FACTA UNIVERSITATIS
Series: **Electronics and Energetics** Vol. 29, N^o 4, December 2016, pp. 721 - 732
DOI: 10.2298/FUEE1604721K

CONTRIBUTION TO CALCULATING THE IMPEDANCE OF GROUNDING ELECTRODES USING CIRCUIT EQUIVALENTS

Andrijana Kuhar, Leonid Grcev

Ss. Cyril and Methodius University, FEIT, Skopje, Macedonia

Abstract. *In this paper the dynamic behavior of grounding electrodes is investigated by calculating their impedance to ground. The calculations are performed by implementing popular modelling approaches including circuit, transmission line, and electromagnetic field (EMF) model. The attention of this paper is given to the circuit based (CBM) method. The results from the rigorous EMF model are used as reference in the process of validity range determination of the other models. Numerically obtained curves for the frequency-dependent impedance to ground are presented in several figures for various electrode lengths and soil characteristics.*

Key words: *equivalent circuits, EMF model, grounding electrode, impedance*

1. INTRODUCTION

Equivalent circuits are frequently used in the analysis of grounding systems due to the simplicity of modelling they offer [1]-[5].

The impedance to ground is one of the most important characteristics of any grounding system. In this paper the circuit based method (CBM) from [5] is used to determine the grounding impedance of a perfect conductor placed in imperfect ground, using the thin wire approximation. The obtained results for the harmonic impedance to ground of the conductor are compared to results obtained by implementation of a lumped R-L-C circuit, a transmission line (TL) model, and the referent EMF model from [6].

A thorough verification process of the CBM method implemented in this paper has been previously performed by the authors [7], [10].

Received January 23, 2016; received in revised form March 9, 2016
Corresponding author: Andrijana Kuhar
Ss. Cyril and Methodius University, FEIT, Skopje, Macedonia
(email: kuhar@feit.ukim.edu.mk)
* An earlier version of this paper was presented at the 12th International Conference on Applied Electromagnetics (ПЕС 2015), August 31 - September 2, 2015, in Niš, Serbia [1].

2. METHODS FOR DETERMINATION OF THE GROUNDING IMPEDANCE

2.1. CBM model

The CBM method was first implemented by Otero, Cidras and Alemo in 1999 [5]. This method operates in the frequency domain and creates an equivalent circuit that takes into account all the inductive, capacitive, and conductive couplings between the conductor segments. Propagation effects on the EM fields are not considered in [5]. A newer expanded version of the CBM method was proposed by Visacro and Silveira in 2004 [8], in which the authors introduced time propagation by multiplying the classical CBM equations with the term $e^{-\gamma r}$, where γ is the propagation constant of the medium and r is the distance between the point of interest and the source point. The enhanced CBM method from [8] was called Hybrid Electromagnetic Model (HEM).

The system analyzed in this paper is consisted of a perfect conductor placed in imperfect ground. The length of the conductor is noted as l, and the radius is noted as a. The conductor is segmented for the needs of the CBM method and one of the segments is presented in Fig. 1.

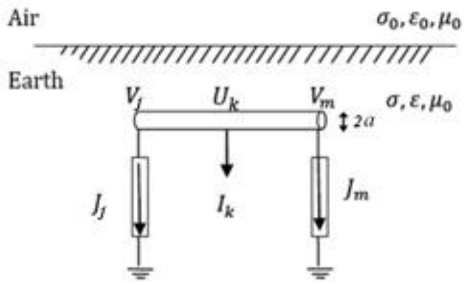


Fig. 1 Geometry of the conductor

In Fig.1 σ, ε and μ are the conductivity, permittivity and permeability of the soil, while the according characteristics of the air are σ_0, ε_0 and μ_0. Indexes j and m mark the number of the corresponding node, while index k marks the corresponding segment. The thin wire approximation requires the length of the conductor to be significantly larger than its radius ($l>>a$).

