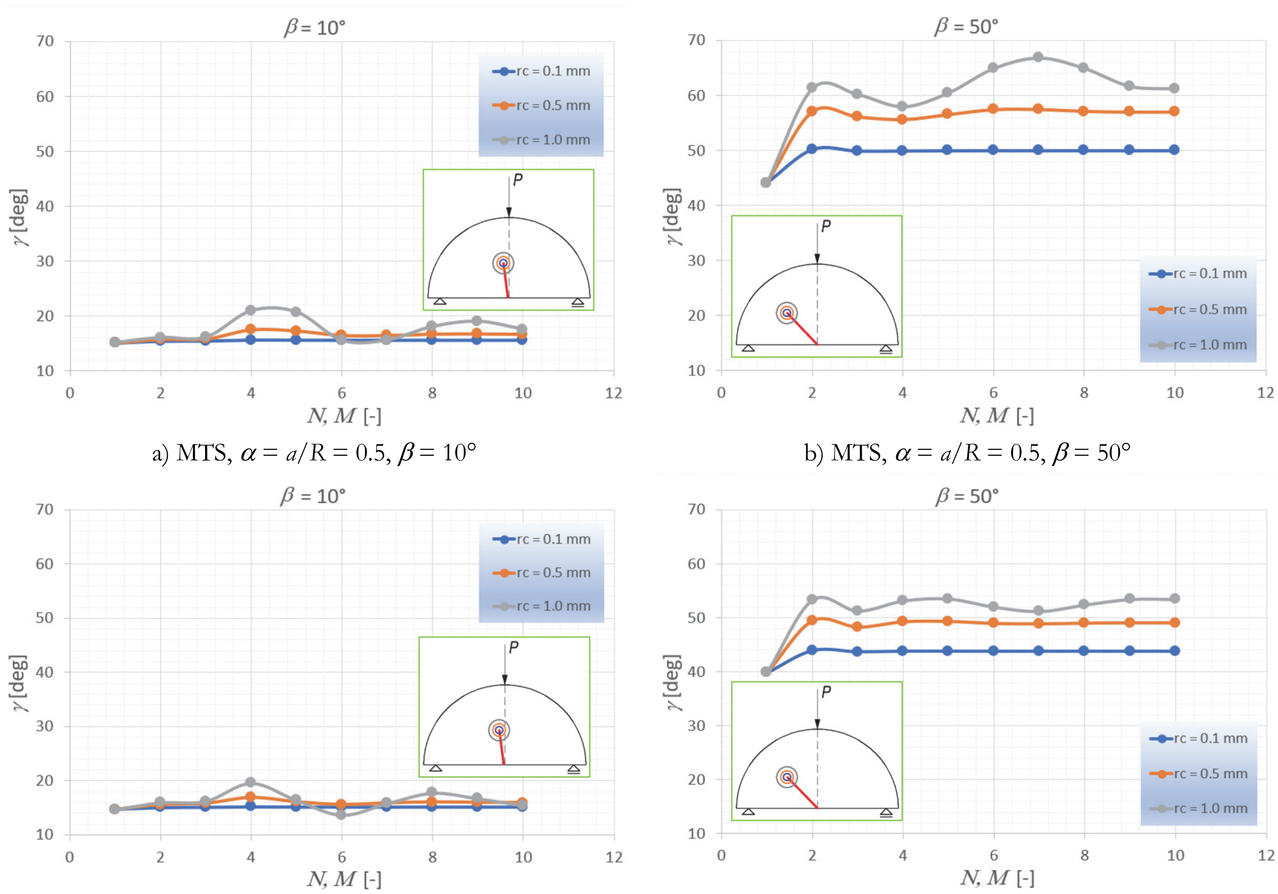
d) SED, $\alpha = a/R = 0.2$, $\beta = 50°$

Figure 5: Crack deflection angles γ obtained on the SCB specimens for short cracks ($a/R = 0.2$) via multi-parameter MTS and SED criterion applied at critical distances of 0.1, 0.5 and 1.0 mm for crack inclination angles 10° and 50° and considering up to 10 initial terms of Williams's expansion.

If the suggested approach is not able to describe the crack path in the fine-grained composite based on the alkali-activated slag as a quasi-brittle material, other fracture criteria (that are more suitable for non-brittle fracture and mixed-mode conditions) should be applied, such as maximum average tangential stress (MATS) criterion [29] or others. Another possibility is to use so-called Equivalent Material Concept (EMC) suggested recently by Torabi [30] and consider a virtual brittle material exhibiting linear elastic behaviour instead of the real quasi-brittle one; details on this concept can be found in the referred literature.

Conclusions

Paper deals with a parametric study on a mixed-mode I+II specimen in order to find a suitable geometrical configuration of a SCB disc for proving importance of considering the higher-order terms of the Williams' expansion during approximation of crack-tip stress field. The analysis shall be performed on a novel material: fine-grained composite based on the alkali-activated slag (an alternative to ordinary portland-cement-based materials, reducing the environmental impact of building industry). The crack deflection angle is studied via multi-parameter MTS and SED criteria for various geometrical configurations and several recommendations are concluded based on the results obtained. The largest differences between the classical single-parameter fracture mechanics and the multi-parameter concept are observed in SCB specimens with larger cracks and larger initial deflection angles β. Thus, the real fine-grained composite specimens will be produced with these parameters.



a) MTS, $\alpha = a/R = 0.5$, $\beta = 10°$ b) MTS, $\alpha = a/R = 0.5$, $\beta = 50°$

c) SED, $\alpha = a/R = 0.5$, $\beta = 10°$ d) SED, $\alpha = a/R = 0.5$, $\beta = 50°$

Figure 6: Crack deflection angles γ obtained on the SCB specimens for larger cracks ($a/R = 0.5$) via multi-parameter MTS and SED criterion applied at critical distances of 0.1, 0.5 and 1.0 mm for crack inclination angles 10° and 50° and considering up to 10 initial terms of Williams's expansion.

Acknowledgement

Financial support from the Czech Science Foundation (project No. 18-12289Y) and from the Faculty of Civil Engineering, Brno University of Technology (project No. FAST-S-19-5896) is gratefully acknowledged.

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