# RECONSTRUCTION OF ERODED AND VISUALLY COMPLICATED ARCHAEOLOGICAL GEOMETRIC PATTERNS: MINARET CHOLI, IRAQ 

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#### Abstract

Visually complicated patterns can be found in many cultural heritages of the world. Islamic geometric patterns present us with one example of such visually complicated archaeological ornaments. As long-lived artifacts, these patterns have gone through many phases of construction, damage, and repair and are constantly subject to erosion and vandalism. The task of reconstructing these visually complicated ornaments faces many practical challenges. The main challenge is posed by the fact that archaeological reality often deals with ornaments that are broken, incomplete or hidden. Recognizing faint traces of eroded or missing parts proved to be an extremely difficult task. This is also combined with the need for specialized knowledge about the mathematical rules of patterns' structure, in order to regenerate the missing data. This paper presents a methodology for reconstructing deteriorated Islamic geometric patterns; to predict the features that are not observed and output a complete reconstructed two-dimension accurate measurable model. The simulation process depends primarily on finding the parameters necessary to predict information, at other locations, based on the relationships embedded in the existing data and in the prior-knowledge of these relations. The aim is to build up from the fragmented data and from the historic and general knowledge, a model of the reconstructed object. The proposed methodology was proven to be successful in capturing the accurate structural geometry of many of the deteriorated ornaments on the Minaret Choli, Iraq. However, in the case of extremely deteriorated samples, the proposed methodology failed to recognize the correct geometry. The conceptual framework proposed by this paper can serve as a platform for developing professional tools for fast and efficient results.


## 1. INTRODUCTION

The Choli Minaret is dated to the Atabag period (1190-1232), during the reign of Muzaffaruddin Al-Kawkaboori, the king of Erbil. The structure was built from low burnt bricks and gypsum based renders and mortars. Due to the long term effect of deterioration, the essential part of the architecture has disappeared and thus the leaning minaret is the last survival of the past mosque. The current shape of the minaret covers the lower, heptagonal/octagonal base (about 12 m high) and the upper broken cylindrical part (about 24 m high) with double spiral staircase inside. A large extent of precious historic fragments of renderings and embossments were identified in the lower part of the object. Particularly large scale findings of Egyptian blue ceramic decoration in niches are considered as utmost important. All fragments of decorations were seriously affected by weathering and mechanical damages (Figure 1). To preserve these delicate artifacts, it is important to understand their geometry, mathematics and generating principles. Producing accurate virtual reconstructions of these ornaments can help professionals to test the different preservation scenarios and make the best decisions. However, the task of reconstructing these deteriorated patterns faces many practical challenges. The main challenge is posed by the fact that these patterns are broken, incomplete or hidden. Recognizing faint traces of eroded or missing parts proved to be an extremely difficult task. This is also combined with the need for specialized knowledge of patterns' structure, in order to build up from the fragmented data and from the historic and general knowledge, an accurate model of the deteriorated ornament. This paper presents a methodology for reconstructing these deteriorated geometric patterns; to predict the features that are not observed and output a complete reconstructed two-dimension accurate measurable model. The simulation process depends primarily on finding the parameters necessary to predict information, at other locations, based on the relationships embedded in the existing data and in the prior-knowledge of these relations. By incorporating the mathematical rules of patterns' structure, this process involves creating a hypothetical geometrical model, which can be fitted to the available partial data to fill in the missing gaps. The
reconstruction process is designed to measure certain patterns' variables, which are used within a mathematical formula to produce the whole system.


Figure 1: The Choli Minaret, Iraq, a: an image showing the lower part of the minaret, b: Rectified images of the different ornaments under investigation.