The solution for the potential distribution of the electrode is based on conventional nodal analysis which is represented in the following matrix equations

$$\begin{aligned}&[\boldsymbol{I}_s]=[\boldsymbol{Y}][\boldsymbol{V}]\\&[\boldsymbol{Y}]=([\boldsymbol{Q}]^{\boldsymbol{T}}[\boldsymbol{G}][\boldsymbol{Q}]+[\boldsymbol{A}]^{\boldsymbol{T}}[\boldsymbol{Z}]^{-1}[\boldsymbol{A}])\end{aligned} \tag{1}$$

where [$\boldsymbol{Y}$] is the admittance matrix, [$\boldsymbol{Z}$] is the impedance matrix, [$\boldsymbol{G}$] is the conductance matrix, [$\boldsymbol{Q}$] is a matrix containing relations between nodal and segment potentials and [$\boldsymbol{I}_s$] is the current source vector. Equation (1) results with the potential distribution [$\boldsymbol{V}$] (of every node) along the conductor. The conductor is excited by a harmonic current source at the first node and the impedance to ground is calculated as a ratio of the conductor's first node voltage and current. The presence of air-earth interface is taken into account by implementing the quasi-dynamic image theory [9]. For the purpose of this research the

CBM method is implemented in the classical form [5], and in the hybrid form [8]. More details of the extraction of impedances, conductances from Maxwell's equations and the approximated relation between nodal and segment potentials can be found in [7].

The validity of the CBM method applied on complex grounding networks has been thoroughly investigated by the authors [7], [10]. One such network is the E. Balaidos II substation grounding (located close to the city of Vigo, Spain). The analyzed network has dimensions 80 x 60 m and is constructed with 107 horizontal copper bars of 1.28 cm diameter buried at 80 cm depth and a set of 67 vertical copper clad steel rods of 1.4 cm diameter and 2.5 m length. 3D view of the grounding grid and the profile on the surface of the ground used for calculations is presented in Fig. 2. Small portion of the verification results will be presented in Fig. 3

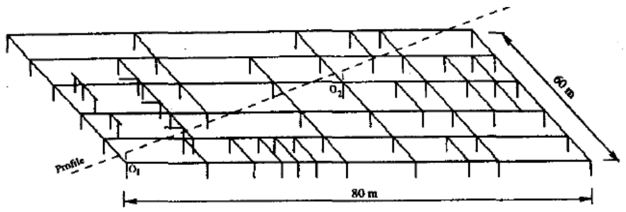


Fig. 2 Geometry of the Balaidos grounding network and the position of the calculation profile.

The verification process included results for the potential along the profile when injected current frequency is low (50 Hz) and higher (100 kHz) [10]. The current was injected in a corner of the grounding grid (point O1 from Fig. 2). The resistivity of the soil was ρ=50 Ωm. The obtained results are compared graphically in Fig. 3.

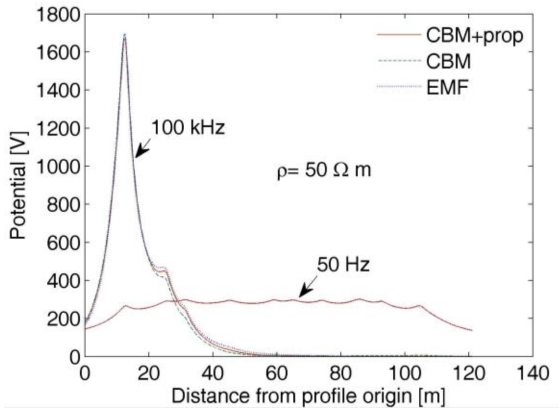


Fig. 3 Potential along the profile.

2.2. Lumped circuit (R-L-C) approach

The second method that was compared in this paper is the lumped circuit (R-L-C) approach. The main purpose of this method is to equivalent the grounding conductor with its input impedance or impedance to remote neutral ground. At low frequencies, the input impedance is represented by a single resistor and at high frequencies by a lumped R-L-C circuit, Fig. 4.

