Focussed on Steels and Composites for Engineering Structures

Visual Abstract

Combination of Intermittent Search Strategy and an Improve Particle Swarm Optimization algorithm (IPSO) for model updating

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ABSTRACT. Modality and intermittent search strategy in combination with an Improve Particle Swarm Optimization algorithm (IPSO) to detect damage structure via using vibration analysis basic principle of a decline stiffness matrix a structure is presented in the study as a new technique. Unlike an optimization problem using a simplistic algorithm application, the combination leads the promising results. Interestingly, the PSO algorithm solves the optimal problem around the location determined previously, whereas Eagle Strategy (ES) is the charging of locating the position in intermittent space for the PSO algorithm to search local. ES can be easy to deal with its problem via drastic support of Levy flight. As know that the PSO algorithm has a fast search speed, yet the accuracy of the PSO algorithm is not as good as expected in many problems. Meanwhile, the combination is powerful to solve two trouble (1. Avoiding local optimization, 2. Receiving the result more accurate). The paper compares the results obtained from the PSO algorithm with the combination of IPSO and ES in some problems, as well as between experiment and FEM to demonstrate the effectiveness as mentioned. In which natural frequencies is used as the objective function to solve this optimization problem. The results present that the combination of IPSO and ES is quite effective.

KEYWORDS. IPSO; Eagle strategy; Damage detection; Inverse problem.

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INTRODUCTION

From ancient times to the present, it is always a desire for Structural Health Monitoring (SHM) from simple structure to the complex, such as a bridge, or even a full skyscraper. However, detecting structural damage in operational status still encounters some difficulties, because environmental varieties impose challenges in real engineering applications and can request heavy computational efforts in the damage evaluation and potential maintenance. With the drastic development of many optimization algorithms to cope with the complex trouble, a lot of solutions applied to the algorithms for dealing with the problem are becoming more and more effective and popular. Generally speaking, the meta-heuristic optimization algorithms include some advantages as follows:

- Simplify in an idea and operational method.
- Application in most fields from routine life to engineering even politics.
- The obtained results meet the requests that were originally set out.

Because of the above advantages, optimization algorithms play a vital role to evaluate both the severity and location of damage structures in the SHM and damage identification field. Many studies employed this method for damage identification in SHM field. For instance, Samir Khatir et al. [1] used BAT algorithm to apply damage at a specific element(s) of the considered beams. The damage is illustrated by a change in Young's modulus, and the determination of severity is constructed as an optimization problem applied objective function based on the Modal Scale Factor and changes the vibration of the structure. A process including steps to detect and identify positions damage of beam-like structures based on the BAT algorithm is applied. In addition to that Parsa Ghannadi et al. [2] also indicated a method using natural frequencies and mode shapes in damage detection. In which, mode expansion techniques are used to cope with the incompleteness of mode shapes, and the authors used the GWO and Harris hawks optimization (HHO) to evaluate the results. The data is collected by experimental analysis of a cantilever beam [3], a Laboratory model of a truss tower, and sensor locations [4], and an Experimental steel frame [5]. Meanwhile, Hoang et al. [6] employed an improvement of PSO (EHVPSO) to identify damage for 3D transmission tower.

As it is known that particle swarm optimization (PSO) [7] is the classic algorithm. With the development of PSO in 1995, James Kennedy et al. marked a significant progress of optimization field. Since its development, PSO leads and inspires a lot of meta-heuristic optimization algorithms, such as Cuckoo Search (CS)[8], Bat Algorithm (BA)[9], Grey Wolf Optimizer (GWO)[10], Gravitational Search Algorithm (GSA)[11] , Salp Swarm Algorithm (SSA) [12], etc. In fact, PSO can search candidate solutions quickly, yet the results of the algorithm are usually not good enough for some problems requested the high accuracy. To solve such limit we combine two advantages of eagle strategy (ES) with an improvement PSO (IPSO) to deal with the problem in this study. The detail of the combination is presented in the next section.

In this study, a new damaged element is detected base on a change in Young modulus's structure between test results and simulation. In which the optimal algorithm (IPSO) combines of eagle strategy (ES) to solve the minimum optimization problem.

METHODOLOGY

An issue asked by most scientists, namely young scientists, is: What algorithm is the most effective into a lot of current algorithms for optimization?

A quite simple question, yet the answer is not easy. There is a lot of numbers that cause that answer is more complex. One of which is that the real-world problems are too many distinct variables, whereas some of them are simplistic. Thus, it is difficult to have a single approach that can solve most kinds of problems. In other words, it is a so-called no-free-lunch (NFL) theorem. It means each algorithm is simple to fit some specific problems. It is not reasonable if the complex algorithm to deal with a simple problem. And it is incapable if the difficult problem is coping with a simple algorithm.

Such reason why ES and IPSO are employed to deal with the problem of this research. With the combined advantage of each member, ES and IPSO create a method for searching effectively and accurately.

The search strategy of eagle strategy (ES)

Modality and intermittent search strategy or eagle strategy is strategy searching used in conjunction with meta-heuristic optimization algorithms, not an algorithm. ES is proposed in 2010 by Xin-She Yang et al. [13, 14]. It is many points quite similar to the random walk. However, two main different support ES outstanding rather than random walk is:

- Levy flight is used to explore global space instead of a random wander.

– ES applies each algorithm for different purposes.

Obviously, in a solution space to search candidate solutions has many values leading to local optimization. To solve the trouble, ES illustrates searching in the global solution via Levy flight. If the result is promising, it is exploratory an intensive by other algorithm optimization. Continue like this until the process either meets the originally established need or ends all iterations.

