Characteristic Behavior of Ion Acoustic Wave in A dusty Plasma

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Abstract

In this research, we make an attempt to derive theoretically 1-D linear dispersion relation of ion-acoustic waves in uniform unmagnetized dusty plasma valid in the long wavelength limits. This equation matched previously special equation of acoustic modes of a general form in magnetized dusty plasma. Depending on previously mentioned experimental data, we numerically consider various parameters that affect the properties of these waves in K^+ dusty plasma. The study has shown that the presence of dust grains is to modify the properties of ion acoustic waves and affect the behavior of the plasma in which they are immersed.

Introduction

Recently there has been a growing interest in the physics of dusty plasma (1). The motivation for studying dusty plasmas is due to the realization of their occurrence in both the laboratory and space. A dusty plasma is roughly defined as a normal electron-ion plasma with an additional charged component of micron or sub-micron sized dust particles. This extra component of such charged dust particles increases the complexity of the plasma system even further. This is why a dusty plasma is also referred to as a "complex plasma" (2). Under some cases the dust particles could achieve large charge numbers. They can have large mass of order of $10^6 - 10^{10}$ times the proton mass. Dust particles are usually negatively charged due to the flow of the electron and ion plasma currents to their surface, but become negatively charged due to the greater mobility of the electrons. Furthermore, the dusty plasma system can support a variety of physical phenomena. The presence of massively charged dust particles in a plasma can affect its dispersion properties. For example, the charged dust is shown to modify the properties of ion-acoustic waves through the quasineutrality condition $n_{io} = n_{eo} + Z_{do} n_{do}$ and the strong inequality $n_{eo} \ll n_{io}$, where n_{so} is the background particle number density of the species $s\,(i, e, d)$ with for ions (electrons) dust grains, Z_{do} is the number of electrons collected by dust grain surface and e is electron charge. Previous theoretical and experimental studies regarding low-frequency electrostatic waves in magnetized dusty plasma were obtained a common linear dispersion relation using a multifluid analysis (3, 4, 5). The common linear dispersion relation yields a phase velocity for dust ion acoustic mode valid in the long wavelength limit, that is $k\lambda_{De} \ll 1$, where k and λ_{De} are the wave number and Debye length for electrons respectively (5):

$$\frac{\omega}{k_z} = \left[\frac{KT_i}{m_i} + \frac{KT_e}{m_i(1-\epsilon Z_d)}\right]^{1/2} = C_{s,d} \qquad \text{..................} \qquad [1]$$

where $\epsilon = \frac{n_{do}}{n_{io}}$ and T_s is the species s temperature. The ω ,m_i, K are the frequency, ion mass and Boltzmann constant, respectively. Note that the dust ion acoustic wave (DIAW) spectrum is similar to the usual ion-acoustic wave spectrum for a plasma with $n_{io} = n_{eo}$; however, in dusty plasmas we usually have $n_{io} \gg n_{eo}$. So, obviously, a dusty plasma cannot support the usual ion-acoustic waves, but the modified DIA waves. In our case we are interested to study this behavior. Firstly, we derive the dispersion relation of DIAW in unmagetized plasma, and then solve numerically this relation for various parameters.

Basic Equations

We consider unmagnetized dusty plasma as consisting of three-component system of ions, electrons, and cold dust grains. For hot and isothermal electrons obeying the ideal gas law, one can adopt Boltzmann type electron density distribution. The system of equations for the components is as follows:

The number density of the electrons is:

$$n_e = n_{eo}\left(\frac{e\phi}{KT_e}\right) \qquad \text{................ ....} \qquad [2]$$

where n_e and ϕ denote the electron number density, and the electrostatic plasma potential, respectively. Assuming a hydrodynamical description of the ion-fluid and the dust-fluid to be non-realistic, we can write the basic equations which govern the dynamics of the ions and the dust particles as:

The equation of continuity for ions is: (6)

$$\frac{\partial n_i}{\partial t} + \nabla.(n_i \mathbf{v}_i) = 0 \qquad \text{.............} \qquad [3]$$

The equation of motion for the ions is: (7)

$$m_i n_i \left[\frac{\partial \mathbf{v}_i}{\partial t} + (\mathbf{v}_i \cdot \nabla)\mathbf{v}_i\right] = e n_i \mathbf{E} - \nabla p \qquad \text{...............} \qquad [4]$$

The equation of continuity for the dust as equation [3] is:

$$\frac{\partial n_d}{\partial t} + \nabla.(n_d \mathbf{v}_d) = 0 \qquad \text{................} \qquad [5]$$