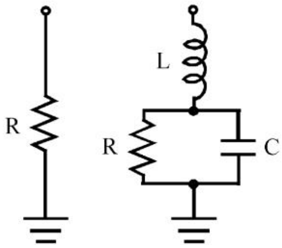


Fig. 4 Low- and high-frequency equivalent lumped circuit of the grounding conductor.

The expressions used for the circuit parameters of a vertical grounding rod are taken from reference [11].

$$
\begin{aligned}
\boldsymbol{R} &= \frac{1}{2\pi \boldsymbol{l}\boldsymbol{\sigma}} \log \frac{2\boldsymbol{l}}{\boldsymbol{a}} (\Omega) \\
\boldsymbol{C} &= 2\pi\varepsilon \boldsymbol{l} / \log \frac{2\boldsymbol{l}}{\boldsymbol{a}} (\boldsymbol{F}) \\
\boldsymbol{L} &= \frac{\mu_0 \boldsymbol{l}}{2\pi} \log \frac{2\boldsymbol{l}}{\boldsymbol{a}} (\boldsymbol{H})
\end{aligned}
\tag{2}
$$

2.3. TL model

The third method being compared in this paper is the TL or distributed circuit parameters method, Fig. 5. This method assumes transverse-electromagnetic (TEM) propagation on a perfect infinite conductor in a homogeneous medium and neglects the effects of the earth-air interface.

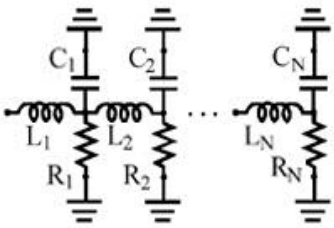


Fig. 5 Discrete approximation of the distributed circuit representation of the grounding conductor.

The distributed circuit parameters (per length) are obtained from the lumped circuit parameters (2) in the following manner

$$\begin{aligned} R' &= \frac{1}{G'} = R \cdot l \,(\Omega m) \\ C' &= \frac{C}{l} (F/m) \\ L' &= \frac{L}{l} (H/m) \end{aligned} \tag{3}$$

Fig. 3 presents a discrete approximation of the distributed-parameter circuit, where each segment of the grounding conductor is represented by a R-L-C section. Identical parameters are used for each section. The impedance to ground of the grounding electrode is in fact, the input impedance of the transmission line open at the lower end [12].

$$\begin{aligned} Z &= Z_0 \cdot \coth \gamma l \\ Z_0 &= \sqrt{\frac{j\omega L'}{(G' + j\omega C')}} \\ \gamma &= \sqrt{j\omega L'(G' + j\omega C')} \end{aligned} \tag{4}$$

2.4. EMF approach

The EMF method is used to investigate the validity range of the methods presented in the previous subsections. The referent results are obtained by rigorous Method of moments calculations of the Mixed potential integral equation [8] implemented on the identical system. For the perfect conductor from Fig. 1, the above mentioned equation in the frequency domain expresses the *z* component (tangential) of the electric field as

$$E_z = -j\omega \int_l G_A I(z') dz' + \frac{1}{j\omega} \frac{d}{dz} \int_l G_V \frac{dI(z')}{dz'} dz' \tag{5}$$

The longitudinal current *I(z')* is then expanded as a linear combination

$$I(z') = \sum_{n=1}^{N} f_n I_n \tag{6}$$

where I_n are the unknown current values on every segment and f_n are triangular basis functions.

The following matrix equation yields the current distribution [5], [13].

