Soft (1,2)*-Omega Separation Axioms and Weak Soft (1,2)*-Omega Separation Axioms in Soft Bitopological Spaces

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Abstract

In the present paper we introduce and study new classes of soft separation axioms in soft bitopological spaces, namely, soft (1,2)*-omega separation axioms and weak soft (1,2)*-omega separation axioms by using the concept of soft (1,2)*-omega open sets. The equivalent definitions and basic properties of these types of soft separation axioms also have been studied.

Keywords: Soft (1,2)*-ω-open sets, soft (1,2)*-ω-$\tilde{T}_i$-spaces, soft (1,2)*-α-ω-$\tilde{T}_i$-spaces, soft (1,2)*-pre-ω-$\tilde{T}_i$-spaces, soft (1,2)*-b-ω-$\tilde{T}_i$-spaces, and soft (1,2)*-β-ω-$\tilde{T}_i$-spaces, for $i = 0, \frac{1}{2}, 1, 2$.

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Introduction

Soft set theory was firstly introduced by Molodtsov [1] in 1999 as a new mathematical tool for dealing with uncertainty while modeling problems in computer science, economics, engineering physics, medical sciences, and social sciences. In 2011 Shabir and Naz [2] introduced and studied the concept of soft topological spaces. In 2014 Senel and Çagman [3] investigated the notion of soft bitopological spaces over an initial universe set with a fixed set of parameters. In 2018 Mahmood and Abdul-Hady [4] introduced and studied new types of soft sets in soft bitopological spaces called soft (1,2)*-omega open sets and weak forms of soft (1,2)*-omega open sets such as soft (1,2)*-α-ω-open sets, soft (1,2)*-pre-ω-open sets, soft (1,2)*-b-ω-open sets and soft (1,2)*-β-ω-open sets. The main purpose of this paper is to introduce and study new types of soft separation axioms in soft bitopological spaces called soft (1,2)*-omega separation axioms and weak soft (1,2)*-omega separation axioms by using the notion of soft (1,2)*-omega open sets such as soft (1,2)*-ω-$\tilde{T}_i$-spaces, soft (1,2)*-α-ω-$\tilde{T}_i$-spaces, soft (1,2)*-pre-ω-$\tilde{T}_i$-spaces, soft (1,2)*-b-ω-$\tilde{T}_i$-spaces, and soft (1,2)*-β-ω-$\tilde{T}_i$-spaces, for $i = 0, \frac{1}{2}, 1, 2$. Moreover we study the fundamental properties and equivalent definitions of these types of soft separation axioms.

1. Preliminaries:

Throughout this paper U is an initial universe set, $P(U)$ is the power set of U, P is the set of parameters and $C \subseteq P$.

Definition (1.1) [1]: A soft set over U is a pair (H, C), where H is a function defined by $H: C \to P(U)$ and C is a non-empty subset of P.

Definition (1.2)[5]: A soft set (H, C) over U is called a soft point if there is exactly one $e \in C$ such that $H(e) = \{u\}$ for some $u \in U$ and $H(e') = \varphi$, $\forall\, e' \in C \setminus \{e\}$ and is denoted by $\tilde{u} = (e, \{u\})$.

Definition (1.3)[5]: A soft point $\tilde{u} = (e, \{u\})$ is called belongs to a soft set (H, C) if $e \in C$ and $u \in H(e)$, and is denoted by $\tilde{u} \tilde{\in} (H, C)$.

Definition (1.4) [5]: A soft set (H, C) over U is called countable (finite) if the set $H(e)$ is countable (finite) $\forall\, e \in C$.

Definition (1.5)[6]: A soft set (H, C) over U is called a null soft set with respect to C if for each $e \in C$, $H(e) = \varphi$, and is denoted by $\tilde{\varphi}_C$. If $C = P$, then (H, C) is called a null soft set and is denoted by $\tilde{\varphi}$.

Definition (1.6)[6]: A soft set (H, C) over U is called an absolute soft set with respect to C if for each $e \in C$, $H(e) = U$, and is denoted by $\tilde{U}_C$. If $C = P$, then (H, C) is called an absolute soft set and is denoted by $\tilde{U}$.

Definition (1.7)[6]: Let (H_1, C_1) and (H_2, C_2) be soft sets over a common universe U. Then we say that:

(1) (H_1, C_1) is a soft subset of (H_2, C_2) denoted by $(H_1, C_1) \tilde{\subseteq} (H_2, C_2)$ if $C_1 \subseteq C_2$ and $H_1(e) \subseteq H_2(e)$ for each $e \in C_1$.