The equation of motion for the dust as in equation [4] is:

$$m_d n_d \left[\frac{\partial \mathbf{v}_d}{\partial t} + (\mathbf{v}_d \cdot \nabla)\mathbf{v}_d\right] = -Q_d n_d \mathbf{E} \qquad \text{.............} \qquad [6]$$

Possion's equation is: (7)

$$\epsilon_o \nabla \cdot \mathbf{E} = e n_e - e n_i - Q_d n_d \quad \ldots\ldots\ldots\ldots\ldots \quad [7]$$

Here n_i, n_d, m_i, m_d represent the number densities of the ions (dust particles) and the ion (dust particle) mass respectively. $\mathbf{v}_i$, $\mathbf{v}_d$ denote, respectively, the velocities of the ion-fluid and the dust-fluid along the direction of propagation of the wave. ϵ_o Q_d, are the permittivity the dust charge, respectively, such that $Q_d = -eZ_d$, Z_d being the charge number on the dust and $E = -\nabla\phi$.

Assuming one dimensional space and that all the perturbations has Fourier solutions:

$$n(x,t) = \hat{n} \exp\, i(kx - \omega t)\ ,\ v(x,t) = \hat{v} \exp\, i(kx - \omega t)\ ,\ \phi(x,t) = \ddot{\phi} \exp\, i(kx - \omega t)$$

In the absence of charge perturbation, the set of lineralized equations are given by:

$$\widetilde{n_e} = n_{eo} \frac{e\tilde{\phi}}{KT_e} \quad \ldots\ldots\ldots\ldots\ldots\ \ldots\ldots\ldots \quad [8]$$

$$-i\omega \widetilde{n_i} + ikn_{io}\widetilde{v_i} = 0 \quad \ldots\ldots\ldots\ldots\ldots \quad [9]$$

$$-i\omega \widetilde{v_i} = -\frac{e}{m_i} ik\tilde{\phi} - \frac{3KT_i}{m_i n_{io}} ik\widetilde{n_i} \quad \ldots\ldots\ldots \quad [10]$$

$$-i\omega \widetilde{n_d} + n_{do} ik\widetilde{v_d} = 0 \quad \ldots\ldots\ldots\ldots \quad [11]$$

$$-i\omega \widetilde{v_d} + \frac{Q_d}{m_d} ik\tilde{\phi} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ \ldots\ldots \quad [12]$$

$$-\epsilon_o k^2 \tilde{\phi} = e\widetilde{n_e} - e\widetilde{n_i} - \widetilde{n_d} Q_d \quad \ldots\ldots\ldots\ldots\ldots \quad [13]$$

where the tilt ($\sim$) indicates perturbed quantity. We obtain from this set of equations a linear dispersion relation of low frequency ion acoustic waves in a dusty plasma:

$$\frac{\omega^2}{k^2} = \left[\frac{3KT_i}{m_i} + \frac{KT_e}{m_i}\left(1 + Z_d \frac{n_{do}}{n_{eo} m_i}\right) + \frac{n_{do} Z_d^2}{m_d}\right] \frac{1}{k^2 \lambda_{De}^2 + 1} \quad \ldots\ldots\ldots\ldots \quad [14]$$

By low frequencies we mean frequencies on the order of or less than Ω_p , the ion plasma frequency. In long wave limit $k \to 0$ the term $k\lambda_{De} \ll 1$, meaning that the wavelength is much larger than the Debye length, yielding that the phase velocity is

$$\frac{\omega}{k} = \left[\frac{3KT_i}{m_i} + \frac{KT_e}{m_i}\left(1 + Z_d \frac{n_{do}}{n_{eo} m_i}\right) + \frac{n_{do} Z_d^2}{m_d}\right]^{1/2} \quad \ldots\ldots\ldots\ldots \quad [15]$$

This equation is similar to dimensionless equation [8] of Nejoh (8) derived by choosing appropriate physical scales applied on equations (1) - (6). However, dimensionless equation, of course, immediately recovers the dispersion relation [15] upon restoring dimensions, but clearly some extra qualitative information is admittedly hidden in the dimensionless relation. Rearranging equation [15], using the ratio $\epsilon = \frac{n_{do}}{n_{io}}$ and quasineutrality condition, we get the following equation:

$$\frac{\omega}{k} = \left[\frac{3KT_i}{m_i} + \frac{KT_e}{m_i(1-\epsilon Z_d)} + \frac{n_{do} Z_d^2}{m_d}\right]^{1/2} \quad \ldots\ldots\ldots\ldots\ldots \quad [16]$$