$$[Z][I] = [-Z_s I_s] \tag{7}$$

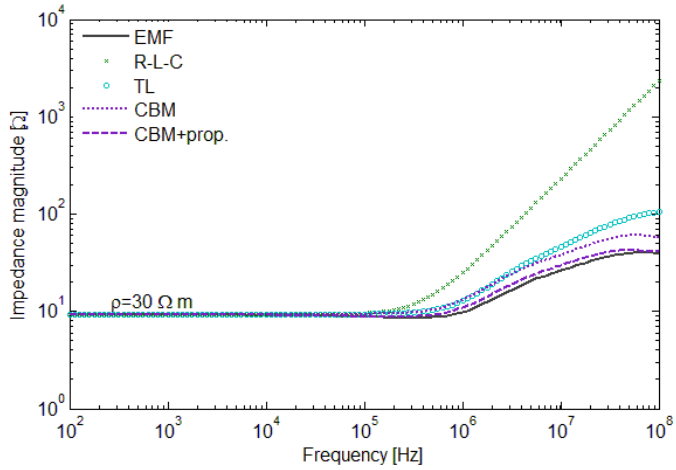
where the array [I] represents the currents to be determined, [Z] is the impedance matrix of mutual impedances between each of the current elements, [$-Z_S I_S$] represents the energization array, and Is is the injected (source) current. The elements of the matrix [Z] are calculated between the observation segment *m* and the source segment *n*, as

$$z_{nm} = \frac{V_{nm}}{I_n} = \frac{-1}{I_n} \int_{l_m} E_{nz} dl_m \tag{8}$$

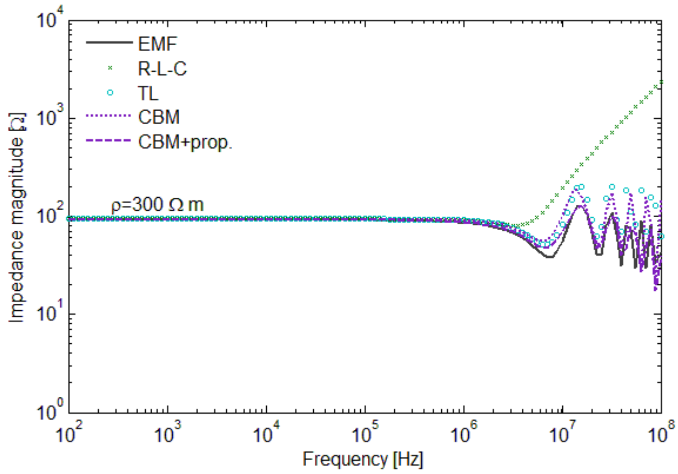
More details of the MPIE solution can be found in [13].

3. RESULTS

In the first part of this section, the comparison between the curves of the impedance to ground obtained by the different methods is presented.



a)

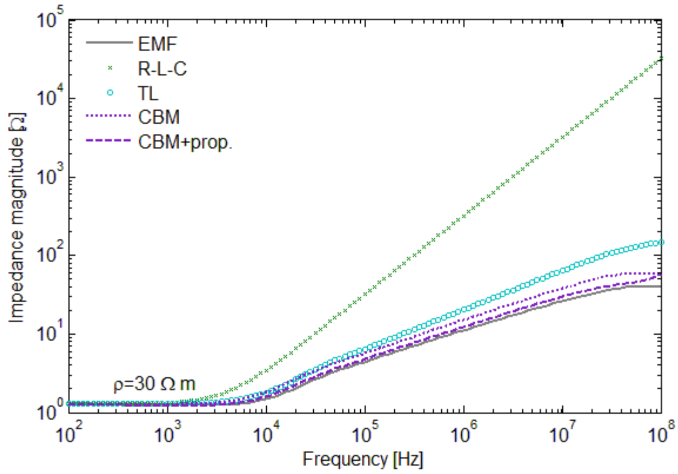


b)

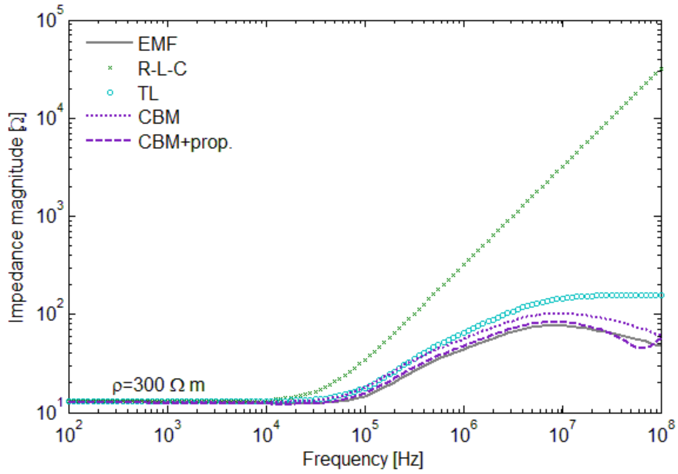
Fig. 6 Impedance to ground of a 3m long vertical grounding rod