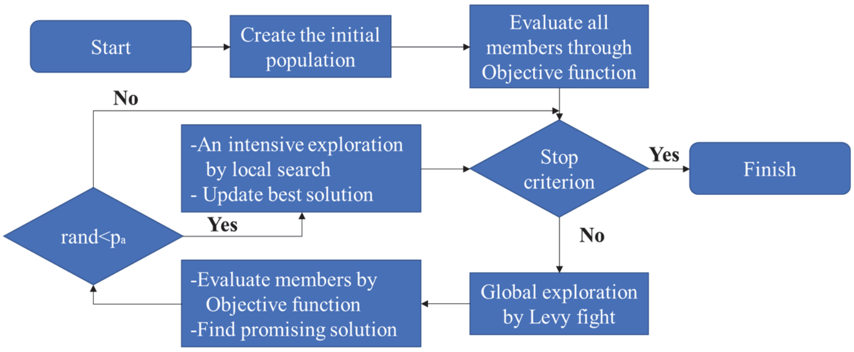


Figure 1: Flowchart of the ES

The Particle Swarm Optimization algorithm (PSO)

The PSO algorithm simulates the each member and social intelligence of birds to navigate. This algorithm operates with two vectors: velocity (1) and location (2) (see Fig. 2). The velocity vector controls the speed and movement direction of each member.

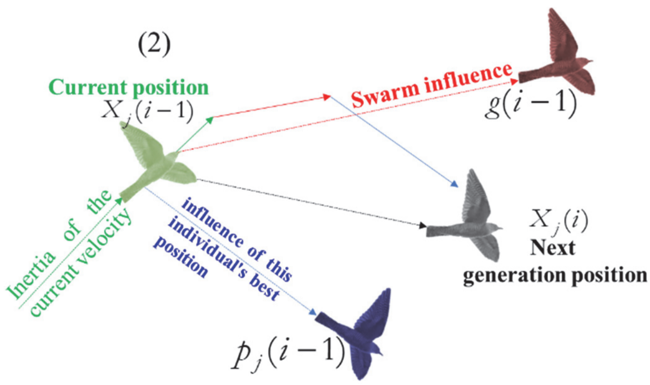


Figure 2: Simulating the operation of PSO

$$Vx_j^d(i) = \omega Vx_j^d(i-1) + c_1 \times rand \times \left(p_j^d(i-1) - X_j^d(i-1)\right) + c_2 \times rand \times \left(g^d(i-1) - X_j^d(i-1)\right) \quad (1)$$

$$X_j^d(i) = X_j^d(i-1) + Vx_j^d(i-1) \quad (2)$$

Symbol	Description
$X_j^d(i)$	Illustrates the d^{th} parameter in the location vector of the j^{th} member in the i^{th} iteration
$Vx_j^d(i)$	Presents the d^{th} parameter in the velocity vector of the j^{th} member in the i^{th} iteration
g	Shows the best location of swarm at current
p	Indicates the best location of the member at the current
$c_1; c_2$	Constants (in this paper is $c_1 = c_2 = 2$)
rand	Value in from 0 to 1
ω	The inertia weight base on iteration and calculated as follows (3)

Table 1: Factor description for PSO

We can see that Eqn. (1) has three main parts with three distinct meanings. In the first part of the Eqn. (1), $\omega Vx_j^d(i-1)$ shows capacity global exploration and local exploration, respectively, ω in [0.9; 0.5] and ω in [0.5; 0.4]. The rest of parts represent the intelligence of an individual and swarm as shown in Fig. 3.

$$\omega = Max_{Vx} - \frac{i \times (Max_{Vx} - Min_{Vx})}{Max\_iteration} \tag{3}$$

In most situations $Max_{Vx} = 0.9, Min_{Vx} = 0.4$

$$Vx_j^d(i) = \boxed{\omega Vx_j^d(i-1)} + \underbrace{\boxed{c_1 \times rand \times \left(p_j^d(i-1) - X_j^d(i-1)\right)}}_{\text{For individual}} + \underbrace{\boxed{c_2 \times rand \times \left(g_j^d(i-1) - X_j^d(i-1)\right)}}_{\text{For swarm}}$$

$$Vx_j^d(i) = \text{Mutant factor} + \text{Intelligence of algorithm}$$

Figure 3: Intelligence of PSO in the search space

Improve Particle Swarm optimization algorithm (IPSO) for combination with ES

As mention above, in Eqn. (1) the first section $\omega Vx_j^d(i-1)$ is in charge of discovering a new promising area (mutant factor) of the PSO. However, with a combination of ES, the part does not become to excess even having side effects in some situations. Consequently, $\omega Vx_j^d(i-1)$ is rejected in the circumstance. From now on, IPSO became an algorithm that uses only the intelligence of a swarm through the best location of the member (p) and the best location of a swarm (g). Of course, the Levy flight is more outstanding rather than a random wander [13, 14]. Therefore, Levy flight is added to increase the accuracy. In schematic of Fig. 4, it indicates the simulation of IPSO. In which, *Xtrue* is the global optimization. Vector velocity is written as following:

