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Original scientific paper

GUIDING ACCURACY OF THE WATT COMPLIANT COGNATE MECHANISMS

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Abstract. *In kinematics, cognate linkages are those that ensure the same input-output relationship or coupler curve geometry, while being dimensionally dissimilar. Compliant mechanisms gain some or all of their mobility from the relative flexibility of their joints rather than from the rigid-body joints only, which is the case with classical mechanisms. In this paper the guiding accuracy of the compliant cognate Watt four-bar linkages has been presented. The compliant cognate mechanisms have been developed as the counterparts of the rigid-body Watt mechanism, where the coupler point can be guided on an approximate rectilinear path. The guiding accuracy of the "coupler" point on the path segment (minimal deviation between exact rectilinear and realized path) has been calculated for the new-designed compliant mechanisms.*

Key Words: *Compliant Mechanisms, Four-bar Linkages, Cognate Linkages, Watt Mechanism, Coupler Point Guiding*

1. INTRODUCTION

Compliant mechanisms gain some or all of their mobility from the relative flexibility of their joints rather than from the rigid-body joints only, which is the case with classical mechanisms. There are many advantages of using the compliant joints in the mechanism structure: a mechanism can be built in one piece, the weight can be reduced and wear, clearance, friction, noise and need for lubrication can be eliminated. Therefore, they are suitable for application in micromachining. On the other hand, because of the elastic deformation, the motion range of these flexible segments is restricted [1].

There are many papers considering the structure and function of the compliant joints and compliant mechanisms. A number of authors have established basic nomenclature and classification for the components of compliant mechanisms [2, 3, 4]. A method to aid the design of a class of compliant mechanisms wherein the flexible sections (flexural pivots) are small in length compared to the relatively rigid sections has been also

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suggested [2, 5]. Ananthasuresh and Kota [6] have presented a formal structural optimization technique called the homogenization method in order to design flexible structures (compliant mechanisms). Böttcher et al. [7] have introduced new ideas of technically realizable joints from nature and their integration into elastically movable structures for motion tasks in positioning and manipulating engineering.

The influence of the geometry as well as the material type of the compliant joints on guiding accuracy of some point of the compliant mechanisms has been also analyzed [8].

Pavlović and Pavlović [9] have presented the original designs of the compliant parallel-guiding mechanisms.

There are many papers that offer information on large motion compliant mechanisms. Mackay et al. [10] have introduced metrics for large-displacement linear-motion compliant mechanisms. Dirksen et al. [11] have presented the topology synthesis of large displacement compliant mechanisms with specific output motion paths. The large-range compliant mechanisms have been also investigated [12–14].

Several papers are dedicated to the consideration of the pantograph apparatus as a basis for developing a compliant mechanism. Merriam et al. [15] have described a statically balanced concept and have demonstrated its optimization, testing, and implementation for a haptic pantograph mechanism. A number of authors have presented how compliant pantograph mechanisms can be used for linear displacement applications [16, 17]. Patil et al. have dealt with a large motion displacement analyzed with an FEM simulation. For the model shown in [18], the pantograph apparatus is not included but a similar double parallel mechanism is shown and used to create a specific desired motion. Stojiljković et al. [19] have presented the designing of the compliant mechanism based on two symmetrical pantograph rigid-body mechanisms in order to achieve large scissors-like motion.

The designing and guiding accuracy of the compliant cognate four-bar linkages developed as the counterparts of the rigid-body Roberts–Chebyshev mechanism, have been also presented [20, 21, 22].

The frequent method to synthesize a compliant mechanism is to design it as the counterpart of the rigid-body linkage being able to realize pre-defined function of the compliant mechanism (rigid-body replacement method). This paper deals with the design and guiding accuracy analyze of the compliant cognate four-bar linkages. The compliant cognate mechanisms have been developed as the counterparts of the rigid-body Watt mechanism, where the coupler point can be guided on an approximate rectilinear path.

