Visual Abstract

Alternative estimation of effective Young's Modulus for Lightweight Aggregate Concrete LWAC

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ABSTRACT. The prediction of effective mechanical properties of composite materials using analytical models is of significant practical interest in situations in which tests are impossible, difficult, or costly. Many experimental and numerical works are attempting to predict the elastic properties of Lightweight Aggregate Concrete (LWAC). In order to choose the optimized prediction composite model, the purpose of this paper is to appraise the effective Young's modulus of LWAC using two-phase composite models. To this effect, results of previous experimental research have used as a platform, upon which, 07 two-phase composite models were applied. The outcomes of this comparative analysis show that not all two-phase analytical models can be directly used for predicting Young's modulus of LWAC. The Maxwell, Counto1 and Hashin-Hansen models are in close concordance with the experimental Young's modulus of all LWAC used for comparison in this study (119 values). They were found more appropriate for reasonable prediction of elasticity modules of the LWAC.

KEYWORDS. Analytic model; Concrete; Young's modulus; Lightweight aggregate; Two-phase.

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INTRODUCTION

Recently, special attention has been paid to the development of Lightweight Aggregate Concrete (LWAC) [1, 2, 3] which offers many advantages as a building material, including low weight, easier construction and better resistance compared with ordinary concrete. Lightweight Aggregate Concrete (LWAC) primarily improves the thermal and sound insulation properties of buildings next to its basic applications [4]. The lightweight concrete are created by substituting the natural aggregates with the lightweight aggregates (LWA), which are classified into two fundamental categories: natural (like pumice, diatomite, volcanic ash, etc.) and manufactured (such as perlite, extended schist, clay, slate, sintered powdered fuel ash (PFA), etc.) [3, 5]. Beside its technical and financial interests, LWAC can be integrated into the demarche of sustainable improvement by utilizing in specific artificial aggregates which are lighter than natural aggregates [6].

The Young's modulus (elastic modulus) is a very important material property which is measured directly on concrete. Engineers need to know the value of this parameter to conduct any computer simulation of structure. Various experimental works have concerned the study of behavior of LWAC [1, 7, 8, 9, 10, 11, 12]. However from an experimental point of view, this is not always easy. Therefore when the tests are impossible, difficult, costly, or time-consuming, the research about prediction models for the elastic modulus using properly validated composite models is of great practical interest.

The aim of the composites materials approach is to develop a model that will enable expression of average properties of the mixtures through properties and volume fractions of its constituents [11]. Diverse explicit models of the literature are utilized. Their application to the prediction of LWAC behaviors shows a wide dissimilarity between the different approaches particularly when the volume fraction of reinforcement is more than 40% and when the contrast between the phases grows [9].

For this purpose and to distinguish the most appropriate two-phase composite model for predicting LWAC's effective modulus of elasticity, the estimation of the Young's modulus of LWAC using two-phase composite models was applied. Furthermore, an efficient and accurate model is useful to reduce the cost and duration of the experimental mix design studies.

In this present work, a large bibliography data for different LWAC tested experimentally and published in the literature are used: De Larrard [7], Yang and Huang [8], and Ke Y et al. [9].

For LWAC test results investigated in this study, the volume fraction V_g of the lightweight aggregate varies from 0% (the matrix) to 47.8% and the contrast of the characteristics of the phases E_g/E_m (Young's modulus of lightweight aggregate and matrix) varies between 0.20% and 95% except for four types of concretes for which this ratio exceeds 1 because of a very low value of E_m ($E_g > E_m$) [7].

In order to determine the models likely to yield the lowest number of errors; the results of effective Young's modulus of LWAC obtained by using 07 two-phase composite models were compared with the experimental results obtained by De Larrard [7], Yang and Huang [8] and Ke Y et al. [9] (119 values) and discussed. Therefore, prediction possibilities using composite material models in determination of modulus of elasticity were sought and some suggestions were made accordingly to a statistical study.

PREDICTION OF ELASTIC MODULUS FOR LWAC

Two-phase composite models

More attention has been paid to lightweight aggregate concrete. The weakest component of LWAC is not the cement matrix or the interfacial transition zone (ITZ) but the aggregates. Therefore, the research about prediction model for LWAC's Young modulus is valuable for the concrete application [6]. Lo and Cui [13] illustrate that the "Wall effect" does not exist on the surface of expanded clay aggregates in lightweight concrete by SEM and BSEI imaging, resulting in a better bond and much more slender interfacial zone than the ordinary concrete [14]. So, materials which are produced can be considered a two-phase composite material.

The purpose of the composites materials approach is to develop a model that will enable expression of average properties of the mixtures through properties and volume fractions of its constituents [1, 11]. We look for the models to estimate the Young modulus for Lightweight Aggregate Concrete (LWAC) in terms of the properties and volume fractions of its constituents. These include the mortar matrix and the lightweight aggregate as reinforcing material. Before analyzing Lightweight Aggregate Concrete as a composite material, some assumptions must be considered.

First, the heterogeneous composite material (LWAC) is considered to be comprised of only two linear-elastic phases (the mortar and the lightweight aggregate). Second, the unit cell is assumed sufficiently large to account for the heterogeneity of the system, and the deferring geometry of the phases. However, it is extremely small so that the composite is described homogeneous on a macro scale [10, 15, 16]. Fig. 1 presents the models for an idealized unit cell of a two-phase composite material [10, 11, 17].

The LWAC comprises a dispersed phase of lightweight aggregate with a Young's modulus E_g and volume fraction V_g and a continuous phase of the mortar matrix, with a Young's modulus E_m and volume fraction V_m.

As explained by Gilormini and Brechert [18], the choice of a model is governed by several parameters including the geometry of the heterogonous medium, the mechanical contrast between the phases (E_g/E_m) and the volume fraction of reinforcement (V_g). Therefore, the equivalent homogenous behavior of LWAC depends of the characteristics of the mortar (matrix, phase m) and lightweight aggregate (dispersed phase, phase g).

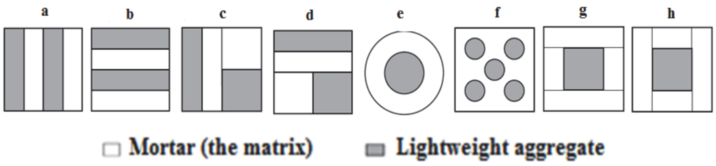

Figure 1: Composite models: (a) Voigt model, (b) Reuss model, (c) Popovics model, (d) Hirsch-Dougill model, (e) Hashin-Hansen model, (f) Maxwell model, (g) Counto1 model, (h) Counto2 model.

Voigt model [10, 19]:
$$E_{c\_Voigt} = E_m V_m + E_g V_g . \tag{1}$$

Reuss model [10, 19]:
$$E_{c\_Reuss} = \frac{E_m E_g}{E_g + V_g \left(E_m - E_g\right)} \tag{2}$$

Popovics model [10, 20]:
$$E_{c\_Popovics} = \frac{1}{2}\left(E_c^{Voigt} + E_c^{Reuss}\right). \tag{3}$$

Hirsch-Dougill model [10, 15, 21]:
$$E_{c\_Hirsch} = \frac{1}{2}\frac{1}{\left(\frac{1}{E_{c\_Voigt}} + \frac{1}{E_{c\_Reuss}}\right)} \tag{4}$$

Hashin-Hansen model [10, 11, 22]:
$$E_{c\_Hashin} = E_m\left[\frac{\left(E_m + E_g\right) + \left(E_g - E_m\right)V_g}{\left(E_m + E_g\right) - \left(E_g - E_m\right)V_g}\right]. \tag{5}$$