Fig. 6 a) and b) present the impedance to ground for 3m long vertical grounding conductor, calculated by implementation of CBM method, compared to curves obtained by the other methods from literature [6]. The radius of the analyzed conductor is 1.25 cm. The value of soil resistivity ($\rho=1/\sigma$) is a) 30 Ωm and b) 300 Ωm. The length of the conductor is increased 10 times in Fig. 7. As shown in the figures, the CBM method is implemented in two ways – first without time propagation, and secondly by including time propagation effects in the equivalent circuit parameters. It is visible from Figs. 6 and 7 that there is an excellent agreement between the results obtained by the CBM method including time propagation and the referent EMF method. The differences between all the other impedance curves and the above mentioned method are clearly significantly larger, especially for high frequencies. The CBM method takes into account not only the self characteristics of each segment but also the coupling among different segments. That is the main reason for the high precision of CBM compared with other less accurate circuit methods, some of which are tested in this paper.

The second part of the section provides a parametric analysis of the grounding impedance of a horizontal conductor buried in imperfect ground, in terms of soil resistivity and conductor length. Classical and enhanced CBM is compared in the latter case.

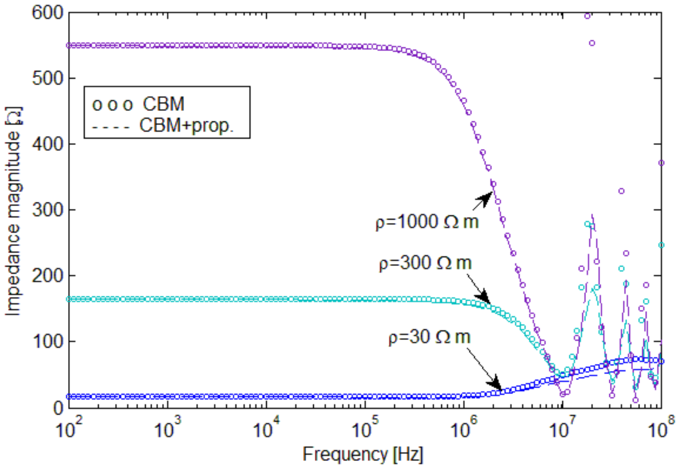


a)

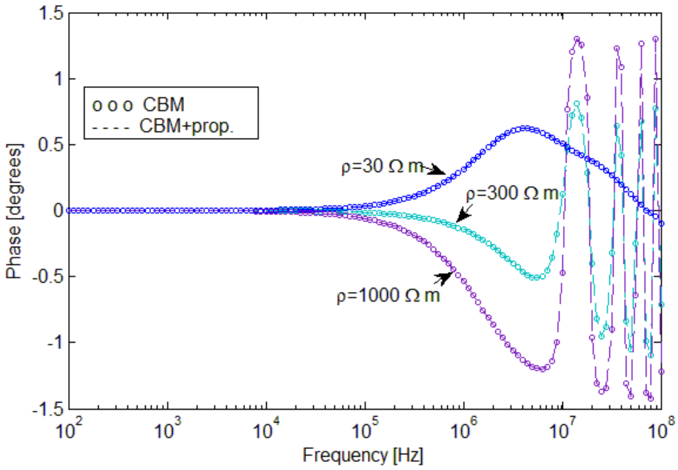


b)