The aim of this paper is to analyze the guiding accuracy of the "coupler" point on the path segment (minimal deviation between exact rectilinear and realized path) for the new-designed compliant mechanisms.

2. WATT RIGID-BODY FOUR-BAR LINKAGE

The coupler point C of the Watt four-bar linkage (Fig. 1), located in the middle of the coupler, can be guided on an approximate rectilinear path segment [6].

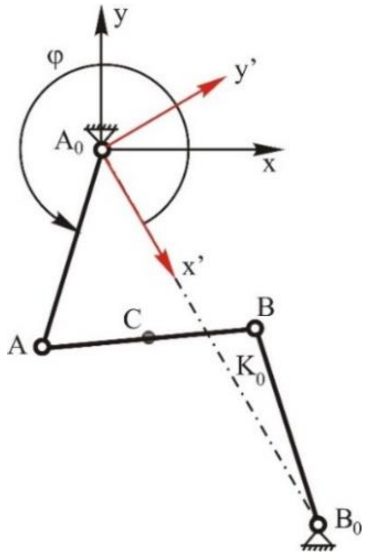


Fig. 1 Watt four-bar linkage

The links lengths of this mechanism as well as the position of the input crank, that enable realizing of horizontal displacement of coupler point C of Δx_C = 5 mm, with the small angular displacement of the input crank ($\Delta\varphi \leq 5^o$) and the minimal difference between realized and exact rectilinear path (Δy_C), has been shown as following [6]:

$$a = \overline{A_0A} = 61.406\ mm \tag{1a}$$

$$b = \overline{B_0B} = a \tag{1b}$$

$$c = \overline{AB} = 1.04a \tag{1c}$$

$$d = \overline{A_0B_0} = 2.1a \tag{1d}$$

$$\overline{AC} = \overline{BC} = \frac{c}{2} \tag{1e}$$

$$\varphi = 313 \div 318\ ^o \tag{1f}$$

$$\Delta y_C = 6.4\ \mu m \tag{1g}$$

The cognate four-bar linkages are those that ensure the same input-output relationship or coupler curve geometry, while being dimensionally dissimilar.

The Roberts–Chebyshev theorem states that each coupler curve can be generated by three different four-bar linkages. These four-bar linkages can be constructed using similar triangles and parallelograms.

Fig. 2 shows two cognate Watt rigid-body four-bar linkages: $A_0A_1CB_1K_0$ and $B_0B_2CA_2K_0$, introduced in this paper, where coupler point C, located in the end of the coupler, can be also guided on an approximate rectilinear path segment.

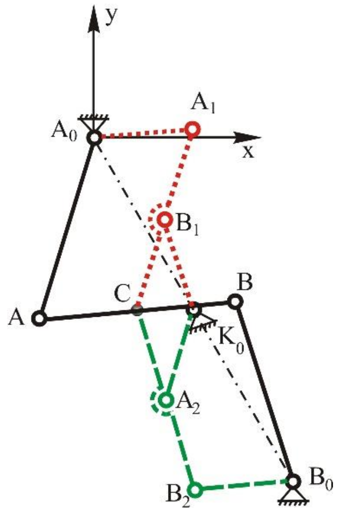


Fig. 2 The cognate Watt four-bar linkages

Geometrical relations defining link positions are:

$$\overline{A_0A_1} \parallel \overline{AC} \wedge \overline{A_0A} \parallel \overline{A_1C} \tag{2a}$$

$$\overline{B_0B_2} \parallel \overline{BC} \wedge \overline{B_0B} \parallel \overline{B_2C} \tag{2b}$$

$$\overline{AC}:\overline{CB} = \overline{A_1B_1}:\overline{B_1C} \Rightarrow \overline{B_1C} = \frac{1}{2}\overline{A_1C} \tag{2c}$$

$$\overline{BC}:\overline{CA} = \overline{B_2A_2}:\overline{A_2C} \Rightarrow \overline{A_2C} = \frac{1}{2}\overline{B_2C} \tag{2d}$$

$$\overline{AC}:\overline{CB} = \overline{A_0K_0}:\overline{K_0B_0} \Rightarrow \overline{A_0K_0} = \frac{1}{2}\overline{A_0B_0} \tag{2e}$$