(2) The soft union of two soft sets (H_1, C_1) and (H_2, C_2) over a common universe U is the soft set (H, C), where $C = C_1 \cup C_2$, and $\forall e \in C$,

$$H(e) = \begin{cases} H_1(e) & \text{if } e \in C_1 - C_2 \\ H_2(e) & \text{if } e \in C_2 - C_1 \\ H_1(e) \cup H_2(e) & \text{if } e \in C_1 \cap C_2 \end{cases}$$

And we write $(H, C) = (H_1, C_1) \tilde{\cup} (H_2, C_2)$.

(3) The soft intersection of two soft sets (H_1,C_1) and (H_2,C_2) over a common universe U is the soft set (H,C), where $C=C_1\cap C_2$, and $\forall e\in C$, $H(e)=H_1(e)\cap H_2(e)$, and we write $(H,C)=(H_1,C_1)\tilde{\cap}(H_2,C_2)$.

(4) The soft difference of two soft sets (H_1,C_1) and (H_2,C_2) over a common universe U is the soft set (H,C), where $C=C_1\cap C_2$, and $\forall e\in C$, $H(e)=H_1(e)-H_2(e)$, and we write $(H,C)=(H_1,C_1)-(H_2,C_2)$.

Definition (1.8)[2]: A soft topology on U is a collection $\tilde{\tau}$ of soft subsets of $\tilde{U}$ having the following properties:

(i) $\tilde{\varphi}\tilde{\in}\tilde{\tau}$ and $\tilde{U}\tilde{\in}\tilde{\tau}$.

(ii) If $(H_1,P),(H_2,P)\tilde{\in}\tilde{\tau}$, then $(H_1,P)\tilde{\cap}(H_2,P)\tilde{\in}\tilde{\tau}$.

(iii) If $(H_j,P)\tilde{\in}\tilde{\tau}, \forall j\in\Omega$, then $\bigcup_{j\in\Omega}(H_j,P)\tilde{\in}\tilde{\tau}$.

The triple $(U,\tilde{\tau},P)$ is called a soft topological space over U. The members of $\tilde{\tau}$ are called soft open sets over U. The complement of a soft open set is called soft closed.

Definition (1.9)[3]: Let U be a non-empty set and let $\tilde{\tau}_1$ and $\tilde{\tau}_2$ be two soft topologies over U. Then $(U,\tilde{\tau}_1,\tilde{\tau}_2,P)$ is called a soft bitopological space over U.

Definition (1.10)[3]: A soft subset (H,P) of a soft bitopological space $(U,\tilde{\tau}_1,\tilde{\tau}_2,P)$ is called soft $\tilde{\tau}_1\tilde{\tau}_2$-open if $(H,P)=(H_1,P)\tilde{\cup}(H_2,P)$ such that $(H_1,P)\tilde{\in}\tilde{\tau}_1$ and $(H_2,P)\tilde{\in}\tilde{\tau}_2$. The complement of a soft $\tilde{\tau}_1\tilde{\tau}_2$-open set in $\tilde{U}$ is called soft $\tilde{\tau}_1\tilde{\tau}_2$-closed.

Definitions (1.11)[7]: A soft bitopological space $(U,\tilde{\tau}_1,\tilde{\tau}_2,P)$ is called a soft (1,2)*-$\tilde{T}_0$-space if for any two distinct soft points $\tilde{x}$ and $\tilde{y}$ of $\tilde{U}$, there exists a soft $\tilde{\tau}_1\tilde{\tau}_2$-open set in $\tilde{U}$ containing one of the soft points but not the other.

Definition (1.12)[7]: A soft bitopological space $(U,\tilde{\tau}_1,\tilde{\tau}_2,P)$ is called a soft (1,2)*-$\tilde{T}_{1/2}$-space if every soft singleton set in $\tilde{U}$ is either soft $\tilde{\tau}_1\tilde{\tau}_2$-open or soft $\tilde{\tau}_1\tilde{\tau}_2$-closed.