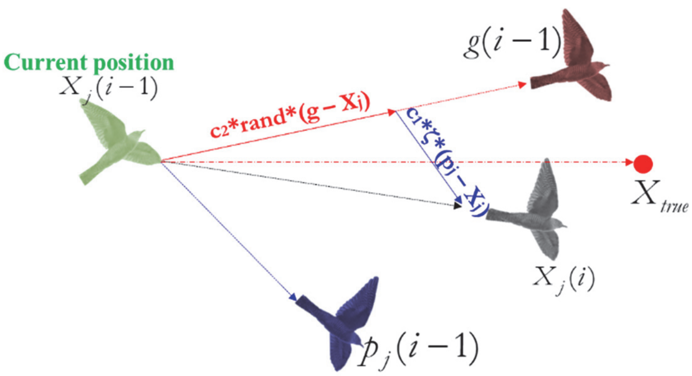


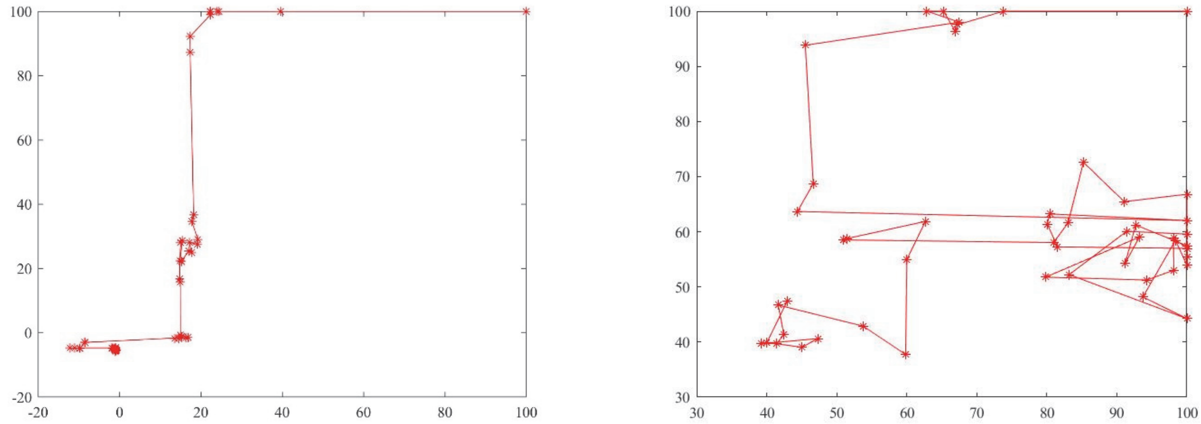
Figure 4: Schematic of IPSO.

$$Vx_j^d(i) = \zeta \times \left(p_j^d(i-1) - X_j^d(i-1)\right) + c_2 \times rand \times \left(g^d(i-1) - X_j^d(i-1)\right) \tag{4}$$

where $\zeta = c_1 \times Levy(d) \times rand$; and d is dimensions.

Fig. 5 illustrates the updating location process of a point starting at (100, 100) using PSO and IPSO in combination to ES. Clearly, with an improvement at vector Vx, IPSO is noticeably more efficient than the pure PSO.

The detail of effectiveness of the improvement is illustrated below, including evaluation, comparison between PSO, and PSO, IPSO combination with ES.



(a) ES-IPSO (b) ES-PSO

Figure 5: ES-IPSO in 50 consecutive steps starting at the origin (100, 100) compares ES-PSO.

RESULTS AND DISCUSSION

In this part, ES-IPSO is employed to handle a set of experiment for different problems. Specifically, two popular experimental sets from artificial benchmark to real-world problems, from simple to complex functions, are used. The first test consists of 13 classical benchmark functions [15-18] classified into two distinct groups, in which each group has a strong individual point to test the optimum capacity of the algorithm. The second one is a real structural problem which is chosen from the many previous researches [2, 5] for model updating and predicting damage.

Classical Benchmark functions

In this subsection, the combination ES with IPSO is benchmarked on 13 classical benchmark functions. Additionally, PSO is also added to compare to ES-PSO and ES-IPSO to examine the accuracy of the algorithm. The functions are shown in Tab. 2. An experiment with population (N) 50 members and 200 iterations (T) is adopted to examine the efficacy of ES-IPSO compared with ES-PSO, PSO in solving benchmark functions. It should also be noted that PSO and IPSO in conjunction with ES (ES-PSO, ES-IPSO) are set up a distinct population (N_local) and iteration (T_Local) number for the local search process. In this case N_Local and T_Local are chosen equal 15. Benchmark functions have been carried out simulations after implementing these algorithms using Matlab. At the same time, each algorithm has been 100 runs so as to carry out meaningful statistical analysis. The result of ES-IPSO is compared with PSO, ES-PSO, and which are illustrated in the Fig. 6.

The example from F1 to F7 represent for unimodal benchmark functions, which are single objective, but the search space is quite large. Multimodal functions, it is the other way around, namely F8 to F13, with many local optimum areas as well as higher difficult, which makes them proper to benchmark the exploration ability of ES-IPSO in a smaller but more challenging search space.

As we can see in the figure above that ES-IPSO is more accurate than ES-PSO and fully superior to the original PSO. Several functions (F1 to F7 and F11 to F13) show outstanding accuracy when using ES in combination with PSO or IPSO algorithm. In which IPSO is superior to PSO when combining ES for evaluating in the most of the functions.