The fluid analysis just presented yields a dispersion relation with a purely real ω . The approaches used here as well as that used in Ref. (5) determine which modes are possible but do not generally provide information regarding wave growth or damping. Equation [16] is similar to equation [1], but the third term appeared due to collective behavior of dust particles. Also the ratio of specific heat is taken here as $\gamma_i = 3$, since the ions suffer one dimensional compressions in the plane waves as we have assumed. Taking the limit of $m_d \to 0$ for static dust background, equation [16] recovers equation [1]. Note that in equation [1] of Ref. (5) it is considered that the three component of plasma are immersed in uniform magnetic field oriented along the z-axis of Cartesian coordinate system. Thus the wave number would be in this direction k_z. The physical interpretation of this low frequency ion-acoustic mode is that the restoring force comes from the pressure of inertialess electrons, whereas the ion mass provides the inertia to set up wave motions. (9)

Dependence of Phase Velocity on the Density of Species

It is known that the wave speed of ion acoustic wave is determined by the temperature and mass of the ion species. However, in dust ion acoustic wave the contribution of the high dust charge Z_d and the relative concentration ϵ is retained. Previous experimental studies of DIWS (5) show that the phase velocity may vary considerably over the range of ϵZ_d. We investigate the density effect of all species on the phase velocity of the waves. We extend the calculations to point out the effect of varying the parameters involved in the dispersion relation for DIAW.

Dependence of Phase Velocity on the Dust Density

Figure (1) shows the phase velocity dependence on the dust density for different values of electron density or/and ion density. The phase velocity is normalized to its respective value in the absence of dust $\epsilon Z_d = 0$. Because of the condition $n_{io} = n_{eo} + Z_{do} n_{do}$ in dusty plasma the phase velocity increases with dust particle density due to the unequal number of electrons and ions in dusty plasmas. As the fraction of dust density increases the wave phase velocity increases. While the relation of phase velocity with the dust density looks somehow linear it is exactly quadratic. This can be shown if one were to extend the range of dust density to a higher value. However, the figure indicates that as electron density increases for the same values of dust density the phase velocity decreases. This increase in electron density leads to an increase in the ion density due to condition of inequality in density of dusty plasma.

Dependence of Phase Velocity on the Ion density

Figure (2) shows the decrease of phase velocity with ion density. The figure also shows that as the charge number of the dust grain increases for the same values of ion density the phase velocity increases.

Dependence of Phase Velocity on the Ratio of Dust to Ion Density

Figure (3) shows the variation of the relative concentration ϵ of the dust versus normalized phase velocity for different values of a charge number. As the relative value of dust concentration ϵ increases the wave phase velocity of the DIAW increases. The reason of this increase can be noticed by writing the linearized momentum equation for the ions in the form $\frac{m_i n_{io} \partial \tilde{n_i}}{\partial t} = -[KT_i + KT_e/(1-\epsilon Z_d)] \times (\partial \tilde{n_i} \partial x)$. The left hand side term is the force per unit volume on a typical ion fluid element in the presence of the wave perturbation, and the right hand side of the equation is the acoustic restoring force per unit volume on the fluid element, which increases with the increase of ϵ. An increase in the restoring force then gives rise to an increase in the wave phase velocity. Physically, as more and more electrons become attached to the immobile dust grains, there are fewer available to provide neutralization for the ion space charge perturbations (5). The figure shows that the greater charge number accumulated on the dust grains the lower the ϵ value that shifts the diagram of phase velocity to left hand side. Therefore, the dust charge number on the dust grains changes the properties of these waves and its characteristics. Note that the phase velocity of DIAW is larger than the usual ion-acoustic velocity. On the other hand, one can estimate a similar behavior of the DIAW frequency with the density of species for a constant wave number.

The Dependence of Phase Velocity on the Dust Parameter ϵZ_d

Figure (4) shows the phase velocity dependence of DIAW on the dust parameter ϵZ_d that is the percentage of negative charge on the dust in K^+ dusty plasma for various values of electron temperature. The phase velocity increases with increasing ϵZ_d as mentioned by previous studies; however increasing electron temperature for a finite ion temperature is to increase the phase velocity higher. The curves of DIAW almost behave in a similar manner; however Figure (5), shows that the effect of increasing ion temperature on DIAW is to increase the phase velocity, but at higher values of ϵZ_d all the curves are converged. The factor ϵZ_d plays a major role in determining the phase velocity. Increasing of this factor yields higher phase velocity.