Fig. 7 Impedance to ground of a 30m long vertical grounding rod



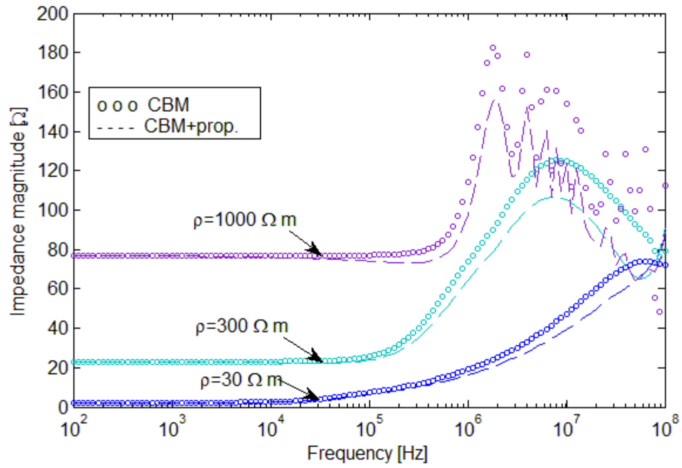
a)



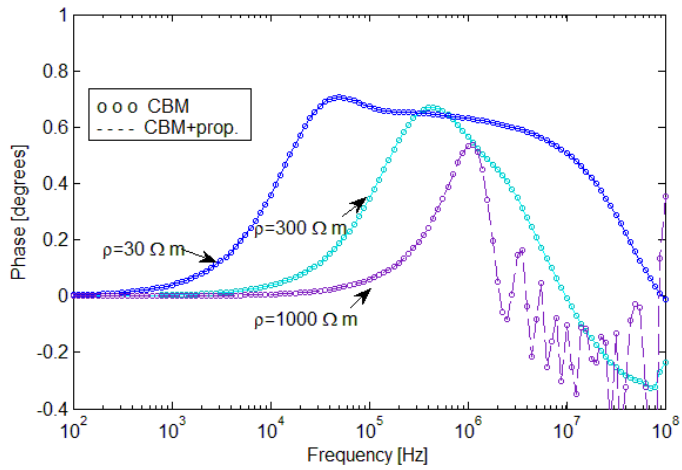
b)

Fig. 8 Impedance to ground of a 3m long horizontal grounding conductor

The dependency of the grounding impedance on the specific soil resistivity is shown in the following figures. Fig. 8 a) and b) presents the magnitude and phase of the grounding impedance for a 3m long horizontal grounding conductor and Fig. 9 for a 30m long horizontal conductor, respectively. The radius of the analyzed conductor is 1.25 cm, the depth of burial is 80 cm and the relative permittivity of soil is set to ε_r=10.



a)

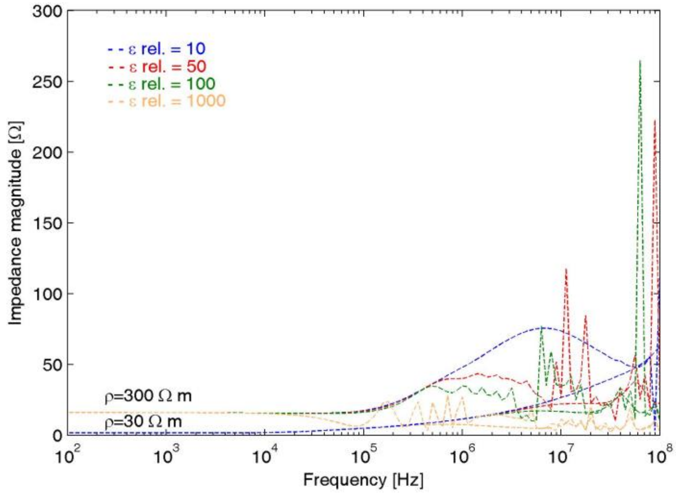


b)