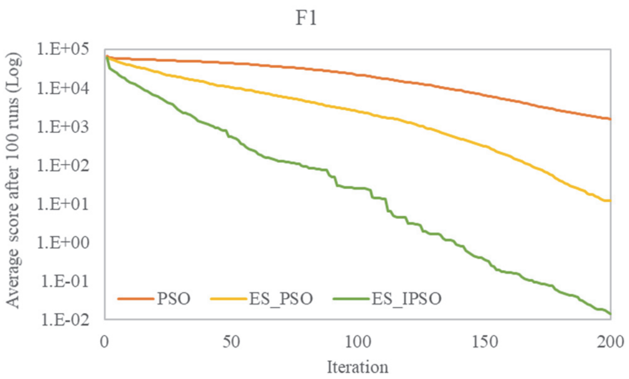
Compared to the functions mentioned above, the rest of the functions (F8 to F10) also show that applying ES in an optimal process provides acceptable results, despite the fact that ES does not totally dominate the original version PSO at F8 and F10. However, based on these all above considerations, it has stressed that the effectiveness of combining ES is far outweigh the limitations.

For a fair comparison, we use the number of runs of the objective function as the evaluation criterion. It can see from Fig. 6 that the results of ES-IPSO most benchmark functions in 50^{th} iteration is outstanding than pure PSO in 200^{th} iteration. It means that ES-IPSO only uses the number of runs T_1=50×50+0.2×50×15×15 = 4750 (0.2

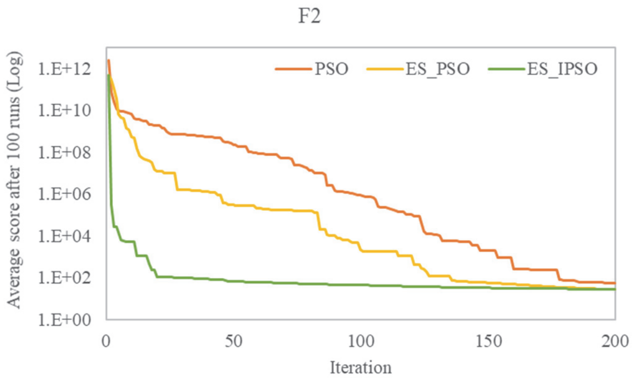
represents p_a - local search factor) to get better results of PSO at $T_2 = 50 \times 200 = 10000$. In other word, T_1 is only a half of T_2. So now we can conclude that the combination of ES with improvement PSO (IPSO) are effective, fast and high performing for getting the desirable result.

Function (Fi)	Range	Dimension (d)	Min Fi
$F_1 = \sum_{i=1}^{n} (x_i)^2$	[-100, 100]	30	0
$F_2 = \sum_{i=1}^{n} \lvert x_i \rvert + \prod_{i=1}^{n} \lvert x_i \rvert$	[-10, 10]	30	0
$F_3 = \sum_{i=1}^{n} \left(\sum_{j-1}^{i} x_j \right)^2$	[-100, 100]	30	0
$F_4 = \max_i \{ \lvert x_i \rvert, 1 \le i \le n \}$	[-100, 100]	30	0
$F_5 = \sum_{i=1}^{n-1} \left(100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2 \right)$	[-30, 30]	30	0
$F_6 = \sum_{i=1}^{n} (x_i + 0.5)^2$	[-100, 100]	30	0
$F_7 = \sum_{i=1}^{n} i x_i^4 + rand[0,1)$	[-1.28, 1.28]	30	0
$F_8 = \sum_{i=1}^{n} -x_i \sin \sqrt{\lvert x_i \rvert}$	[-500, 500]	30	-418.9829x5
$F_9 = \sum_{i=1}^{n} (x_i^2 - 10\cos(2\pi x_i) + 10)^2$	[-5.12, 5.12]	30	0
$F_{10} = -20 \exp\left(-0.2 \sqrt{\frac{1}{n} \sum_{i=1}^{n} x_i^2} \right) - \exp\left(\frac{1}{n} \sum_{i=1}^{n} \cos(2\pi x_i) \right) + 20 + e$	[-32, 32]	30	0
$F_{11} = \frac{1}{4000} \sum_{i=1}^{n} x_i^2 - \prod_{i=1}^{n} \cos\left(\frac{x_i}{\sqrt{i}} \right) + 1$	[600, 600]	30	0
$F_{12} = \frac{\pi}{n} \left\{ 10\sin(\pi y_1) + \sum_{i=1}^{n-1} (y_i - 1)^2 \left[1 + 10\sin^2(\pi y_{i+1}) \right] + (y_n - 1)^2 \right\}$ $+ \sum_{i=1}^{n} u(x_i, 10, 100, 4)$ $where: y_i = 1 + \frac{(x_i + 1)}{4}$	[-50, 50]	30	0
$F_{13} = 0.1\{\sin^2(3\pi x_1) + \sum_{i=1}^{n} (x_i - 1)^2 \left[1 + 10\sin^2(3\pi x_i + 1) \right] +$ $+ (x_n - 1)^2 [1 + \sin^2(2\pi x_n)]\} + \sum_{i=1}^{n} u(x_i, 5, 100, 4)$	[-50, 50]	30	0

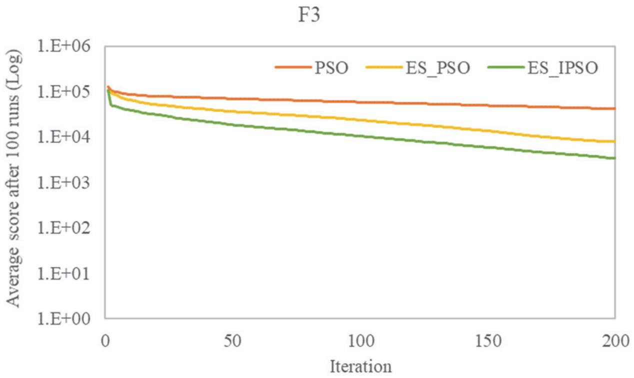
Table 2: Classical Benchmark Functions.