## 2. LITERATURE REVIEW

### 2.1 Islamic geometry

The rise of Islamic culture in the seventh century has marked the beginning of a new artistic, decorative and sacred tradition [1]. This artistic tradition was completely inspired by a deep religious philosophical and cosmological approach, which embodied all aspects of life and manifested itself in every product [2,3]. The use of geometric patterns is one of the chief characteristics that give the Islamic artistic heritage its distinct identity. For more than thirteen centuries they acted as unifying factors. They have linked the architectural products from all over the Islamic world, extending across Europe, Africa and Asia [4,5]. Geometry as an abstract art form was developed in part due to the discouragement of images in Islam on basis that it could lead to idolatry [6]. These visually diverse formations grow out from the same spiritual origin to represent the multiple manifestation of the divine $[7,8]$. Islamic geometric patterns were applied to all kinds of materials: metal work, woodworks, ceramics, textiles, carpets, stone, fabric and miniatures. The universal application of these patterns implies that they were created based on solid formal methods. The act of designing and applying these patterns was considered a form of worship and encapsulated a divine religious experience. These artists and the methods they used were secretive, and only few passed on this tradition until it was lost [9,10]. The vast variety of geometric formation and the strict rules of its generation reveal an important inner dimension of Islamic tradition: "unity in multiplicity and multiplicity in unity" [11]. Islamic designs were constructed by using a compass and a straight edge; therefore the circle becomes the foundation for Islamic patterns [12,13,14]. This "conventional" method emphasizes the symbolic relationship between the global dimension and its center. The generating force of patterns lies in the center of the circle, which represents the point at which all Islamic patterns begin. It is the symbol of a religion that emphasizes one God, the center of universe [15,16].

### 2.2 Periodic Islamic patterns

This decorative art is generated from a discrete geometrical unit using the circle as its basis, and then applying the principles of repetition, and symmetry to it [17,18,19]. Although each pattern has its own distinct geometrical design, the vast varieties of ornamental compositions are based on a simple constitutive geometry, which is generated from a limited number of simple base grids of polygons [20]. Mathematically these grids are known as regular tessellations, in which one regular polygon is repeated to fill the plane. The main basic grids are: a) Basic grids based on the equilateral triangles and hexagons and its multiples. b) Basic grids based on the squares, octagons and their multiples. c) Basic grids based on the pentagon and its multiples. d) Basic grids based on nine fold. e) Basic grid based on seven fold. f) Basic grids based on eleven fold. g) Basic grids based on a combination of basic categories. Figure 2 demonstrates the construction process of an octagon-square based pattern.

### 2.3 Aperiodic Islamic patterns

Recently, Islamic patterns, with five-fold symmetries and non-periodic geometry, have been discovered in some medieval Islamic patterns [21]. These non-periodic patterns exhibit a complicated long-range translational order that is not periodic and a long-range orientational order that does not have rotational point symmetry. This discovery has attracted a significant scientific interest into understanding the structural principles of Islamic formations. Some scientists have suggested that by the 1200 C.E. a conceptual breakthrough occurred in the way that Muslim artists conceived and implemented these patterns; shifting from the conventional-global method of using a compass and a straight edge, to a new perception, in which Islamic patterns were reconceived as tessellations of different types of prescribed tiles [22]. These scientists indicated that building theses complex non-periodic patterns would have required the application of a complex set of mathematical rules, which they believe, are beyond the grasp of the Muslim artists. Others have argued against this localized tiling system, which is challenging centuries of proven "conventional" knowledge as well as doubting the conceptual abilities of the Muslim artists to comprehend the global construction rules of these complex patterns. Inspired by the conventional view of Islamic geometry, recently, the first global construction method that is able to describe the long range order of aperiodic formations was proposed [23]. Al Ajlouni (2011), proposed a global multi-level hierarchical framework that is able to describe the long-range translational and orientational order of aperiodic formations. The proposed model shows that geometric arrangements of the non-periodic formations are determined entirely by one hierarchical framework, which works in perfect concert with the "conventional view" of pattern geometry. It suggests that the position of geometrical units, locally and globally, is defined by one global framework, and not tiled based on local tiling system. This hierarchical principle, presents a new methodology for understanding and constructing complicated Islamic patterns, which might generate a new perspective into the history, craft, construction and creativity of the Muslim Artists.