Coordinates of the cognate mechanism joints have been calculated based on plane analytic geometry, respectively:

- joint A_1

$$x_{A1} = \frac{y_{A0} - y_C + \frac{y_{A0}-y_A}{x_{A0}-x_A}x_C - \frac{y_C-y_A}{x_C-x_A}x_{A0}}{\frac{y_{A0}-y_A}{x_{A0}-x_A} - \frac{y_C-y_A}{x_C-x_A}} \tag{3a}$$

$$y_{A1} = y_C + \frac{y_{A0}-y_A}{x_{A0}-x_A}(x_{A1} - x_C) = y_{A0} + \frac{y_C-y_A}{x_C-x_A}(x_{A1} - x_{A0}) \tag{3b}$$

- joint B_1

$$x_{B1} = \frac{1}{2}(x_{A1} + x_C) \tag{3c}$$

$$y_{B1} = \frac{1}{2}(y_{A1} + y_C) \tag{3d}$$

- joint B_2

$$x_{B2} = \frac{y_{B0} - y_C + \frac{y_{B0}-y_B}{x_{B0}-x_B} x_C - \frac{y_C - y_B}{x_C - x_B} x_{B0}}{\frac{y_{B0}-y_B}{x_{B0}-x_B} - \frac{y_C-y_B}{x_C-x_B}} \tag{3e}$$

$$y_{B2} = y_C + \frac{y_{B0}-y_B}{x_{B0}-x_B}(x_{B2} - x_C) = y_{B0} + \frac{y_C - y_B}{x_C - x_B}(x_{B2} - x_{B0}) \tag{3f}$$

- joint A_2

$$x_{A2} = \frac{1}{2}(x_{B2} + x_C) \tag{3g}$$

$$y_{A2} = \frac{1}{2}(y_{B2} + y_C) \tag{3h}$$

- joint K_0

$$x_{K0} = \frac{1}{2}(x_{A0} + x_{B0}) \tag{3i}$$

$$y_{K0} = \frac{1}{2}(y_{A0} + y_{B0}) \tag{3j}$$

3. WATT COMPLIANT FOUR-BAR LINKAGE

Fig. 3 shows a notch joint (circular flexure hinge) as a characteristic type of the compliant joint [6]. This compliant joint is fully determined by two parameters: the width of relatively rigid segments w_R and the width of relatively elastic segments w_E.

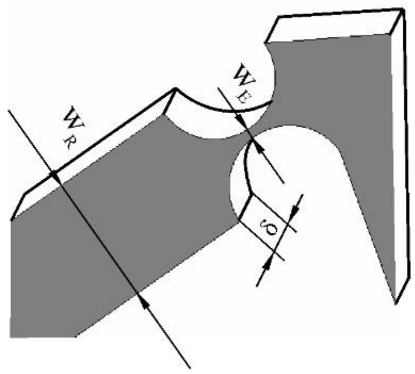


Fig. 3 A notch compliant joint

Based on the rigid-body Watt mechanism, a compliant Watt mechanism with notch joints was developed (Fig. 4a). Compliant joints A_0 and A have been oriented in the direction of rigid-body input crank A_0A, while compliant joints B_0 and B have been oriented in the direction of rigid-body follower B_0B.