Maxwell model (dispersed phase) [10, 15]:
$$\begin{cases} E_{c\_Maxwell} = E_m\left[\dfrac{1 + 2V_g(\alpha - 1)/(\alpha + 2)}{1 - V_g(\alpha - 1)/(\alpha + 2)}\right] \\ \alpha = \dfrac{E_g}{E_m} \end{cases}. \tag{6}$$

Counto1 mod [17, 23]:
$$E_{c\_Counto1} = E_m\left[1 + \frac{V_g}{\sqrt{V_g} - V_g + \frac{E_m}{\left(E_g - E_m\right)}}\right]. \tag{7}$$

Counto2 model [17]:
$$E_{c\_Counto2} = E_m\left[1 + \frac{V_g}{\frac{E_g}{\left(E_g - E_m\right)} - \sqrt{V_g}}\right]. \tag{8}$$

Bache and Nepper-Christensen model [15, 24]:
$$E_{c\_Bache} = E_m^{V_m} \times E_g^{V_g} \tag{9}$$

The models of Voigt (Eq. and Reuss (Eq.2) provide the upper and lower bound of effective properties, respectively. It has been indicated [11, 19] that the upper bound relation of the parallel phase 'Voigt Model' might be applied as a first approximation to LWAC when $E_g < E_m$. However, the relation of the series phase 'Reuss model' validates the results of normal weight concrete with $E_g > E_m$ [11, 19].

The biphasic models of Popovics (Eq. 3) and Hirsch-Dougill (Eq. 4) originally designed for composites with particles (like concrete) [25], propose elastic modulus of the composite by combining the Voigt and Reuss models. Hirsh [21] derived an equation to express the elastic modulus of concrete in terms of empirical constant, and also provided some experimental results for the elastic modulus of concrete with different aggregates.

The model composite spheres was introduced by Hashin [25]. This model consists of a gradation of size of spherical particles embedded in a continuous matrix [26]. Hansen [19] evolved mathematical models to predict the elastic modulus of composite materials based on the individual elastic modulus and volume portion of the components. From the concentric model, Hashin-Hansen model (Eq. 5) supposes that the Poisson ratios of all phases and the composite are equal ($\nu_c=\nu_m=0.2$) [10, 19].

The dispersed phase model "Maxwell model", Eq. (6), describes concrete as a dispersed phase composite material [10, 11]. As a concentric model [10], Zhou et al. [17] indicate that a more realistic Counto1 model (Eq.7) can be considered (Fig. 1g). Another version of Counto's model (Eq.8) [17] is presented in Fig. 1h. The strength-based Bache and Nepper-Christensen model (Eq. 9), gives a geometric average of component properties in relation to their volume fractions V_m and V_g. This is a mathematical model with no physical meaning [17].

Experimental data from published literature

In this section, the bibliography data for different Lightweight Aggregate Concrete (LWAC) tested experimentally by De Larrard [7], Yang and Huang [8] and Ke Y et al. [9] are compiled in Tabs. 1, 2 and 3, respectively. The mechanical properties E_m, E_g are the Young's modulus of the matrix (mortar: phase m) and lightweight aggregate (dispersed phase, phase g), respectively. The Young's modulus of the composite obtained experimentally by De Larrard [7], Yang and Huang [8] and Ke Y et al. [9] are $E_{exp\_De\ Larrard}$, E_{exp_Yang} and E_{exp_Ke} respectively.

For LWAC test results by De Larrard [7] compiled in Tab. 1, it can be seen that the volume fraction Vg (the volume fraction of the lightweight aggregate) varies from 25.5% to 47.8% and that the contrast of the characteristics of the phases E_g/E_m varies between 27.74% and 95% except for four types of concretes for which this ratio exceeds 1 because of a very low value of E_m ($E_g > E_m$).

In their experimental program Yang and Huang [8] have tested three types of artificial coarse aggregates with Young's modulus of 6.01, 7.97 and 10.48GPa made of cement and fly ash with various combinations through a cold-pelletizing process. Each type of aggregate was mixed with four types of mortar matrices with a Young's modulus of 29.330, 28.130, 26.440 and 24.870GPa. This corresponds to a contrast ratio E_g/E_m between the two phases ranging from 20.49% to 42.14%. By supposing a Poisson's ratio of 0.2, the strength of coarse aggregate was computed from the elastic moduli of the components and the strength of concrete. The rate of lightweight aggregate volume fraction Vg was between 18% and 36%, the diameter of the gold aggregates assumed as spherical for all concretes tested had a d/D ratio in the order of (5/10) mm (Tab. 2). In their study, concrete was considered as a composite material in which coarse aggregate were embedded in a matrix of hardened mortar.

In the experimental study of Ke Y et al. [9], five LWAs are used: three expanded clay aggregates (A) of quasi-spherical shape (0/4 650A, 4/10 550A, 4/10 430 A) and two aggregates of expanded shale (S) of irregular shape (4/10 520S, 4/8 750 S). The three used matrices (called M8, M9 and M10) are made of Portland cement mortar CEM I 52.5 and normal sand 0/2 mm. Normal, high performance (HP) and very high performance (VHP) mortar matrices, were utilized for the realization of the concrete specimens tested by Ke Y et al. [9]. In their work, the volume fraction of aggregate was 0% (mortar), 12.5%, 25%, 37.5% and 45% with a contrast of the properties varying from 12.26% to 69.61%. The Young's modulus of the three mortar matrices were experimentally determined as 28.6, 33.2 and 35.4 GPa for M8, M9 and M10, respectively, as seen in Tab. 3 [9]. They correspond to a normal, HP and VHP matrix, respectively [9]. Mechanical properties of the lightweight aggregate are shown in Tab. 4 [9].

The elastic modulus of LWAC is estimated by utilizing some composite material models E_{c_anal} like Popovics, Hirsch-Dougill, Hashin-Hansen, Maxwell, Counto1, Counto2, and Bache and Nepper-Christensen (Eqs.(2)-(9)).

This Study try to figure out that these composite material models, mentioned above, got reliable prediction abilities for the modulus of elasticity of LWAC. The modulus of elasticity values were predicted utilizing the composite models and then, the predicted results were compared to the experimental results of De Larrard [7], Yang and Huang [8] and Ke Y et al. [9] respectively.

Ref. grav.	d/D (mm)	V_g	E_g (GPa)	E_m (GPa)	$E_{exp\_De\ Larrard}$ (GPa)
8 Argi 16	4-12	0.414	8	25.2	15.6
8 Isol S	3.15-8	0.414	13.1	25.2	19.2
8 Leca 7j.	4-10	0.414	7.6	23.5	14.1
8 Leca 28j.	4-10	0.414	7.6	25.2	15.7
8 Surex 675	6.3-10	0.414	16.2	25.2	21
8 Galex 7j.	3-8	0.425	33	20.6	25.5
8 Galex 28j.	3-8	0.425	33	21.9	25.8
9 Schiste 15j.	-	0.391	21	24.9	23.9
9 Schiste 28j.	-	0.391	21	25.6	23.4
9 Leca 1j.	4-10	0.414	8.6	11	10
9 Leca 2j.	4-10	0.414	8.6	15	12
9 Leca 7j.	4-10	0.414	8.6	20	13.9
9 Leca 28j.	4-10	0.414	8.6	25.2	16
9 Leca 90j.	4-10	0.414	8.6	31	17.8
9 Surex 1j.	6.3-10	0.414	19	11	14.6
9 Surex 2j.	6.3-10	0.414	19	15	17.1
9 Surex 7j.	6.3-10	0.414	19	20	20.6
9 Surex 28j.	6.3-10	0.414	19	25.2	22.3
9 Surex 90j.	6.3-10	0.414	19	31	22.6
3 LWC1 Crush	10-20	0.403	13.6	33.2	23
3 LWC1 Crush	10-20	0.403	13.6	33.2	24.3
3 LWC1 Pellet	5-20	0.414	14	32.8	22.7
3 LWC1 Pellet	5-20	0.414	14	32.8	24.3
3 HSLWC Pel.	5-20	0.255	14	38.5	27.6
3 HSLWC Pel.	5-20	0.255	14	38.5	28.3
1 Liapor	0-16	0.473	21.5	29.7	25.5
2 Liapor	4-16	0.432	21.5	27.9	24.8
16 Javron	4-10	0.463	16	23.8	19.7
16 G/S -0.2	4-10	0.443	16	24.6	19.9
16 G/S +0.2	4-10	0.478	16	23.1	19.7
16 EAU +	4-10	0.473	16	26.6	20.9
16 EAU –	4-10	0.453	16	21.1	18.1

Table 1: Characteristics of LWAC tested by De Larrard [7].