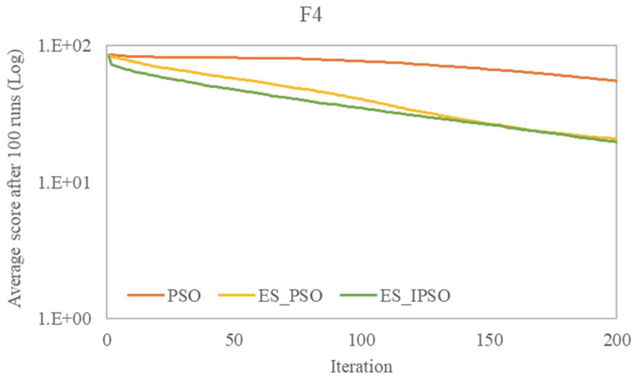
Function 1



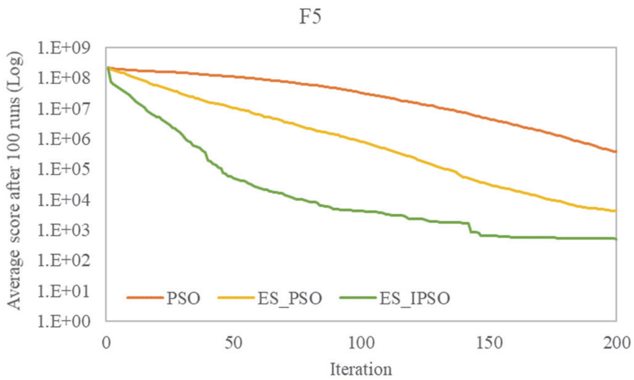
Function 2



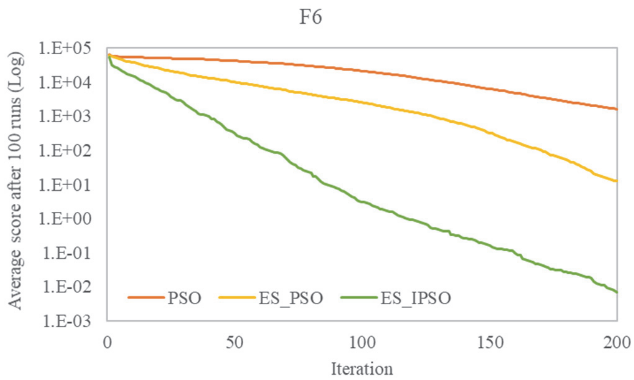
Function 3



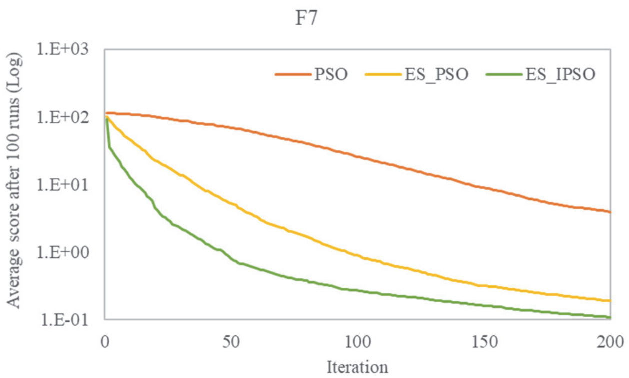
Function 4



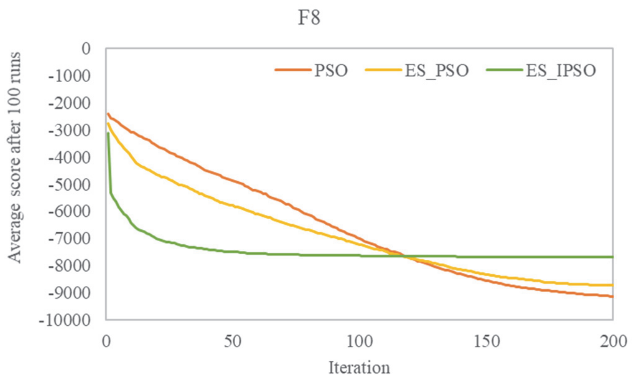
Function 5



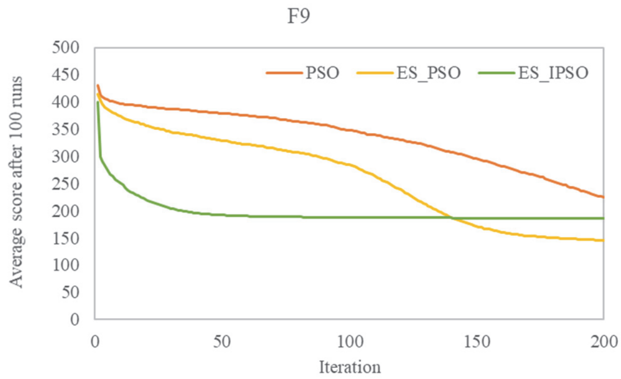
Function 6



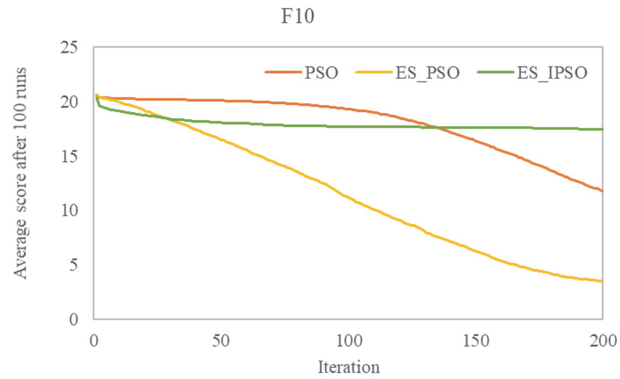
Function 7



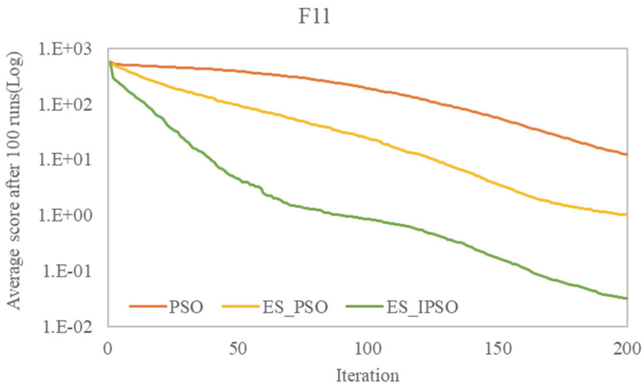
Function 8



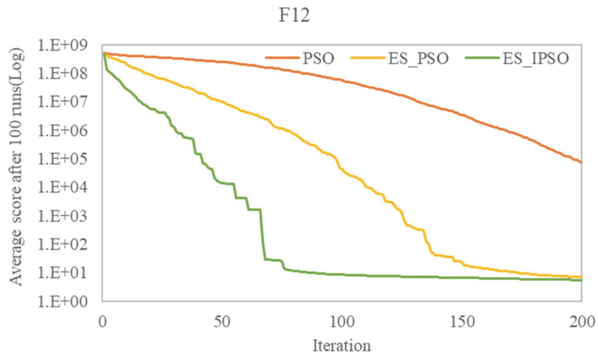
Function 9



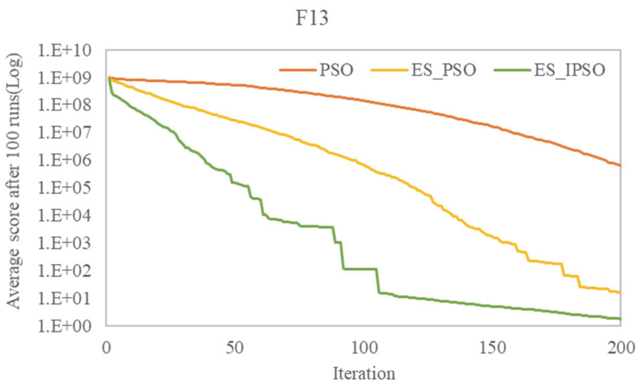
Function 10



Function 11



Function 12



Function 13

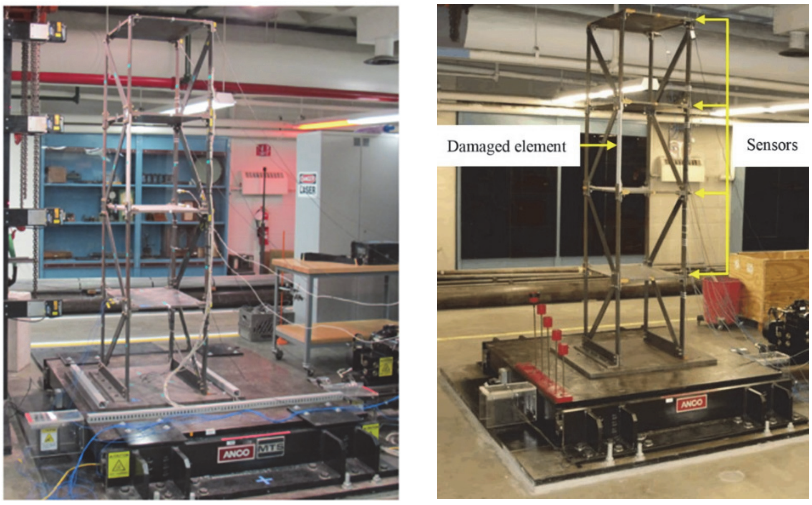
Figure 6: Results of classical benchmark functions

3D steel frame

We can see from the examples above, in combination ES with IPSO is more effective than ES and PSO. Therefore, evaluating between PSO and ES-PSO is presented in the real experiment to compare effectively between PSO and ES-PSO. In this part, a steel frame 4 stories high, as shown in Fig. 4, is performance and evaluated damage level. Input parameters are provided by the experimental modal. To evaluate the accuracy of the inverse problem in both healthy and damaged frame structures the frequencies are provided into the objective function to compare measured and compute. This structure was tested by the experiment at Columbia University. This structure was used to detect structural damage in Refs [2, 5]

This structure was excited on a hydraulic shake table in the frequency scope of [0; 150] Hertz with the maximum acceleration of 3g. The behaviors of the steel frame are determined by piezoelectric accelerometers. The positions' sensor are indicated in the Fig. 4-b. This structure was erected by a system including four steel plates and several bars. It is 53.3 cm high, and each slab plate dimensions is $61\times45.7\times1.27$ cm^3. Each column's cross-sectional dimensions is 5.08×0.95 cm^2.

The components of this steel frame are linked to create the complete system by bolts for easy replacement. A damage was created by decreasing 66 percent of the area of a column at the 3rd story, affect a 22.2 percent go down stiffness at inter-story. The location of damaged bar is presented in the Fig. 7-b. The natural frequencies of the undamaged this structure based on FEM and measurement in the laboratory are provided in the Tab. 3.