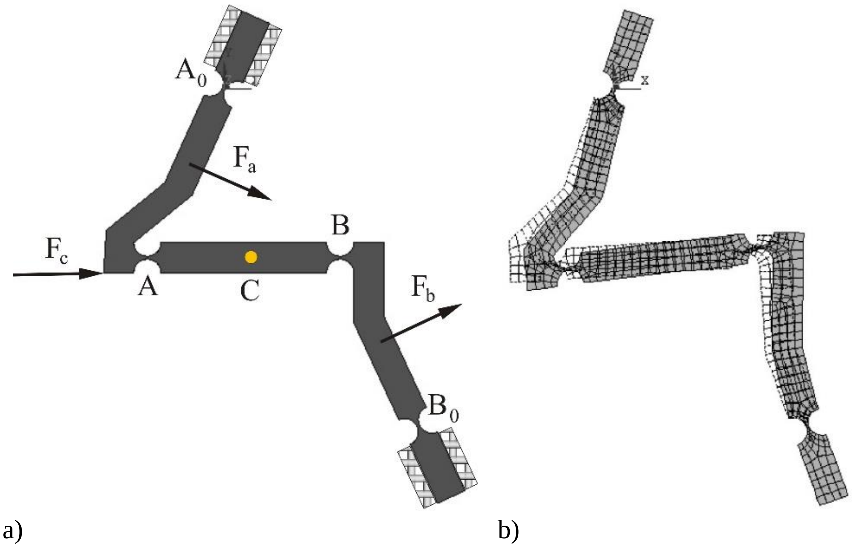


Fig. 4 The basic Watt compliant mechanism with notch joints (circular flexure hinges) in: a) undeformed position, b) deformed position

The input force may act in the middle of the "input crank" (F_a), in the middle of the "follower" (F_b) or at the end of "coupler" (F_c). The deformed position of the compliant Watt mechanism is shown in Fig. 4b [6].

The symmetry axes of the compliant notch joints cross each other at the point 1 (Fig. 5) corresponding to revolute joint of the rigid-body counterpart.

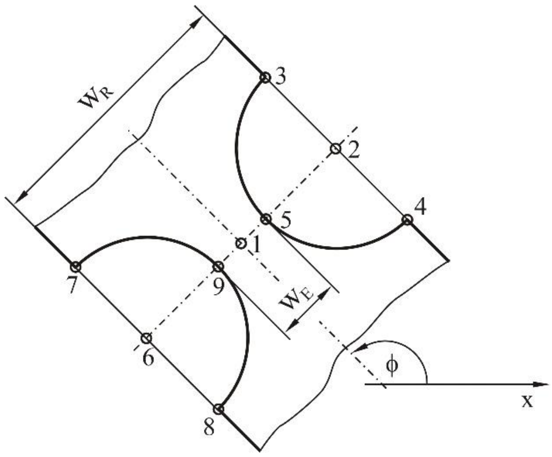


Fig. 5 Characteristic key points of a compliant notch joint

The other characteristic key points of a compliant notch joint (Fig. 5) can be calculated by using the set of equations:

$$\overrightarrow{r_2} = \overrightarrow{r_1} + \frac{w_R}{2} e^{i\left(\phi - \frac{\pi}{2}\right)} \tag{4a}$$

$$\overrightarrow{r_3} = \overrightarrow{r_2} + \frac{w_R - w_E}{2} e^{i\phi} \tag{4b}$$

$$\overrightarrow{r_4} = \overrightarrow{r_3} + \frac{w_R - w_E}{2} e^{i(\phi - \pi)} \tag{4c}$$

$$\overrightarrow{r_5} = \overrightarrow{r_4} + \frac{w_R - w_E}{2} e^{i\left(\phi + \frac{\pi}{2}\right)} \tag{4d}$$

$$\overrightarrow{r_6} = \overrightarrow{r_1} + \frac{w_R}{2} e^{i\left(\phi + \frac{\pi}{2}\right)} \tag{4e}$$

$$\overrightarrow{r_7} = \overrightarrow{r_6} + \frac{w_R - w_E}{2} e^{i\phi} \tag{4f}$$

$$\overrightarrow{r_8} = \overrightarrow{r_6} + \frac{w_R - w_E}{2} e^{i(\phi - \pi)} \tag{4g}$$

$$\overrightarrow{r_9} = \overrightarrow{r_6} + \frac{w_R - w_E}{2} e^{i\left(\phi - \frac{\pi}{2}\right)} \tag{4h}$$

Angle ϕ determines the orientation of a compliant joint in reference to the x-axis (Fig. 5).