Ref. grav.	d/D (mm)	V_g	E_g (GPa)	E_m (GPa)	E_{exp_Yang} (GPa)
A3	5-10	0.18	6.01	29.33	23.020
A4		0.24	6.01	28.13	20.600
A5		0.30	6.01	26.44	18.210
A6		0.36	6.01	24.87	15.800
B3		0.18	7.97	29.33	23.790
B4		0.24	7.97	28.13	21.530
B5		0.30	7.97	26.44	19.010
B6		0.36	7.97	24.87	17.220
C3		0.18	10.48	29.33	24.660
C4		0.24	10.48	28.13	22.580
C5		0.30	10.48	26.44	20.320
C6		0.36	10.48	24.87	18.650

Table 2: Characteristics of LWAC tested by Chung-Chia Yang and Ran Huang [8].

	E_{exp_Ke}	E_0^{Ke}	$E_{0.125}^{Ke}$	$E_{0.250}^{Ke}$	$E_{0.375}^{Ke}$	$E_{0.450}^{Ke}$
M8	0/4 650A	28.588	23.539	20.665	16.743	15.669
	4/10 550A		26.157	21.680	17.900	16.606
	4/10 430A		24.900	21.391	17.293	15.699
	4/10 520S		25.135	22.471	19.428	18.286
	4/8 750S		27.367	26.262	25.281	24.324
M9	0/4 650A	33.183	29.396	23.712	19.871	17.175
	4/10 550A		29.159	24.934	21.358	19.696
	4/10 430A		27.568	23.778	20.818	18.935
	4/10 520S		29.480	26.521	22.188	20.184
	4/8 750S		31.931	30.987	30.146	29.311
M10	0/4 650A	35.397	31.147	26.753	22.427	20.346
	4/10 550A		32.089	27.991	23.684	21.724
	4/10 430A		30.220	26.033	22.296	20.082
	4/10 520S		32.783	27.998	24.340	22.024
	4/8 750S		34.213	33.845	32.945	33.002

Table 3: Characteristics of LWAC tested by Ke Y et al. [9] (GPa).

LWA	0/4 650A	4/10 550A	4/10 430A	4/10 520S	4/8 750S
Eg	6.870	6.790	4.340	6.490	19.900

Table 4: Mechanical properties of lightweight aggregate tested by Ke Y et al. [9] (GPa).

RESULTS AND DISCUSSIONS

Comparative analysis

Comparison between the estimative results of effective elastic modulus of LWAC obtained as a result of calculations of the Eqns. (2-9) and those of experimental data have been presented in Tabs. 5, 6 and 7 respectively.

A confrontation of LWAC Young's modulus between experimental results in [7, 8, 9] and the predictions of 07 composite models material models are shown in Fig. 2, Fig. 3 and Fig. 4 respectively.

The differences between the various predictive composite models and the experimental results in [7, 8, 9] have been computed according to the proportion of reinforcement Vg in LWAC. When the volume fraction of aggregates Vg grows, the errors between the predictions and the experimental results increase for all composite material models. Since the weakest component of LWAC is not the cement matrix but the lightweight aggregates, the effect of volume fraction of lightweight aggregate on Young's modulus of LWAC is very clear. The increase in the volume fraction of lightweight aggregates Vg substantially reduces the Young's modulus of the LWAC.

To compare the experimental and predicted Young's modulus of LWAC, the error percentage $|\Delta E|$. is determined using the following expression:

$$\Delta E(\%) = \left(\frac{E_{c\_anal} - E_{exp}}{E_{exp}}\right) \times 100 \tag{10}$$

$|\Delta E| = \mathrm{Abs}(\Delta E)$, Absolute value of ΔE

It appears for first time that all models are generally suitable for predicting the modulus of elasticity of the LWAC.

Tabs. 8-9-10 give the error percentages of the composite material models and experimental results in [7, 8, 9] respectively. In order to choose the models which have good performances, the error percentages below 10% are chosen as desired range and the model's error percentages below this value are indicated in bold. Therefore, the models which verified this condition have been underlined.

Ref. grav.	$E_{exp\_De\ Larrard}$	$E_{c\_Popovics}$	E_{c_Hirsch}	E_{c_Hashin}	$E_{c\_Maxwell}$	$E_{c\_Counto1}$	$E_{c\_Counto2}$	E_{c_Bache}
8 Argi 16	15.6	15.71	15.35	16.30	16.98	16.76	15.79	15.67
8 Isol S	19.2	19.21	19.16	19.37	19.67	19.57	19.16	19.22
8 Leca 7j.	14.1	14.76	14.44	15.29	15.91	15.71	14.82	14.73
8 Leca 28j.	15.7	15.39	14.98	16.04	16.76	16.52	15.50	15.34
8 Surex 675	21	20.98	20.97	21.04	21.19	21.14	20.93	20.99
8 Galex 7j.	25.5	25.19	25.17	25.09	25.32	25.24	24.96	25.17
8 Galex 28j.	25.8	26.09	26.07	26.02	26.20	26.13	25.91	26.07
9 Schiste 15j.	23.9	23.29	23.29	23.30	23.33	23.32	23.28	23.30
9 Schiste 28j.	23.4	23.69	23.69	23.70	23.73	23.72	23.67	23.69
9 Leca 1j.	10	9.93	9.93	9.94	9.96	9.95	9.92	9.93
9 Leca 2j.	12	11.91	11.89	11.97	12.11	12.06	11.87	11.91
9 Leca 7j.	13.9	14.10	14.00	14.33	14.69	14.57	14.08	14.10
9 Leca 28j.	16	16.17	15.88	16.68	17.31	17.10	16.22	16.15
9 Leca 90j.	17.8	18.32	17.69	19.24	20.20	19.88	18.51	18.23
9 Surex 1j.	14.6	13.82	13.80	13.73	13.90	13.84	13.63	13.79
9 Surex 2j.	17.1	16.54	16.54	16.54	16.57	16.56	16.51	16.54
9 Surex 7j.	20.6	19.58	19.58	19.58	19.58	19.58	19.58	19.58
9 Surex 28j.	22.3	22.42	22.41	22.43	22.50	22.48	22.39	22.42
9 Surex 90j.	22.6	25.30	25.28	25.40	25.63	25.55	25.24	25.31
3 LWC1 Crush	23	23.15	22.95	23.61	24.25	24.05	23.13	23.17
3 LWC1 Crush	24.3	23.15	22.95	23.61	24.25	24.05	23.13	23.17
3 LWC1 Pellet	22.7	23.05	22.88	23.45	24.04	23.84	23.02	23.06
3 LWC1 Pellet	24.3	23.05	22.88	23.45	24.04	23.84	23.02	23.06
3 HSLWC Pel.	27.6	29.44	29.17	30.31	31.08	31.07	29.38	29.75
3 HSLWC Pel.	28.3	29.44	29.17	30.31	31.08	31.07	29.38	29.75
1 Liapor	25.5	25.49	25.49	25.52	25.62	25.58	25.46	25.49
2 Liapor	24.8	24.93	24.93	24.94	25.01	24.98	24.90	24.93
16 Javron	19.7	19.80	19.80	19.84	19.96	19.91	19.77	19.80
16 G/S -0.2	19.9	20.33	20.32	20.38	20.53	20.47	20.29	20.33
16 G/S +0.2	19.7	19.38	19.38	19.41	19.51	19.47	19.35	19.38
16 EAU +	20.9	20.92	20.90	21.00	21.21	21.12	20.87	20.92
16 EAU –	18.1	18.61	18.61	18.63	18.68	18.66	18.59	18.61

Table 5: Modulus of elasticity of LWAC predicted by various composite models compared with the experimental results of De Larrard [7](GPa).