Figure 2: The construction process of an octagon-square based pattern

### 2.4 The Choli ornaments [24]

Islamic architecture and its decorative ornamentalism form an integral unity. The same attitude was applied to the construction of the Choli Minaret in Erbil. Due to the restrictions in Islamic art namely absence of figural art and elimination of certain creativity, decorative design has developed itself to the perfection in the Middle Ages. This art was in fact in its complexity an applied science, which in the fields of mathematics, geometry and optics reached a high level at that time. Islamic ornamentalism widely used spatial zoning and contrasts of light and shadow and emphasised the use of blue colour, the colour of water. All these elements were abundantly applied in the decorative composition of the Choli Minaret. The minaret makes use of the contrast of light and shadow in the spatial arrangement of the brick "mosaic" of the outer surface/shell and blue colour in ornaments in the minaret niches and dividing strips, using glazed tiles. The outer shell ornament formed regular patterns composed of the sizes of bricks. The design of niches and dividing strips also stemmed from the sizes of bricks and their parts, which were complemented with geometric shapes made of glazed tiles (Figure 1). The comprehensive conservation treatment of the minaret was implemented in 20082009 covering also the partial reconstruction of niches. All fragments of ceramic, brick and stucco decorations were seriously affected by weathering and mechanical damages. The principle of intervention was kept in terms of pure conservation of the historic landmark. The stabilization of the whole structure and conservation treatments of all surfaces were carried out without the tendency to reconstruct missing and unknown parts of the minaret. The only exception was the area of lower niches where pure conservation would be confusing from the architectural point of view. The inside area of all niches has been covered by mortar imitating the under layer for ceramic tiles. All remaining original parts were carefully consolidated and retouched. The inner vaults of niches were reconstructed to its original shape to make the architectural frame better identified. Samples of glazed tiles were tested for the purpose of determination of the type of terracotta. All samples displayed same results and proved that the ceramic body was made from ceramic material (terracotta) burned at lower temperatures. The coloured decoration layer is enamel, perhaps with low melting enamel coloured by Cu compounds. The cross section of studied tiles a clear bottom white vitreous layer containing silica was documented. The presence of approx. $18 \%$, sodium, $65 \%$, silica, potassium, calcium and copper was proved to be present in the blue
vitreous substance. The production of so-called Egyptian blue - glaze with copper oxides in strongly alkaline medium, was relatively frequent in the area. The first glazes documented in the Middle East were decorations at Ishtar Gate in Babylon.

## 3. RESEARCH DESIGN

The research at hand encapsulates two approaches within its methodology. Qualitative approach is evident in the process of reading and interpreting the surviving fragments as well as evaluating of the final reconstructions. Quantitative approach is manifested through using mathematical knowledge of pattern generation to produce the geometric models, which are used to simulate the missing information from the partial data. Although this research is generally based on deductive logic, it still uses induction to generalize output beyond the observed instances. The reconstruction process involves "induction, deduction and analogy". This research follows an empirical paradigm in testing its methodology and uses experimentation as its main strategy. The proposed methodology includes three main tasks.

### 3.1 Photogrammetry and image rectification [25]

Calibrated digital cameras, réseau photogrammetric camera RolleiMetric 6006 and total station were used for the basic documentation of the Choli minaret. The geodetic measurements were taken at a temperature of around $45^{\circ} \mathrm{C}$. The total station experienced some problems with the LCD display under this climate. A small provisional geodetic network consisting of 4 points was stabilized by the usage of nails and temporary marks. First of all, network point adjustments have been made on site. Preliminary calculations have been carried out in Erbil for control with satisfactory results of about 7 mm in position. All the necessary control points and object points were measured with accuracy of about 1-2 cm in position. Altogether approximately 250 points were calculated. Sets of 25 digital photogrammetric images have been taken using the Canon 20D digital camera with a resolution of 8 MP. This particular camera was calibrated by using Photomodeler software in the Laboratory of Photogrammetry at the Czech Technical University in Prague. Two zoomequipped lenses used for imaging were calibrated on focal lengths $10 \mathrm{~mm}, 22 \mathrm{~mm}$ and $17 \mathrm{~mm}, 85 \mathrm{~mm}$ respectively. Upon completion of the expedition all images taken on the site were scanned using a professional film scanner (Nikon CoolScan 8000 ) at true 2500 DPI . The outdoor parts of the Choli minaret were measured and processed by using intersection terrestrial photogrammetry in the Photomodeler software (residuals on control points were about 2 cm . All construction and editing of photogrammetrical measured items were processed in AutoCAD.