Fig. 6a shows the first cognate compliant Watt mechanism with notch joints, introduced in this paper, that has been developed based on cognate Watt rigid-body four-bar linkage $A_0A_1CB_1K_0$ (Fig. 6b).

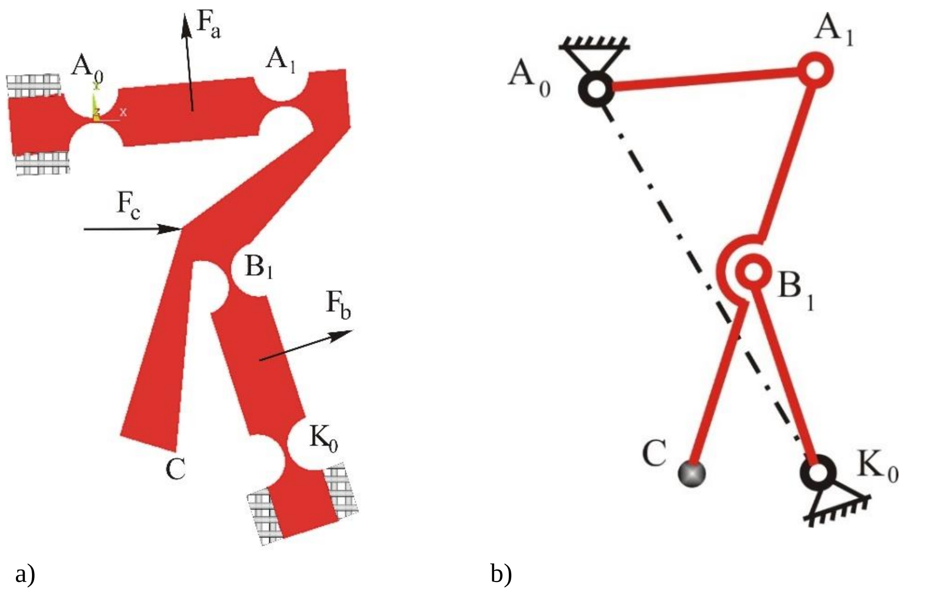


Fig. 6 The first Watt cognate compliant mechanism with notch joints (circular flexure hinges) (a) and its rigid-body counterpart $A_0A_1CB_1K_0$ (b)

Fig. 7a shows the second cognate compliant Watt mechanism with notch joints, introduced in this paper, that has been developed based on the cognate Watt rigid-body four-bar linkage $B_0B_2CA_2K_0$ (Fig. 7b).