Ref. grav.	E_{exp_Yang}	$E_{c\_Popovics}$	E_{c_Hirsch}	E_{c_Hashin}	$E_{c\_Maxwell}$	$E_{c\_Counto1}$	$E_{c\_Counto2}$	E_{c_Bache}
A3	23.020	21.201	20.471	23.102	23.967	24.121	21.589	22.049
A4	20.600	18.879	18.055	20.559	21.501	21.523	19.265	19.422
A5	18.210	16.701	15.920	18.039	18.963	18.860	17.018	16.953
A6	15.800	14.879	14.190	15.905	16.770	16.570	15.124	14.915
B3	23.790	22.635	22.276	23.848	24.530	24.653	22.709	23.198
B4	21.530	20.398	19.987	21.481	22.218	22.236	20.504	20.784
B5	19.010	18.248	17.863	19.106	19.820	19.739	18.340	18.451
B6	17.220	16.445	16.112	17.095	17.754	17.600	16.515	16.510
C3	24.660	24.047	23.898	24.723	25.214	25.305	23.944	24.370
C4	22.580	21.963	21.794	22.568	23.093	23.106	21.900	22.195
C5	20.320	19.900	19.746	20.369	20.867	20.810	19.854	20.030
C6	18.650	18.166	18.039	18.512	18.960	18.854	18.130	18.221

Table 6: Modulus of elasticity of LWAC predicted by various composite models compared with the experimental results of Yang and Huang [8](GPa).

Ref. grav.	E_{exp_Ke}	$E_{c\_Popovics}$	E_{c_Hirsch}	E_{c_Hashin}	$E_{c\_Maxwell}$	$E_{c\_Counto1}$	$E_{c\_Counto2}$	E_{c_Bache}
M8 0/4 650 A	28.588	28.588	28.588	28.588	28.588	28.588	28.588	28.588
	23.539	23.182	22.870	24.522	25.101	25.303	23.253	23.921
	20.665	19.563	18.903	20.996	21.886	21.886	19.833	20.016
	16.743	16.762	15.954	17.908	18.912	18.652	17.044	16.748
	15.669	15.308	14.504	16.234	17.233	16.845	15.556	15.050
M8 4/10 550 A	28.588	28.588	28.588	28.588	28.588	28.588	28.588	28.588
	26.157	23.132	22.810	24.499	25.084	25.288	23.215	23.886
	21.680	19.499	18.820	20.956	21.855	21.855	19.781	19.957
	17.900	16.693	15.863	17.857	18.870	18.607	16.984	16.675
	16.606	15.237	14.413	16.176	17.185	16.793	15.492	14.971
M8 4/10 430 A	28.588	28.588	28.588	28.588	28.588	28.588	28.588	28.588
	24.900	21.195	20.297	23.769	24.561	24.828	21.878	22.586
	21.391	17.227	15.597	19.699	20.895	20.895	18.062	17.845
	17.293	14.366	12.534	16.216	17.543	17.203	15.041	14.099
	15.699	12.906	11.142	14.357	15.667	15.162	13.450	12.240
M8 4/10 520 S	28.588	28.588	28.588	28.588	28.588	28.588	28.588	28.588
	25.135	22.939	22.576	24.414	25.022	25.233	23.067	23.752
	22.471	19.253	18.499	20.808	21.740	21.740	19.583	19.733
	19.428	16.429	15.516	17.662	18.711	18.439	16.756	16.395
	18.286	14.967	14.063	15.959	17.002	16.597	15.250	14.669
M8 4/10 750 S	28.588	28.588	28.588	28.588	28.588	28.588	28.588	28.588
	27.367	27.305	27.304	27.335	27.396	27.421	27.236	27.322
	26.262	26.095	26.091	26.137	26.237	26.237	26.027	26.113
	25.281	24.948	24.942	24.988	25.110	25.077	24.895	24.957
	24.324	24.286	24.280	24.322	24.448	24.397	24.244	24.288
M9 0/4 650 A	33.183	33.183	33.183	33.183	33.183	33.183	33.183	33.183
	29.396	26.167	25.636	28.147	28.904	29.166	26.435	27.253
	23.712	21.778	20.708	23.821	24.978	24.978	22.283	22.383
	19.871	18.468	17.195	20.065	21.363	21.028	18.937	18.384
	17.175	16.763	15.512	18.040	19.328	18.829	17.160	16.336
M9 4/10 550 A	33.183	33.183	33.183	33.183	33.183	33.183	33.183	33.183
	29.159	26.108	25.562	28.123	28.887	29.151	26.392	27.214
	24.934	21.707	20.611	23.780	24.947	24.947	22.228	22.318
	21.358	18.394	17.093	20.012	21.320	20.982	18.874	18.303
	19.696	16.688	15.410	17.981	19.279	18.776	17.094	16.250
M9 4/10 430 A	33.183	33.183	33.183	33.183	33.183	33.183	33.183	33.183
	27.568	23.852	22.477	27.365	28.353	28.684	24.953	25.733
	23.778	19.220	16.848	22.485	23.970	23.970	20.430	19.955
	20.818	15.934	13.338	18.333	19.975	19.555	16.871	15.475
	18.935	14.259	11.782	16.127	17.743	17.121	15.001	13.285
M9 4/10 520 S	33.183	33.183	33.183	33.183	33.183	33.183	33.183	33.183
	29.480	25.881	25.274	28.034	28.824	29.095	26.232	27.060
	26.521	21.435	20.234	23.627	24.830	24.830	22.020	22.067
	22.188	18.113	16.699	19.812	21.158	20.811	18.638	17.996
	20.184	16.405	15.021	17.759	19.093	18.577	16.845	15.923
M9 4/10 750 S	33.183	33.183	33.183	33.183	33.183	33.183	33.183	33.183
	31.931	31.075	31.069	31.170	31.303	31.355	30.943	31.128
	30.987	29.150	29.133	29.276	29.493	29.493	29.031	29.201
	30.146	27.371	27.346	27.490	27.749	27.679	27.287	27.393
	29.311	26.362	26.335	26.466	26.732	26.626	26.298	26.363
M10 0/4 650 A	35.397	35.397	35.397	35.397	35.397	35.397	35.397	35.397
	31.147	27.567	26.907	29.889	30.735	31.026	27.953	28.838
	26.753	22.816	21.515	25.176	26.466	26.466	23.452	23.494
	22.427	19.271	17.742	21.098	22.541	22.169	19.839	19.141
	20.346	17.450	15.953	18.905	20.334	19.781	17.925	16.926
M10 4/10 550 A	35.397	35.397	35.397	35.397	35.397	35.397	35.397	35.397
	32.089	27.504	26.826	29.865	30.718	31.011	27.909	28.796