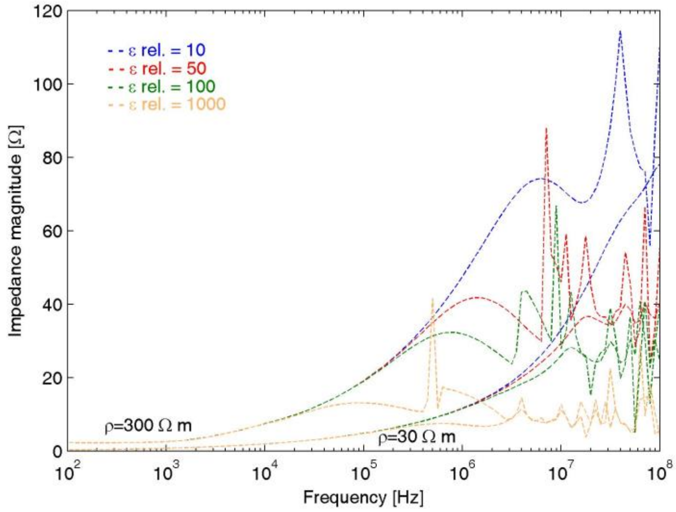
Fig. 9 Impedance to ground of a 30m long horizontal grounding conductor.

It may be observed from Figs. 8 and 9 that as could be expected, the impedance is significantly higher for grounding conductors placed in highly resistive soil.

The third part of this section provides a parametric analysis of the grounding impedance of a horizontal conductor buried in imperfect ground, in terms of soil resistivity and relative permittivity. Classical and enhanced CBM is implemented for horizontal and vertical grounding conductors. Fig. 10 a) and b) presents the magnitude of the grounding impedance for a horizontal grounding conductor with 30 and 300 m length.



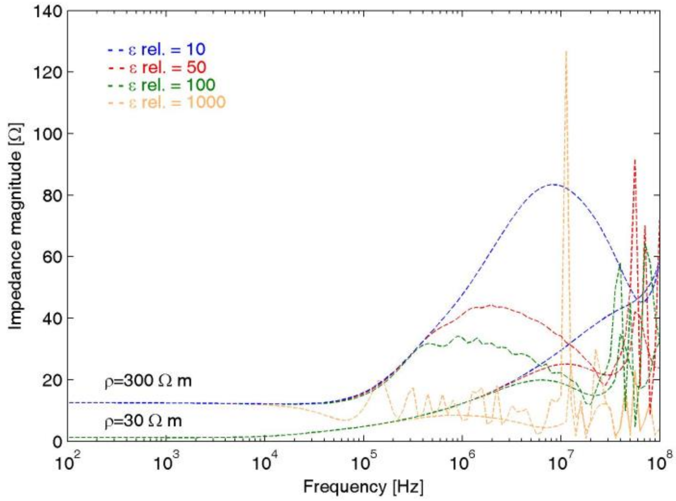
a) *l*=30 m



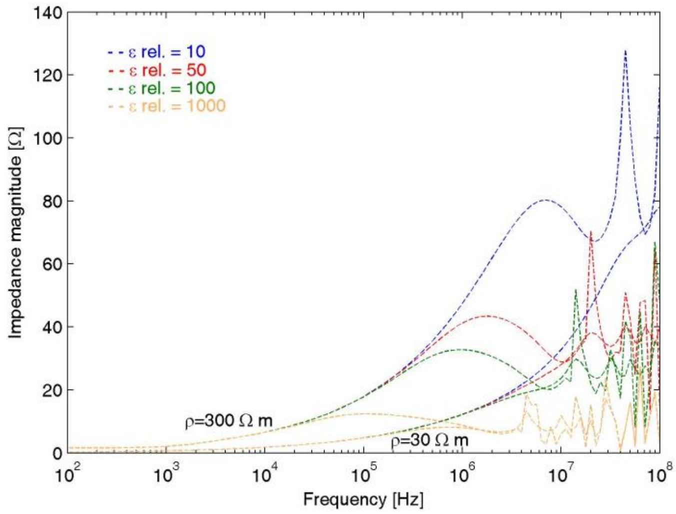
b) *l*=300 m

Fig. 10 Impedance to ground of a horizontal grounding conductor

Fig. 11 a) and b) presents the magnitude of the grounding impedance for a 30 and 300 m long vertical grounding conductor