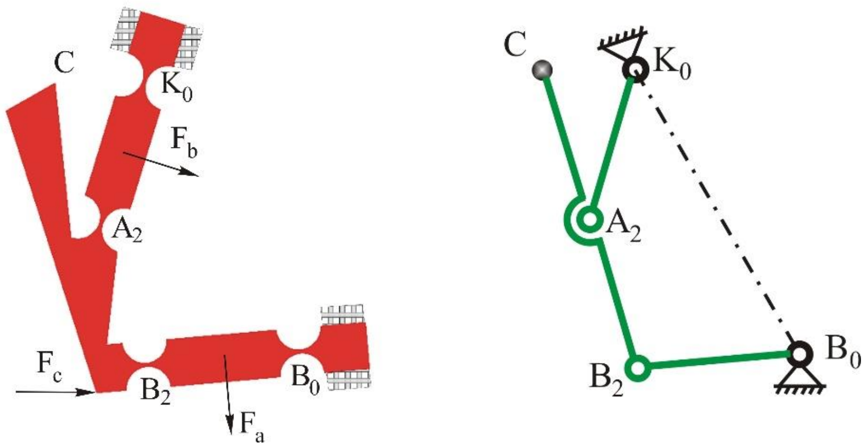
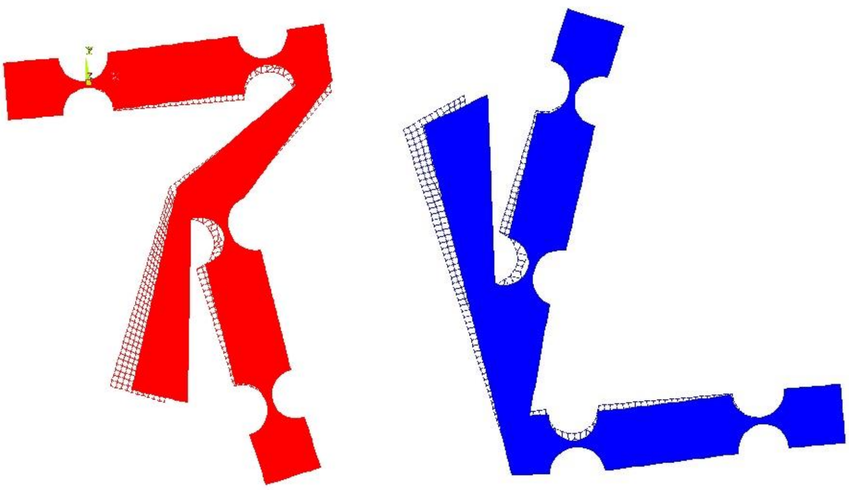


Fig. 7 The second Watt cognate compliant mechanism with notch joints (circular flexure hinges) (a) and its rigid-body counterpart $B_0B_2CA_2K_0$ (b)

The deformed position of the Watt cognate compliant mechanisms with notch joints are shown in Fig. 8.



a) b)

Fig. 8 The first (a) and the second (b) Watt cognate compliant mechanism with notch joints in deformed position

4. GUIDING ACCURACY OF THE WATT COMPLIANT MECHANISMS

The guiding accuracy of "coupler" point C (on the path segment of Δx_C = 5 mm) of the compliant Watt mechanism as well its cognate compliant mechanisms with notch joints were analyzed. The lengths of the mechanism "links" have been defined in equations (1) with the following compliant joint parameters: the width of relatively rigid segments w_R = 10 mm and the width of relatively elastic segments w_E = 1mm.

The position analysis of the compliant mechanisms was performed using the ANSYS Software for material piacryl (modulus of elasticity E = 3700 N/mm^2, bending strength σ_{bs} = 90 N/mm^2) as well as material thickness of δ = 4 mm. This material is not expensive; it is available on the market as well as suitable for further experimental research of these compliant mechanisms. The calculation was performed for the elements with a rectangular cross-sectional area using PLANE 183 (2-D 8-Node Structural Solid) as a characteristic ANSYS element type (Fig. 9).

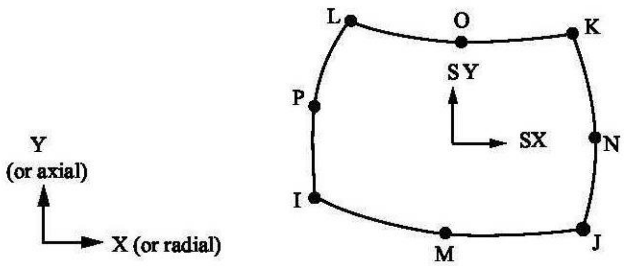


Fig. 9 ANSYS element type PLANE 183 (2-D 8-Node Structural Solid)

Due to the fact that the acting point of the input force does not have to be located on the "input crank" of the compliant mechanism, the guiding accuracies were compared for three different cases of location of input force acting points: in the middle of the "input crank" - F_a, in the middle of the "follower" - F_b and at the "coupler" point nearby the joint A or B– F_c (Fig. 4a, 6a, 7a).