	27.991	22.742	21.411	25.135	26.434	26.434	23.396	23.426
	23.684	19.195	17.633	21.045	22.497	22.123	19.776	19.057
	21.724	17.373	15.847	18.845	20.285	19.727	17.858	16.837
	35.397	35.397	35.397	35.397	35.397	35.397	35.397	35.397
M10	30.220	25.099	23.460	29.096	30.180	30.541	26.427	27.229
4/10 430 A	26.033	20.162	17.394	23.825	25.451	25.451	21.564	20.946
	22.296	16.680	13.683	19.351	21.146	20.687	17.748	16.112
	20.082	14.904	12.055	16.977	18.742	18.063	15.746	13.766
	35.397	35.397	35.397	35.397	35.397	35.397	35.397	35.397
M10	32.783	27.261	26.510	29.775	30.654	30.954	27.743	28.634
4/10 520 S	27.998	22.459	21.007	24.980	26.316	26.316	23.183	23.163
	24.340	18.906	17.218	20.843	22.334	21.950	19.536	18.737
	22.024	17.085	15.439	18.621	20.098	19.527	17.606	16.498
	35.397	35.397	35.397	35.397	35.397	35.397	35.397	35.397
M10	34.123	32.858	32.847	33.001	33.176	33.244	32.695	32.938
4/10 750 S	33.845	30.576	30.546	30.762	31.047	31.047	30.437	30.651
	32.945	28.491	28.449	28.665	29.002	28.912	28.398	28.522
	33.002	27.317	27.273	27.469	27.815	27.677	27.249	27.316

Table 7: Modulus of elasticity of LWAC predicted by various composite models compared with the experimental results of Ke Y et al. [9](GPa).

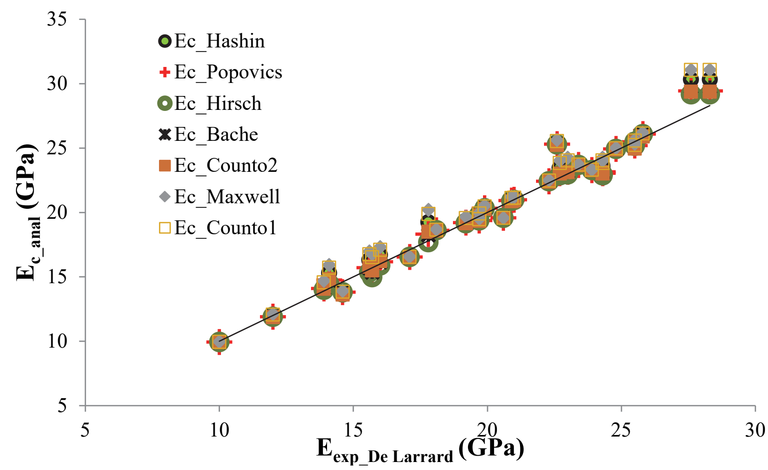

Figure 2: Confrontation of LWAC Young's modulus between experimental results in [7] and the predictions of 07 composite material models.

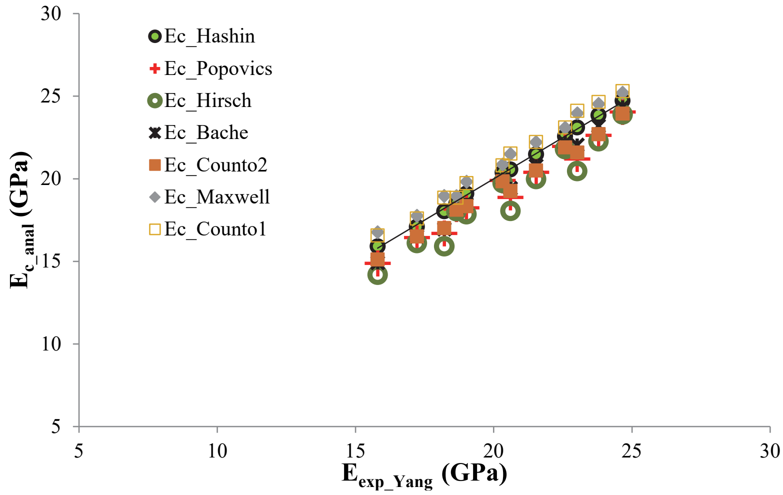

Figure 3: Confrontation of LWAC Young's modulus between experimental results in [8] and the predictions of 07 composite material models.

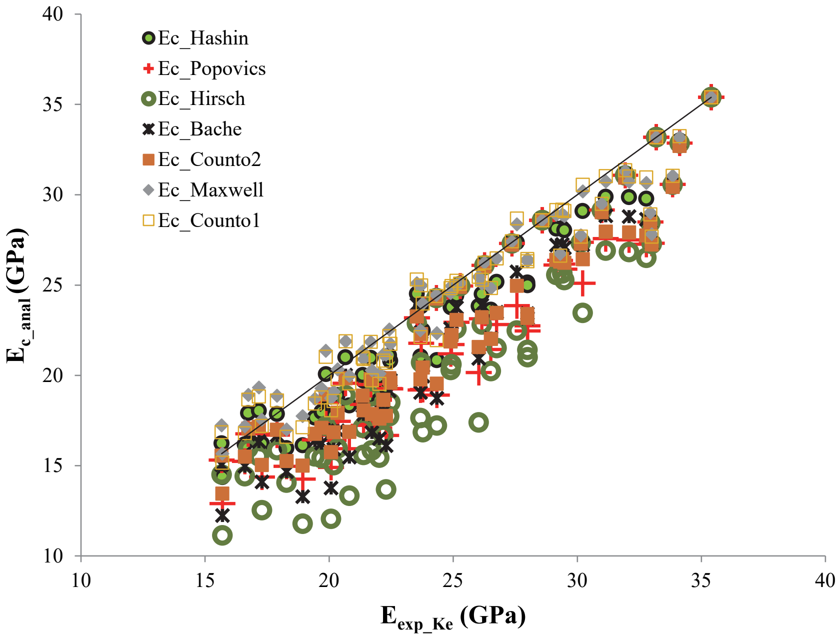

Figure 4: Confrontation of LWAC Young's modulus between experimental results in [9] and the predictions of 07 composite material models.