(a) (b)

Figure 7: The 4 stories shear-type steel frame [5]. (a) Healthy and (b) Damaged

The finite element method updating is employed to modify the story stiffness of the numerical analysis by minimizing the error between the first 4 natural frequencies obtained from the finite element analysis approach and corresponding obtained values of this steel frame from the lab, according to Eqn. (5) using different iterations of optimization algorithm. Convergence evaluation is analyzed to research the efficacy of both optimization approaches.

$$ObjctiveFunction = \sqrt{\sum_{i=1}^{r} \frac{\left(\omega_i^c - \omega_i^m\right)^2}{\left(\omega_i^m\right)^2}} \qquad i = 1;2;...;r \tag{5}$$

where: ω_i^c and ω_i^m illustrate for the natural frequencies the i[th], respectively, computed and measured. Meanwhile, r represents the order number of modes.

In the section, two scenarios are investigated. Firstly, using natural frequencies to find out the stiffness matrix with the mass matrix assumed previously. In the second scenario, the stiffness matrix is used to search both location and severity of damage structures.

We use 100 members and 50 iterations to analyze the problem. In which IPSO is set up N_Local =30 and T_Local =10 when combined with ES.

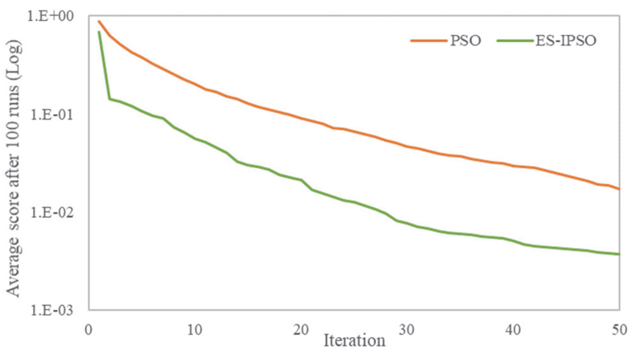


Figure 8: Convergence curve when update stiffness matrix

Natural frequencies	Results of Experiment	PSO	ES-IPSO	% diff PSO	% diff ES-IPSO
ω_1	3.9020	3.8895056	3.90294817	0.320	0.024
ω_2	10.9800	10.9782208	10.98021178	0.016	0.002
ω_3	18.6450	18.7014805	18.64835169	0.303	0.018
ω_4	26.2430	26.2619459	26.24518694	0.072	0.008

Table 3: Natural frequencies of the undamaged steel frame at the best run.

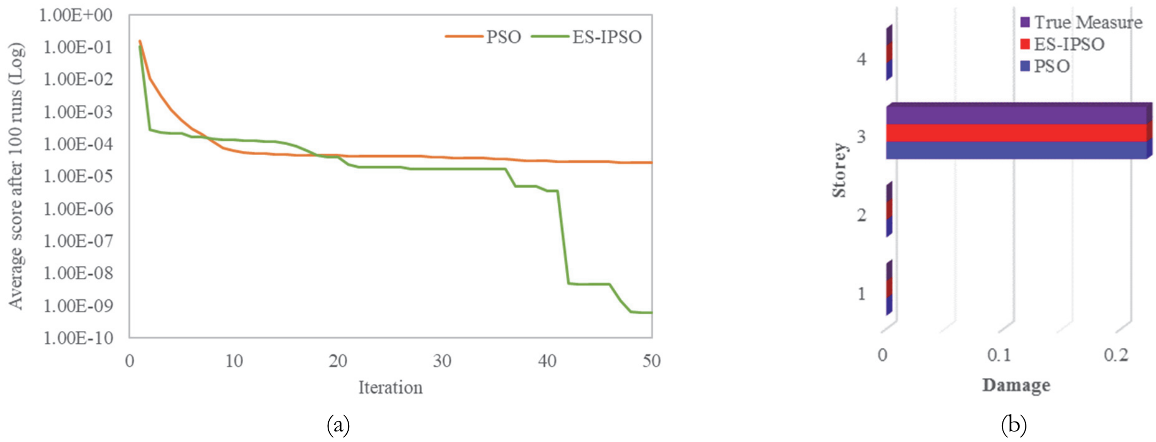


Figure 9: Fitness and damage in structure

Tab. 3 presents the results of optimization process for updating steel frame's stiffness matrix using PSO and ES-IPSO. In which, %diff PSO and %diff ES-PSO illustrate, respectively, the difference of natural frequencies based on updating model using PSO, ES-IPSO and result of the experiment. Although the objective value of both optimization processes (shown in the Fig. 8) is quite small, the results of ES-IPSO shows that natural frequencies received from this new technique are more accurate than pure PSO and almost reach the results of the experiment. Meanwhile, Fig. 9-a shows the process of finding PSO and ES-IPSO step by step whereas Fig. 9-b indicates the location and damage level of this frame in the second scenario. The results of ES-IPSO are promising.

As we can see in the figures above, result of ES-IPSO at 21^{st} iteration is outstanding than PSO at 50^{th} iteration. It means that the number of objective function runs of ES-IPSO is 3360 instead of 5000 by PSO. For this reason, It could be argued that Eagle Strategy and improved PSO is more effective than pure PSO to solve optimization problem, and we can see that in prediction and identification the damaged structure shows superiority of ES-IPSO about time once again.

CONCLUSION

In the paper, the first section, a combination of ES and IPSO is presented. While the second section indicated how to apply some functions to evaluate ES-IPSO compare to PSO and ES-PSO, the final section showed the results used to check the match between prediction and experiment.