The results are shown in Table 1.

Table 1 Guiding accuracy Δy_C [μm] of the Watt compliant mechanisms on the horizontal displacement of Δx_C = 5 mm

	F_a	F_b	F_c
0	2.3	8.8	11.7
1	0.3	6.5	17.8
2	29.2	0.6	1.3

0 - the basic Watt compliant mechanism
1 - the first cognate Watt compliant mechanism
2 - the second cognate Watt compliant mechanism

5. CONCLUSION

The frequent method to synthesize a compliant mechanism is to design it as the counterpart of the rigid-body linkage being able to realize pre-defined function of the compliant mechanism (rigid-body replacement method). This paper deals with the design and guiding accuracy analyze of the compliant cognate four-bar linkages. The basic compliant mechanism as well as two compliant cognate mechanisms have been developed as the counterparts of the rigid-body Watt mechanism, where the coupler point can be guided on an approximate rectilinear path.

The guiding accuracy, that is, the difference between realized and exact rectilinear path, have been analyzed and compared for all above mentioned compliant mechanisms being suitable to realize approximate rectilinear guiding of the coupler point.

The basic compliant model with the input force acting in the middle of the "input crank", the first cognate compliant model with the input force acting in the middle of the "input crank" as well as the second cognate compliant model of the Watt mechanism with the input force acting in the middle of the "follower" and in the end of "follower", realize better rectilinear guiding accuracy than the rigid-body counterpart linkage. The differences between realized and exact rectilinear path of these compliant mechanisms rate in range $\Delta y_C = 0.3 \div 2.3$ μm on the path segment of $\Delta x_C = 5$ mm, comparing with the $\Delta y_C = 6.4$ μm of the rigid-body counterpart linkage. Also, the producing of the rigid-body linkages is not possible without appearing of the tolerance of the links lengths and clearance in the revolute joints, which decrease the guiding accuracy.

It means that the Roberts–Chebyshev theorem, concerning a coupler curve being generated by two different four-bar linkages, should be also applied on the compliant mechanism.

The best guiding accuracy has been provided by the first cognate Watt compliant mechanism ($\Delta y_C = 0.3$ μm on the path segment of $\Delta x_C = 5$ mm) with the input force acting point located in the middle of the "input crank".

Thus the cognate compliant mechanism has realized a smaller deviation between exact rectilinear and realized path than the basic compliant mechanism, and this fact confirms our suggestion that the results offered by the cognate compliant mechanisms should be taken into consideration in the synthesis procedure of the compliant mechanisms.

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REFERENCES

1. Zentner, L. Linβ, S., 2019, *Compliant systems - Mechanics of Elastically Deformable Mechanisms, Actuators and Sensors*, De Gruyter, Oldenbourg, Muenchen, 166 p.
2. Howell, L.L., 2001, *Compliant mechanisms*, John Wiley &Sons, Inc., New York, 480 p.
3. Howell, L.L., Magleby, S.P., Olsen, B.M., *Handbook of Compliant Mechanisms*, John Wiley & Sons Inc., New York, 352 p.
4. Midha, A., Norton, T.W., Howell, L.L., 1994, *On the nomenclature, classification and abstractions of compliant mechanisms*, ASME Journal of Mechanical Design, 116(1), pp. 270-279.