Ref. grav.	Popovics	Hirsch-Dougill	Hashin-Hansen	Maxwell	Counto1	Counto2	Bache and Nepper-Christensen
8 Argi 16	**0.68**	**-1.62**	**4.48**	**8.87**	**7.42**	**1.21**	**0.46**
8 Isol S	**0.05**	**-0.21**	**0.89**	**2.46**	**1.93**	**-0.23**	**0.11**
8 Leca 7j.	**4.65**	**2.40**	**8.44**	12.87	11.40	**5.14**	**4.44**
8 Leca 28j.	**-1.98**	**-4.61**	**2.16**	**6.77**	**5.24**	**-1.29**	**-2.28**
8 Surex 675	**-0.09**	**-0.15**	**0.18**	**0.93**	**0.67**	**-0.33**	**-0.06**
8 Galex 7j.	**-1.21**	**-1.28**	**-1.60**	**-0.69**	**-1.03**	**-2.13**	**-1.30**
Galex 28j.	**1.11**	**1.06**	**0.84**	**1.55**	**1.28**	**0.43**	**1.04**
9 Schiste 15j.	**-2.53**	**-2.53**	**-2.52**	**-2.41**	**-2.44**	**-2.59**	**-2.53**
9 Schiste 28j.	**1.24**	**1.24**	**1.27**	**1.42**	**1.38**	**1.16**	**1.25**
9 Leca 1j.	**-0.66**	**-0.67**	**-0.61**	**-0.38**	**-0.46**	**-0.77**	**-0.66**
9 Leca 2j.	**-0.76**	**-0.90**	**-0.23**	**0.91**	**0.52**	**-1.04**	**-0.71**
9 Leca 7j.	**1.42**	**0.70**	**3.12**	**5.68**	**4.82**	**1.27**	**1.45**
9 Leca 28j.	**1.05**	**-0.76**	**4.27**	**8.20**	**6.90**	**1.36**	**0.92**
9 Leca 90j.	**2.93**	**-0.63**	**8.07**	13.50	11.71	**3.98**	**2.42**
9 Surex 1j.	**-5.36**	**-5.48**	**-5.96**	**-4.79**	**-5.22**	**-6.64**	**-5.53**
9 Surex 2j.	**-3.25**	**-3.26**	**-3.30**	**-3.08**	**-3.15**	**-3.44**	**-3.26**
9 Surex 7j.	**-4.95**	**-4.95**	**-4.95**	**-4.94**	**-4.95**	**-4.96**	**-4.95**
9 Surex 28j.	**0.52**	**0.52**	**0.60**	**0.91**	**0.81**	**0.39**	**0.54**
9 Surex 90j.	11.96	11.87	12.37	13.39	13.04	11.67	12.00
LWC1 Crush	**0.66**	**-0.21**	**2.66**	**5.43**	**4.56**	**0.57**	**0.74**
3 LWC1 Crush	**-4.73**	**-5.55**	**-2.83**	**-0.21**	**-1.03**	**-4.81**	**-4.65**
3 LWC1 Pellet	**1.54**	**0.80**	**3.29**	**5.88**	**5.01**	**1.40**	**1.57**
3 LWC1 Pellet	**-5.15**	**-5.84**	**-3.51**	**-1.09**	**-1.90**	**-5.28**	**-5.12**
3 HSLWC Pel.	**6.65**	**5.68**	**9.82**	12.61	12.57	**6.45**	**7.78**
3 HSLWC Pel.	**4.02**	**3.06**	**7.11**	**9.82**	**9.79**	**3.81**	**5.11**
1 Liapor	**-0.03**	**-0.05**	**0.07**	**0.49**	**0.30**	**-0.17**	**-0.04**
2 Liapor	**0.52**	**0.51**	**0.57**	**0.85**	**0.74**	**0.40**	**0.52**
16 Javron	**0.52**	**0.48**	**0.71**	**1.34**	**1.07**	**0.34**	**0.52**
16 G/S -0.2	**2.16**	**2.11**	**2.41**	**3.14**	**2.86**	**1.94**	**2.17**
16 G/S +0.2	**-1.61**	**-1.64**	**-1.47**	**-0.94**	**-1.18**	**-1.77**	**-1.62**
16 EAU +	**0.09**	**-0.01**	**0.47**	**1.48**	**1.04**	**-0.13**	**0.07**
16 EAU –	**2.84**	**2.83**	**2.91**	**3.22**	**3.09**	**2.72**	**2.84**

Table 8: Error percentages of composite models and experimental results in [7] (%).

It can be observed from Tab. 8 that:
For the Hirsch-Dougill, Popovics, Bache and Nepper-Christensen, Counto2 and Hashin-Hansen models, 31/32 cases lead to |ΔE| smaller than 10%. All these models give a maximum |ΔE| for a contrast equal to E_g/E_m= 61.29% with a volume fraction of the aggregates Vg = 41.4% (aggregate: 9 Surex 90j.).
For the Popovics and Bache and Nepper-Christensen models, 28/32 cases give |ΔE| smaller than 5% and 3/32 cases give |ΔE| smaller than 10%. ΔE ranges from –5.53% to 12.00%.
For the Hirsch-Dougill and Counto2 models, 27/32 cases give |ΔE| smaller than 5% and 4/32 cases give |ΔE| smaller than 10%. ΔE ranges from –5.84% to 11.87%.
For the Hashin-Hansen model, 31/32 cases lead to |ΔE| smaller than 10%. Hence, for 26/32 cases it is smaller than 5%. ΔE ranges from –5.96% to 12.37%.
For the Counto1 and Maxwell models, 28/32 cases lead to |ΔE| smaller than 10%, ΔE ranges from –4.94% to 13.50%.
It is clear from these results that the selected models are able to effectively estimate the Young's modulus of LWAC tested by De Larrard [7] with a max difference |ΔE| equal to 13.50% (obtained by the Maxwell model) using 32 measurements.

Ref. grav.	Popovics	Hirsch-Dougill	Hashin-Hansen	Maxwell	Counto1	Counto2	Bache and Nepper-Christensen
A3	**-7.90**	-11.07	**0.36**	**4.11**	**4.78**	**-6.22**	**-4.22**
A4	**-8.36**	-12.35	**-0.20**	**4.37**	**4.48**	**-6.48**	**-5.72**
A5	**-8.29**	-12.57	**-0.94**	**4.13**	**3.57**	**-6.54**	**-6.90**
A6	**-5.83**	-10.19	**0.66**	6.14	**4.87**	**-4.28**	**-5.60**
B3	**-4.85**	**-6.36**	**0.25**	**3.11**	**3.63**	**-4.54**	**-2.49**
B4	**-5.26**	**-7.17**	**-0.23**	**3.20**	**3.28**	**-4.77**	**-3.47**
B5	**-4.01**	**-6.04**	**0.50**	**4.26**	**3.84**	**-3.53**	**-2.94**
B6	**-4.50**	**-6.44**	**-0.72**	**3.10**	**2.21**	**-4.09**	**-4.12**
C3	**-2.49**	**-3.09**	**0.26**	**2.25**	**2.62**	**-2.90**	**-1.17**
C4	**-2.73**	**-3.48**	**-0.05**	**2.27**	**2.33**	**-3.01**	**-1.70**
C5	**-2.07**	**-2.82**	**0.24**	**2.69**	**2.41**	**-2.29**	**-1.42**
C6	**-2.59**	**-3.28**	**-0.74**	**1.66**	**1.09**	**-2.79**	**-2.30**

Table 9: Error percentages of composite models and experimental results in [8] (%).

Compared with the experimental data of Yang and Huang [8] (Tab. 6, Tab. 9), Bache and Nepper-Christensen, Counto2, Popovics, Hirsch-Dougill, underestimate the measured Young's modulus. On the other hand, the Maxwell and Counto1 models overestimate the Young's modulus measured by Yang and Huang [8].
As seen in Tab. 9, for the Hashin-Hansen and Counto1 models, 12/12 cases give |ΔE| smaller than 5%. |ΔE| ranges from 0.94% to 4.87%. The Maxwell gives 12/12 cases smaller than 10% and 11/12 smaller than 5%. |ΔE| ranges from 1.66% to 6.14%. In all composite models, the error percentages differ between 0.05% and 12.57%.
It can be seen that the most accurate models are those of Hashin-Hansen, Counto1 and Maxwell which give less errors percentages.
The predictions of the LWAC Young's modulus using the 07 composite material models are compared with experimental data of Ke Y et al. [9] (Tab. 7 and Tab.10) in Fig. 4. All selected composite models appear applicable to predict the Young's modulus of LWAC tested by Ke Y et al [9].
For the Maxwell model, 50/75 cases give |ΔE| smaller than 5% and 22/75 cases smaller than 10%. This means that 72/75 cases have |ΔE| smaller than 10%. This model converges on the experimental values measured by Ke Y et al. [9] with an absolute maximum difference |ΔE| of 15.72%.
For the Counto1 model, 47/75 cases lead to |ΔE| smaller than 5% and 23/75 smaller than 10%, which gives 70/75 cases with |ΔE| smaller than 10%, with a maximum difference of 16.14%.
For the Hashin-Hansen model, 59/75 cases give |ΔE| smaller than 10% of which 38/75 cases smaller than 5%. ΔE ranges from 0% to 16.77%.
For the Counto2 model, 36/75 cases have |ΔE| smaller than 10%, of which 27 cases have |ΔE| smaller than 5%, with the maximum difference of 21.59%.
For the Popovics model, 34/75 cases give |ΔE| smaller than 10% with 25/75 cases smaller than 5%. The maximum ΔE is 25.78%.