Based on the results shown herein, the according to conclusions are able to draw as following:

- The results of classical benchmark functions present the outstanding efficacy of ES-PSO or ES-IPSO than pure PSO in terms of exploiting the optimum. This is proof that this combination is proper. Furthermore, the data on ES-IPSO is completely superior to ES-PSO, which determine that improvement PSO is effective.
- ES-IPSO provides the results to predict location damage and severity of the real structure accurately and quickly.

- In comparison original PSO, which spends a lot of loops in the optimization process, ES-IPSO requires a number of iteration less to get the expected result. It means that ES-IPSO is faster or saving time. Especially, for problems with complex FEMs that require a longer computation time is that ES-IPSO becomes more significant.

The study provided one new technique to detect damage in structure by improving PSO (IPSO) and combine to ES for the creation of an effective approach. ES-IPSO not only deals with trouble local optimization but also increases the accuracy of the algorithm.

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REFERENCE

[1] Khatir, S., Belaidi, I., Serra, R., Wahab, M.A., Khatir, T. (2016). Numerical study for single and multiple damage detection and localization in beam-like structures using BAT algorithm, J. Vibroengineering, 18(1), pp. 202–213.
[2] Ghannadi, P., Kourehli, S.S., Noori, M., Altabey, W.A. (2020). Efficiency of grey wolf optimization algorithm for damage detection of skeletal structures via expanded mode shapes, Adv. Struct. Eng., 23(13), pp. 2850–2865, DOI: 10.1177/1369433220921000.
[3] Altunışık, A.C., Okur, F.Y., Kahya, V. (2019). Vibrations of a box-sectional cantilever Timoshenko beam with multiple cracks, Int. J. Steel Struct., 19(2), pp. 635–649, DOI: 10.1007/s13296-018-0152-5.
[4] Weber, B., Paultre, P. (2010). Damage identification in a truss tower by regularized model updating, J. Struct. Eng., 136(3), pp. 307–316, DOI: 10.1061/(ASCE)ST.1943-541X.0000105.
[5] Chatzi, E.N., Smyth, A.W. (2011).Experimental identification of the base excitation of a shear-type structure subjected to ground motion. Proceedings of the 8th International Conference on Structural Dynamics, EURODYN 2011, 8th International Conference on Structural Dynamics, pp. 2507–2514.
[6] Minh, H.-L., Khatir, S., Wahab, M.A., Cuong-Le, T. (2021). An Enhancing Particle Swarm Optimization Algorithm (EHVPSO) for damage identification in 3D transmission tower, Eng. Struct., 242, pp. 112412, DOI: 10.1016/j.engstruct.2021.112412.
[7] Minh, H. L., Khatir, S., Rao, R. V., Abdel Wahab, M., & Cuong-Le, T. (2021). A variable velocity strategy particle swarm optimization algorithm (VVS-PSO) for damage assessment in structures. Engineering with Computers, 1-30, DOI: 10.1007/s00366-021-01451-2.
[8] Cuong-Le, T., Nghia-Nguyen, T., Khatir, S., Trong-Nguyen, P., Mirjalili, S., & Nguyen, K. D. (2021). An efficient approach for damage identification based on improved machine learning using PSO-SVM. Engineering with Computers, 1-16, DOI: 10.1007/s00366-021-01299-6.
[9] Kennedy, J., Eberhart, R. (1948).Ieee, Particle swarm optimization. 1995 Ieee International Conference on Neural Networks Proceedings, 1-61995.
[10] Yang, X.-S., Deb, S. (2009).Cuckoo search via Lévy flights. 2009 World congress on nature & biologically inspired computing (NaBIC), Ieee, pp. 210–214.
[11] Gandomi, A.H., Yang, X.-S., Alavi, A.H., Talatahari, S. (2013). Bat algorithm for constrained optimization tasks, Neural Comput. Appl., 22(6), pp. 1239–1255, DOI: 10.1007/s00521-012-1028-9.
[12] Mirjalili, S., Mirjalili, S.M., Lewis, A. (2014). Grey Wolf Optimizer, Adv. Eng. Softw., 69, pp. 46–61, DOI: 10.1016/j.advengsoft.2013.12.007.
[13] Rashedi, E., Nezamabadi-Pour, H., Saryazdi, S. (2009). GSA: a gravitational search algorithm, Inf. Sci. (Ny)., 179(13), pp. 2232–2248, DOI: 10.1016/j.ins.2009.03.004.
[14] Mirjalili, S., Gandomi, A.H., Mirjalili, S.Z., Saremi, S., Faris, H., Mirjalili, S.M. (2017). Salp Swarm Algorithm: A bio-inspired optimizer for engineering design problems, Adv. Eng. Softw., 114, pp. 163–191, DOI: 10.1016/j.advengsoft.2017.07.002.
[15] Yang, X.S., Deb, S. (2012). Two-stage eagle strategy with differential evolution, Int. J. Bio-Inspired Comput., 4(1), pp. 1-5, DOI: 10.1504/IJBIC.2012.044932.
[16] Yao, X., Liu, Y., Lin, G. (1999). Evolutionary programming made faster, IEEE Trans. Evol. Comput., 3(2), pp. 82–102,

DOI: 10.1109/4235.771163.
[17] Digalakis, J.G., Margaritis, K.G. (2001). On benchmarking functions for genetic algorithms, Int. J. Comput. Math., 77(4), pp. 481–506, DOI: 10.1080/00207160108805080.
[18] Molga, M., Smutnicki, C. (2005). Test functions for optimization needs, Test Funct. Optim. Needs, 101, pp. 48.