5. Howell, L.L., Midha, A., 1994, *Method for the design of compliant mechanisms with small-length flexural pivots*, ASME Journal of Mechanical Design, 116(1), pp. 280-290.
6. Ananthasuresh, G.K., Kota, S., 1995, *Designing compliant mechanisms,* Mechanical Engineering, 117 (11), pp. 93-96.
7. Böttcher, F., Christen, G., Pfefferkorn, H., 2001, *Structure and function of joints and compliant mechanism*, Motion Systems (Collected Short Papers of the Innovationskolleg "Bewegungssysteme" Friedrich-Schiller Universität Jena), Technische Universität Jena, Technische Universität Ilmenau, ShakerVerlag, Aachen, pp. 30-35.
8. Pavlović, N.T., Pavlović, N.D., 2013, *Compliant mechanisms* (in Serbian), Faculty of Mechanical Engineering, University of Niš, Serbia, 217 p.
9. Pavlović, N.T., Pavlović, N.D., 2009, *Compliant mechanism design for realizing of axial link translation*, Mechanism and Machine Theory, 44(5), pp. 1082-1091.
10. Mackay, A.B., Smith, D.G.,Magleby, S.P., Jensen, B.D., Howell, L.L., 2012, *Metrics for evaluation and design of large-displacement linear-motion compliant mechanisms*, J. Mech. Des.,134(1), 011008.
11. Dirksen, F., Berg, T., Lammering, R., Zohdi, T.I.,2012, *Topology synthesis of large-displacement compliant mechanisms with specific output motion paths,* Proceedings in Applied Mathematics and Mechanics (PAMM), 12, pp. 801–804.
12. Mackay, A.B., 2007, *Large-Displacement Linear-Motion Compliant Mechanisms*, Master Thesis, Brigham Young University - Provo,87 p.
13. Pei, X., Yu, J., Zong, G., Bi, S., 2010, *An effective pseudo-rigid-body method for beam-based compliant mechanisms,* Precis. Eng. 34(3), pp. 634–639.
14. Xu, Q., 2016,*Design and Implementation of Large-Range Compliant Micropositioning Systems*, John Wiley & Sons Inc., New York, 274 p.
15. Merriam, E.G., Colton, M., Magleby, S.P., Howell, L.L., 2013, *The design of a fully compliant statically balanced mechanism,* In: Proceedings of the ASME2013 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference. Volume 6A: 37^{th} Mechanisms and Robotics Conference, Portland, Oregon, USA (7 pages)
16. Patil, V., Anerao, P.R., Chinchanika, S.S., 2016, *Finite element analysis of compliant pantograph mechanism,* Int. Eng. Res. J., pp. 65–68.
17. Deshmukh,B.,Pardeshi,S.,Mishra,P.K., 2012, *Conceptual design of a compliant pantograph,* Int. J. Emer. Technol. Adv. Eng. 2(8), pp. 270–275.
18. https://www.compliantmechanisms.byu.edu/wyrd-mechanisms (last access: 01.06.2022)
19. Stojiljković, D., Zentner, L., Pavlović, N.T., Linβ, S., Uhlig, R., 2021, *Modeling, Design and Prototyping of a Pantograph-Based Compliant Mechanism,* In: Zentner L., Strehle S. (eds) *Microactuators, Microsensors and Micromechanisms*, MAMM 2020, Mechanisms and Machine Science, 96, Springer, Cham, pp. 76-88.
20. Pavlović, N.T., Pavlović, N.D., 2018, *Design of Compliant Cognate Mechanisms*, Proceedings of the 4th International Conference "Mechanical Engineering in XXI Century", Niš, pp. 253-258.
21. Stojiljković, D., Pavlović, N.T.,2020, *Influence of Flexure Hinges Design on Guiding Accuracy of Roberts-Tsebysev Compliant Mechanism*, Proceedings of the 5th International Conference "Mechanical Engineering in XXI Century", Niš, pp. 217-220.
22. Stojiljković, D., Pavlović N.T., Milošević M., 2022, *Design of Compliant Mechanism for Rectilinear Guiding with Non-conventional Optimization of Flexure Hinges,* 2021, In: Khang N.V., Hoang N.Q., Ceccarelli M. (Eds.), *Advances in Asian Mechanism and Machine Science*, ASIAN MMS 2021, Mechanisms and Machine Science, 113, Springer, Cham, pp. 370-378.