Ref. grav.	Popovics	Hirsch-Dougill	Hashin-Hansen	Maxwell	Counto1	Counto2	Bache and Nepper-Christensen
M8 0/4 650 A	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**
	4.18	**-2.84**	**1.52**	**6.63**	**7.49**	**1.21**	**1.62**
	1.60	**-8.53**	**5.33**	**5.91**	**5.91**	**4.03**	**-3.14**
	6.96	**-4.71**	**0.12**	12.96	11.40	**1.80**	**0.03**
	3.61	**-7.43**	**2.31**	**9.98**	**7.50**	**0.72**	**-3.95**
M8 4/10 550 A	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**
	6.34	-12.80	11.56	**4.10**	**3.32**	11.25	**-8.68**
	3.34	-13.19	10.06	**0.81**	**0.81**	**8.76**	**-7.95**
	0.24	-11.38	**6.74**	**5.42**	**3.95**	**5.12**	**-6.84**
	2.59	-13.21	**8.25**	**3.49**	**1.13**	**6.71**	**-9.85**
M8 4/10 430 A	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**
	4.54	**-18.49**	14.88	**1.36**	**0.29**	12.14	**-9.29**
	7.91	-27.09	19.47	**2.32**	**2.32**	15.56	-16.58
	6.23	-27.52	16.93	**1.45**	**0.52**	13.02	-18.47
	8.55	-29.03	17.79	**0.20**	**3.42**	14.33	-22.04
M8 4/10 520 S	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**
	2.87	-10.18	**8.74**	**0.45**	**0.39**	**8.23**	**-5.50**
	7.40	-17.68	14.32	**3.25**	**3.25**	12.85	-12.18
	9.09	-20.14	15.44	**3.69**	**5.09**	13.75	-15.61
	12.72	-23.09	18.15	**7.02**	**9.24**	16.60	-19.78
M8 4/10 750 S	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**
	0.12	**-0.23**	**0.23**	**0.11**	**0.20**	**0.48**	**-0.16**
	0.48	**-0.65**	**0.63**	**0.09**	**0.09**	**0.90**	**-0.57**
	1.16	**-1.34**	**1.32**	**0.68**	**0.81**	**1.53**	**-1.28**
	0.01	**-0.18**	**0.16**	**0.51**	**0.30**	**0.33**	**-0.15**
M9 0/4 650 A	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**
	4.25	-12.79	10.99	**1.67**	**0.78**	10.07	**-7.29**
	0.46	-12.67	**8.16**	**5.34**	**5.34**	**6.03**	**-5.60**
	0.98	-13.46	**7.06**	**7.51**	**5.82**	**4.70**	**-7.49**
	5.04	**-9.68**	**2.40**	12.54	**9.63**	**0.09**	**-4.89**
M9 4/10 550 A	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**
	3.55	-12.34	10.46	**0.93**	**0.03**	**9.49**	**-6.67**
	4.63	-17.34	12.94	**0.05**	**0.05**	10.85	-10.49
	6.30	-19.97	13.88	**0.18**	**1.76**	11.63	-14.30
	8.71	-21.76	15.27	**2.12**	**4.67**	13.21	-17.50
M9 4/10 430 A	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**
	0.74	-18.47	13.48	**2.85**	**4.05**	**9.48**	**-6.66**
	5.44	-29.14	19.17	**0.81**	**0.81**	14.08	-16.08
	11.94	-35.93	23.46	**4.05**	**6.07**	18.96	-25.67
	14.83	-37.78	24.69	**6.29**	**9.58**	20.77	-29.84
M9 4/10 520 S	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**
	4.90	-14.27	12.21	**2.23**	**1.31**	11.02	**-8.21**
	10.91	-23.71	19.18	**6.38**	**6.38**	16.97	-16.79
	10.71	-24.74	18.37	**4.64**	**6.21**	16.00	-18.90
	11.97	-25.54	18.68	**5.36**	**7.92**	16.50	-21.07
M9 4/10 750 S	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**
	2.38	**-2.70**	**2.68**	**1.97**	**1.80**	**3.09**	**-2.51**
	5.52	**-5.98**	**5.93**	**4.82**	**4.82**	**6.31**	**-5.76**
	8.81	**-9.29**	**9.21**	**7.95**	**8.18**	**9.48**	**-9.13**
	9.71	-10.15	10.06	**8.80**	**9.16**	10.28	-10.06
M10 0/4 650 A	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**
	4.04	-13.61	11.50	**1.32**	**0.39**	10.26	**-7.41**
	5.89	-19.58	14.71	**1.07**	**1.07**	12.34	-12.18
	5.92	-20.89	14.07	**0.51**	**1.15**	11.54	14.65
	7.08	-21.59	14.24	**0.06**	**2.78**	11.90	-16.81

M10 4/10 550 A	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**
	6.93	-16.40	14.29	**4.27**	**3.36**	13.03	-10.26
	10.20	-23.51	18.75	**5.56**	**5.56**	16.42	-16.31
	11.14	-25.55	18.95	**5.01**	**6.59**	16.50	-19.54
	13.25	-27.05	20.03	**6.63**	**9.19**	17.80	-22.49
M10 4/10 430 A	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**
	3.72	-22.37	16.94	**0.13**	**1.06**	12.55	**-9.90**
	8.48	-33.18	22.55	**2.24**	**2.24**	17.16	-19.54
	13.21	-38.63	25.19	**5.16**	**7.22**	20.40	-27.73
	15.46	-39.97	25.78	**6.67**	10.05	21.59	-31.45
M10 4/10 520 S	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**
	9.18	-19.13	16.85	**6.49**	**5.58**	15.37	-12.66
	10.78	-24.97	19.78	**6.01**	**6.01**	17.20	-17.27
	14.37	-29.26	22.32	**8.24**	**9.82**	19.74	-23.02
	15.45	-29.90	22.42	**8.74**	11.34	20.06	-25.09
M10 4/10 750 S	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**
	3.29	**-3.74**	**3.71**	**2.77**	**2.57**	**4.18**	**-3.47**
	9.11	**-9.75**	**9.66**	**8.27**	**8.27**	10.07	**-9.44**
	12.99	-13.65	13.52	11.97	12.24	13.80	-13.43
	16.77	-17.36	17.22	15.72	16.14	17.43	-17.23

Table 10: Error percentages of composite models and experimental results in [9] (%).

The Bache and Nepper-Christensen and Hirsch-Dougill models underestimate the Young's modulus of LWAC measured in [3]. Bache and Nepper-Christensen model, 43/75 cases give |ΔE| smaller than 10% and ΔE ranges from 31.45% to 1.62%.

For the Hirsch-Dougill model, 29/75 cases give |ΔE| smaller than 10% with 23 cases smaller than 5%.

It can be seen by examining Fig. 4 that the most accurate models are those of Maxwell, Counto1 and Hashin-Hansen which give less errors percentages (Fig. 4 and Tab. 10).

Statistical analysis

In order to confirm what has been announced previously and distinguish the most suitable model for predicting the effective elasticity modulus of the LWAC, a global statistical study was carried out on all the experimental values of the three researchers (119 measures). To this effect, the mean values and standard deviation for all composite models used in this study and experimental data are calculated as seen in Tab. 10.

	Popovics	Hirsch-Dougill	Hashin-Hansen	Maxwell	Counto1	Counto2	Bache and Nepper-Christensen
Mean Values	-6.90	-9.66	-2.72	0.29	-0.23	-5.94	-6.42
Standard deviation	8.24	11.14	5.72	5.27	5.32	7.12	8.46

Table 10: Mean values and standard deviation of composite models and all experimental data in [7, 8, 9].

Fig. 5 shows the normal distribution approximation of error percentage for all 07 composite analytical models. Every estimator has a pick on the mean value and a standard deviation presented by a tight or wide curve. As expected, the Maxwell, Counto1 and Hashin-Hansen composite models provide a good prediction of experimental Young's modulus of all LWAC tested by De Larrard [7], Yang and Huang [8] and Ke Y et al. [9] (119 values) with a maximum volume fraction of aggregates Vg equal to 49.37%.