### 3.2 Reading and interpreting the surviving fragments

A periodic geometric pattern is defined mathematically as "a planar arrangement of line segments that together delineate copies of a small number of different shapes" [26]. To understand these patterns mathematically, it is important to study their abstractions; through which all rendering effects and colors are discarded and only the basic line structure is kept. The process starts by mapping the basic line structure of the surviving line fragments through extracting their center lines. The produced skeleton represents the minimum information needed to preserve the general structural of the available data. This abstraction is done manually and with the aid of AutoCAD tools. These line abstractions are then used as the basic template for pattern analysis and interpretation. The interpretation process involves deconstructing these abstract geometric formations to their elementary components, and then investigating the rules that organize the relations between geometric primitives (points and lines). The definition of certain intersection points, edges and shape symmetries are essential to the interpretation process. In this process certain parameters, within the pattern's structure, are defined, measured and then used to identify the generating basic grid and the repeated star units. Different mathematical models are then tested to see which one fits the available parameters. These models are checked against the surviving fragments to define the correct match.
A) Defining and generating the basic Grid

Periodic Islamic patterns are constructed based on a limited number of basic grids, which are generated from patterns of circles [27]. The complex geometric patterns are all elaborations of simpler constructions of circles, which are often used to determine the basic grids $[28,29]$. The process of defining the basic grid starts by locating all points of shape symmetry within the pattern. These points represent the center points of the circles that constitute the basic grid. The structure around this area is then mapped to define the polygon edges and divisions. The careful arrangement of these polygons generates the general structure of the basic grid. Figure 3a shows the basic grid of 14 folds and 11 folds, generated for one deteriorated ornament on the Choli minaret.
B) Defining and generating the repeated star units.

Defining the repeated star unit involves interpreting the line information contained within each polygon. Each repeated star unit is constructed using single polygon. The star unit is formed by an array of lines connecting either the
intersection points of the polygon's sides, or connecting the mid points of the polygons sides. The repeated star can be as simple as one array of lines or a combination of two or more simple stars (arrays). Figure 3b demonstrates the construction process of the two repeated star units used to generate the sample pattern on the Choli Minatare.

### 3.3 Generating the final reconstruction

The final reconstruction is generated by combining the repeated star units and the basic grid of polygons. As shown in Figure 4 a , the fourteen-fold star unit and the eleven-fold star unit are inserted into the grid of polygons, guided by the lines of the surviving fragments. Star unit are rotated and positioned according the laws of pattern's symmetry. Each of these star's arrays are then extended beyond the edges of their polygon to meet other arrays and form the connection areas between all polygons (Figure 4b). The final step involves trimming the edges of the pattern to fill the frame of the original ornament and adding a thickness to the line pattern to match the original pattern (Figure 4c).


Figure 3: The process of defining and generating the basic grid and the repeated star unit


Figure 4: The process of generating the final reconstruction

## 4. RESULTS AND DISCUSSION

The proposed methodology was tested on twelve different deteriorated ornaments on the Choli minaret. The proposed methodology was successful in capturing the accurate structural geometry of nine patterns (Figure 5). All of which are periodic patterns. However, in the case of extremely deteriorated samples, the proposed methodology failed to reconstruct the patterns; in most cases, surviving evidence was not sufficient to read the basic grid or the repeated star unit. However, in one case, where the surviving evidence seemed to be sufficient, the deteriorated fragments did not follow a periodic logic and therefore the methodology failed to render any results. Based on these results, three basic grids were identified; five patterns were generated based on six folds symmetries. All of these patterns where located on the upper part of all niches. Three patterns were generated based on four and eight folds and one pattern was generating based on a combination of eleven and fourteen folds. All of which were located in the lower part of the niches.


Figure 5: The final reconstructed patterns of the sampled data

## 5. CONCLUSIONS

The proposed methodology was proven to be successful in capturing the accurate structural geometry of the sampled data. The key challenge was evident in arriving at accurate interpreting of the surviving data. In addition, the need for specialized knowledge, skills and deep understanding of pattern generating principles are crucial to the analysis and testing of the different reconstruction options. A deeper understanding of the generating principles of different types of Islamic patterns (i.e., periodic, aperiodic, etc.) is much needed. More investigation into their mathematics, techniques, craft and symbolic significance is essential to arriving at the best preservation decision. The proposed methodology relies heavily on the subjective nature of our perceptual power in understanding shape complexity and depicting its color differences. The problem with such methods is related to the subjective and limited human ability of recognizing faint traces of subtle color evidence. Digital techniques offer many advantages over the human eye in terms of
recognizing subtle differences in light and color. The conceptual framework proposed by this paper can serve as a platform for developing digital pattern recognition tools for fast and efficient results.

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