a) l=30 m



b) l = 300 m

Fig. 11 Impedance to ground of a vertical grounding conductor

4. CONCLUSIONS

It may be concluded from the presented research that the CBM method is a solid choice for determining the grounding impedance of buried single conductors, in terms of accuracy. Even the classical form of CBM that doesn't include time propagation of EM fields provides much more accurate results than the other investigated methods, compared to the referent results. It may be observed in the presented figures that a very high agreement exists between the referent results and those obtained by implementation of enhanced CBM method (HEM). The higher accuracy of this method compared to other circuit methods is mainly due to the coupling of segments that CBM (and HEM) take into account.

The high precision of the CBM method in addition with the relative simplicity and high speed of its application shows that it may be one of the best choices in the analysis of grounding systems.

REFERENCES

[1] A. Kuhar and L. Grcev, "Calculating the impedance of grounding electrodes using circuit equivalents", In Proc. of the 12th International Conference on Applied Electromagnetics - ΠΕC 2015, Niš, Serbia, 2015.
[2] R. Velazquez and D. Mukhedkar, "Analytical modelling of grounding electrodes transient behavior", *IEEE Trans. Power Apparatus and Systems*, vol. 103, pp. 1314-1322, 1984.
[3] F. Napolitano, M. Paolone, A. Borghetti, C. A. Nucci, F. Rachidi, V. A. Rakov, J. Schoene and M. A. Uman, "Interaction Between Grounding Systems and Nearby Lightning for the Calculation of Overvoltages in Overhead Distribution Lines", IEEE Trondheim PowerTech, 2011.
[4] P. Yutthagowith, A. Ametani: "Application of a hybrid electromagnetic circuit method to lightning surge analysis", IEEE Trondheim PowerTech, 2011.
[5] A. F Otero, J. Cidras and J. L. Alemo, "Frequency-dependent grounding system calculation by means of a conventional nodal analysis technique" *IEEE Transactions on power delivery*, vol. 14, no. 3, pp. 873-878, 1999.
[6] L. Grcev and M. Popov, "On high-frequency circuit equivalents of a vertical ground rod", *IEEE Transactions on power delivery*, vol. 20, no. 2, pp. 1598-1603, 2005.
[7] R. Jankoski, A. Kuhar and L. Grcev, "Application of the electric circuit approach in the analysis of grounding conductors", In Proc. of the 5th International Symposium on Applied Electromagnetics – SAEM'2014, Skopje, Macedonia, 2014.
[8] S. Visacro, F.H. Silveira, "Evaluation of current distribution along the lightning discharge channel by a hybrid electromagnetic model", *Journal of Electrostatics*, vol. 60, pp. 111-120, 2004.
[9] V. Arnautovski-Toseva, L. Grcev, "Electromagnetic Analysis of Horizontal Wire in Two-layered Soil", *Journal of Computational and Applied Mathematics*, vol. 168, no. 1-2, pp. 21-29, 2004.
[10] A. Kuhar, L. Ololoska-Gagoska and L. Grcev, "Numerical analysis of complex grounding systems using circuit based method", In Proc. of the XII International Conference – ETAI, Ohrid, Macedonia, 2015.
[11] R. Rudenberg, Electrical shock waves in power systems, Cambridge, MA: Harvard Univ. Press, 1968.
[12] S. Bourg, B. Sacepe and T. Debu, "Deep earth electrodes in highly resistive ground: Frequency behavior", In Proc. of the IEEE Int. Symp. Electromagnetic Compatibility, 1995, pp. 584-589.
[13] L. Grcev, B. Markovski, V. Arnautovski-Toseva, A. Kuhar, K. El Khamlichi Drissi, K. Kerroum: "Modeling of horizontal grounding electrodes for lightning studies", In Proc. of the European electromagnetics, EUROEM 2012, Toulouse, France, July 2-6, 2012.