It is clear from curves of Fig. 5 that the best curves that fit experimental data are respectively Maxwell, Counto1 and Hashin-Hansen models because the mean values are closest to zero than others. It is also important to notice that the standard deviation of both models (Maxwell 5.27, Counto1 5.32 and Hashin-Hansen 5.72) are tight which indicates that there is a high concentration of estimated values around of zero.

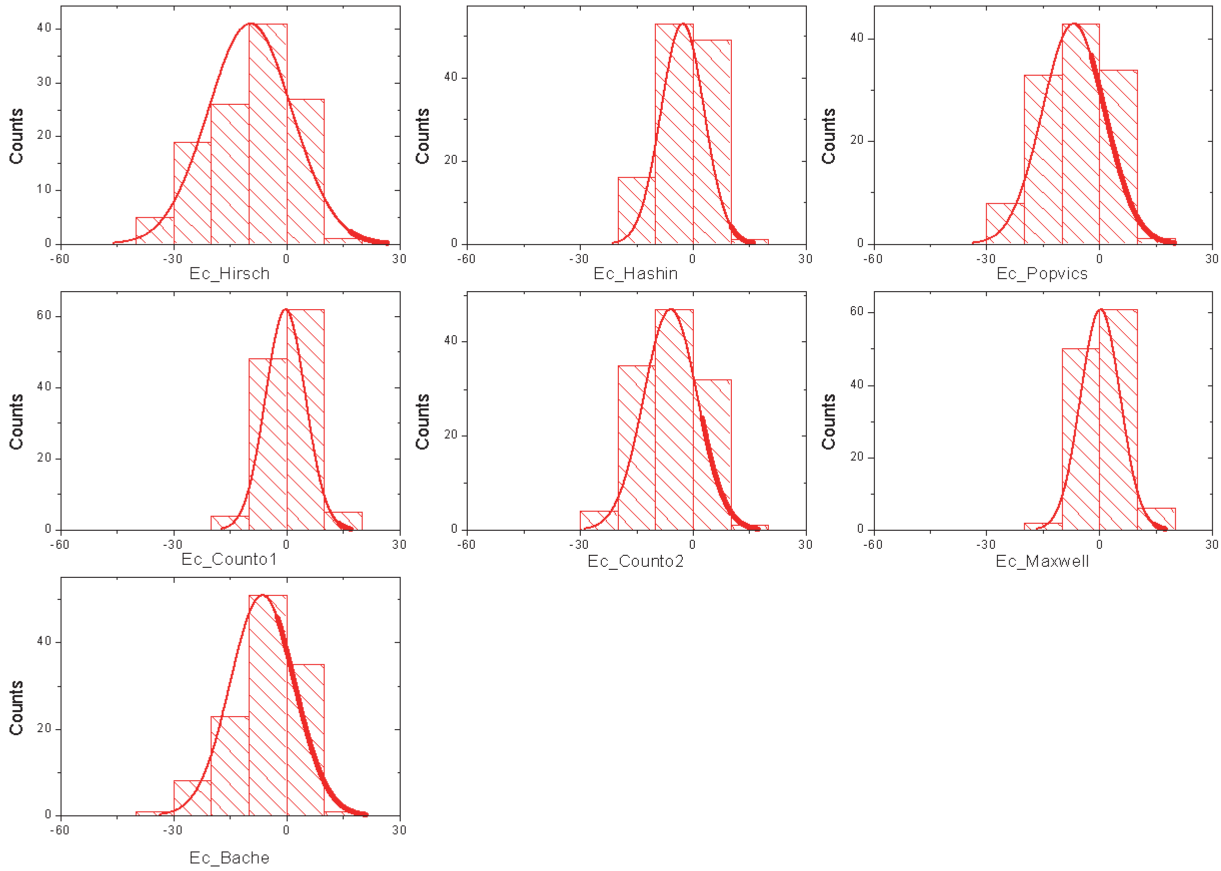

Figure 5: Error percentage distribution approximation to normal statistic low of each composite material model.

Conclusion

The modulus of elasticity is a very important mechanical parameter, its determination sometimes involves impossible, difficult or costly tests, the alternative use of the biphasic laws in these cases appears very interesting but the choice of a model and not another remains a question which requires a precise examination and strongly depends on the type of materials chosen.

In order to choose the optimized prediction composite model for Lightweight Aggregate Concrete, the purpose of this paper was to appraise the effective Young's modulus of LWAC using two-phase composite models. From the obtained numerical predictions, as confronted to existing experimental data and analytical results, the main findings are summarized below:

When the Young's modulus of lightweight aggregates E_g is much less than the Young's modulus of the mortar matrix in the lightweight aggregate concrete E_m, Hirsch-Dougill models remain distant from experimental results and cannot be applied to predict the modulus of elasticity of LWAC.

Using Popovics, Counto2 and Bache-Nepper Christensen composite models may not always produce accurate results.

For 119 experimental values of Young's modulus for LWAC, the Maxwell, Counto1 and Hashin-Hansen seem the most reasonable for this purpose.

The Maxwell model takes into account in the calculation of the effective elastic modulus of the contrast between the two phases (the mortar matrix and the light aggregates) represented by the coefficient α (E_g/E_m) which made it possible to simulate the materials well and offered consequently more precise results if compared with other models. Thus, the precision of this prediction model demonstrates its effectiveness and potential application as a model for Lightweight Aggregate Concrete. The Maxwell model remains close from the experimental values with a man value error equal to 0.29 and a standard deviation equal to 5.27. In addition the Counto1 and Hashin-Hansen models provide a good prediction of

experimental Young's modulus of all LWAC tested by De Larrard [7], Yang and Huang [8] and Ke Y et al. [9] (119 values) with a maximum volume fraction of aggregates Vg equal to 49.37%.
In conclusion, it can be suggested that additional studies about investigation for predicting modulus of elasticity of LWAC, may contribute to confirm the reliability and the accuracy of Maxwell, Counto1 and Hashin-Hansen models.

NOMENCLATURE

E: Young's modulus
E_g: Young's modulus of lightweight aggregate (dispersed phase)
E_m: Young's modulus of matrix (mortar)
V_g: Volume fraction of aggregate (dispersed phase)
V_m: Volume fraction of matrix (mortar)
E_c: Young's modulus of composite
E_{c_Voigt}: Young's modulus of composite using Voigt model (upper bound)
E_{c_Reuss}: Young's modulus of composite using Reuss model (lower bound)
E_{c_Hashin}: Young's modulus of composite using Hashin-Hansen model
E_{c_Hirsch}: Young's modulus of composite using Hirsch-Dougill model
$E_{c\_Popovics}$: Young's modulus of composite using Popovics model
$E_{c\_Maxwell}$: Young's modulus of composite using Maxwell model
α: Empirical factor
$E_{c\_Counto1}$: Young's modulus of composite using Counto1 model
$E_{c\_Counto2}$: Young's modulus composite using Counto2 model
E_{c_Bache}: Young's modus of composite using Bache and Nepper-Christensen model
$E_{exp_{De}Larrard}$: Young's modulus of LWAC tested by De Larrard and Le Roy (1995)
E_{exp_Yang}: Young's modulus of LWAC tested by Yang and Huang (1998)
E_{exp_Ke}: Young's modulus of LWAC tested by Ke Y et al (2010)
υ_c: Poisson's ratio of composite
υ_m: Poisson's ratio of matrix (mortar)
υ_g: Poisson's ratio of aggregate (dispersed phase)
d: Smallest diameter of aggregates in concrete
D: Largest diameter of aggregates in concrete
E_{c_anal}: Young's modulus predicted from analytic model
ΔE: Error percentage
$|\Delta E|$: Absolute value of error percentage

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