The Dependence of Phase Velocity on the Effective Electron Temperature

It is understood that as the dust parameter increases the phase velocity increases. Thus, the behavior of effective electron temperature $KT_e/(1-\epsilon Z_d)$ for other fixed parameters with the phase velocity is to increase it. Figure (6) shows the behavior of the phase velocity with effective electron temperature $KT_e/(1-\epsilon Z_d)$ for two values of ion temperature. The effect of effective electron temperature on the phase velocity with increasing ion temperature for other fixed parameters is to shift up the curve or increasing the phase velocity. Similarly for constant wave number the frequency of DIAW behaves in the same manner versus the effective electron temperature.

Dependence of the Phase Velocity of DIAW on the Electron and Ion Temperature

Figure (7) shows the effect of varying electron temperature on the frequency of DIAW while figure (8) shows the effect of varying ion temperature on the frequency. The two figures are plotted for various values of dust parameter. Figure (7) is plotted with fixed ion temperature while figure (8) is plotted with fixed electron temperature and other fixed parameters for both. The figures indicate an increase in phase velocity with an increased value of electron temperature (fig. 7) and ion temperature (fig. 8). Also, the figures show that the phase velocity of DIAW increases with dust parameter as described above. The relation of the phase velocity with electron temperature is nonlinear for entire values of dust parameter. However, this nonlinearity is not sharp for the relation of phase velocity with ion temperature as in phase velocity with electron temperature.

Effect of Ion Mass on the Phase Velocity of DIAW

We consider the case of existing dust grains in different plasmas to examine the ion mass effect for various plasmas. For simplicity we assume all parameters are the same for different ion plasmas. Figure (9) shows the variation of phase velocity with ion mass number for two different values of dust parameter. Obviously, the phase velocity of lighter ion plasma is higher than the heavier ion. However, there is a comparison in the phase velocity for lighter and heavier ion for two different values of dust parameter. While the difference in phase velocity for the light ion is clearly seen with two different values of dust parameter, the difference is very low for heavy ion. But detailed calculations indicate that the higher value of dust parameter still yields higher phase velocity of DIAW.

The Dispersion Relation

Figure (10) is a plot of wave number versus the frequency for different values of dust parameter. The wave shows a linear dispersion which have low frequency ion-acoustic wave in dusty plasma. The figure shows an increase in the frequency with increased dust parameter. This is presumably due to increased phase velocity as the dust parameter increased since the wave number is fixed. Obviously, the DIAW frequency is much larger than the dust plasma frequency ω_{pd}, however, since the phase velocity is essentially constant, the ion plasma frequency Ω_p must be well above the highest wave frequency. Hence stationary dust grains do not respond to the DIA waves. This is the reason of taking the limit $m_d \to 0$ in equation [16]. In this case, the dust grains simply provide an immobile charge-neutralizing background. On the other hand, the DIAW is a fast wave with its phase velocity lying between electron and ion thermal velocities.

Conclusions

In this report the presented dispersion equation of ion-acoustic waves in uniform unmagnetized dusty plasma is studied by changing various parameters involved. Immersing dust grains in a plasma have a significant effects in modifying the ion acoustic waves through the quasineutrality condition. As the fraction of dust density or/and the relative concentration of the dust grains the wave phase velocity of DIAW increases. Also as the fraction of negative charge per unit volume on the dust or dust parameter increases the wave phase velocity increases. For finite ion temperature the phase velocity increases with electron temperature. Although the effect of ion temperature is to increase the phase velocity the curves of phase velocity versus ion temperature converged at higher values of dust parameter. The phase velocity of DIAW increases nonlinearly with effective electron temperature. Furthermore, the phase velocity of DIAW in lighter ion plasma is higher than the heavier one. The dispersion relation of DIAW yields low frequency of ion-acoustic wave in dusty plasma in comparison with ion plasma frequency.

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السلوك المميز لموجة الآيون السمعية في بلازما الغبار

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الخلاصة

في هذا البحث، تمت نظريا محاولة اشتقاق علاقة التفريق لموجات الايون السمعية ببعد واحد في بلازما الغبار غير ممغنطة للأطوال الموجية الطويلة. لقد وجد إنها توافق معادلة خاصة للأنماط السمعية اشتقت سابقا من معادلة عامة للإنماط العاملة في بلازما الغبار الممغنطة . بالإعتماد على البيانات التجريبية المتوفرة ، تم تغيير المعلمات المختلفة التي تؤثر على صفات هذه الموجات في بلازما ايون البوتاسيوم الغبارية. وتوصلنا الى ان تواجد حبيبات الغبار في البلازما يعدل من صفات موجات الآيون السمعية ويؤثرعلى سلوك البلازما.