Definition (1.13)[7]: A soft bitopological space $(U,\tilde{\tau}_1,\tilde{\tau}_2,P)$ is called a soft (1,2)*-$\tilde{T}_1$-space if for any two distinct soft points $\tilde{x}$ and $\tilde{y}$ of $\tilde{U}$, there exists a soft $\tilde{\tau}_1\tilde{\tau}_2$-open set in $\tilde{U}$ containing $\tilde{x}$ but not $\tilde{y}$ and a soft $\tilde{\tau}_1\tilde{\tau}_2$-open set in $\tilde{U}$ containing $\tilde{y}$ but not $\tilde{x}$.

Definition (1.14)[7]: A soft bitopological space $(U,\tilde{\tau}_1,\tilde{\tau}_2,P)$ is called a soft (1,2)*-$\tilde{T}_2$-space if for any two distinct soft points $\tilde{x}$ and $\tilde{y}$ of $\tilde{U}$, there are two soft $\tilde{\tau}_1\tilde{\tau}_2$-open sets (H,P) and (K,P) in $\tilde{U}$ such that $\tilde{x}\tilde{\in}(H,P)$, $\tilde{y}\tilde{\in}(K,P)$ and $(H,P)\tilde{\cap}(K,P)=\tilde{\varphi}$.

Definition (1.15) [4]: A soft subset (H,P) of a soft bitopological space $(U,\tilde{\tau}_1,\tilde{\tau}_2,P)$ is called soft (1,2)*-omega open (briefly soft (1,2)*-ω-open) if for each $\tilde{x}\tilde{\in}(H,P)$, there exists a soft $\tilde{\tau}_1\tilde{\tau}_2$-open set (O,P) in $\tilde{U}$ such that $\tilde{x}\tilde{\in}(O,P)$ and $(O,P)-(H,P)$ is a countable soft set. The complement of a soft (1,2)*-ω-open set is called soft (1,2)*-omega closed (briefly soft (1,2)*-ω-closed).

Clearly, every soft $\tilde{\tau}_1\tilde{\tau}_2$-open set is soft (1,2)*-ω-open, but the converse in general is not true we can see in the following example:

Example (1.16): Let $U=\{1,2,3\}$ and $P=\{p_1,p_2\}$, and let $\tilde{\tau}_1=\{\tilde{U},\tilde{\varphi},(H_1,P)\}$ and $\tilde{\tau}_2=\{\tilde{U}, \tilde{\varphi},(H_2,P)\}$ be soft topologies over U, where $(H_1,P)=\{(p_1,\{U\}),(p_2,\{1,2\})\}$ and $(H_2,P)=$

$\{(p_1,\{U\}),(p_2,\{1,3\})\}$. The soft sets in $\{\tilde{U},\tilde{\varphi},(H_1,P),(H_2,P)\}$ are soft $\tilde{\tau}_1\tilde{\tau}_2$-open sets in $\tilde{U}$. Thus $(U,\tilde{\tau}_1,\tilde{\tau}_2,P)$ is a soft bitopological space and $(H,P)=\{(p_1,\{U\}),(p_2,\{1\})\}$ is a soft (1,2)*-ω-open set in $\tilde{U}$, but is not soft $\tilde{\tau}_1\tilde{\tau}_2$-open.

Definition (1.17) [4]: Let $(U,\tilde{\tau}_1,\tilde{\tau}_2,P)$ be a soft bitopological space and $(H,P)\tilde{\subseteq}\tilde{U}$. Then:

(i) The soft (1,2)*-omega closure (briefly soft (1,2)*-ω-closure) of (H,P), denoted by (1,2)*-$\omega cl(H,P)$ is the intersection of all soft (1,2)*-ω-closed sets in $\tilde{U}$ which contains (H,P).

(ii) The soft (1,2)*-omega interior (briefly soft (1,2)*-ω-interior) of (H,P), denoted by (1,2)*-$\omega int(H,P)$ is the union of all soft (1,2)*-ω-open sets in $\tilde{U}$ which are contained in (H,P).

Theorem (1.18) [4]: If $(U,\tilde{\tau}_1,\tilde{\tau}_2,P)$ is a soft bitopological space, and $(H,P),(K,P)\tilde{\subseteq}\tilde{U}$. Then:

(i) $(H,P)\tilde{\subseteq}(1,2)^*\text{-}\omega cl(H,P)\tilde{\subseteq}\tilde{\tau}_1\tilde{\tau}_2 cl(H,P)$.

(ii) $(1,2)^*\text{-}\omega cl(H,P)$ is soft (1,2)*-ω-closed set in $\tilde{U}$.

(iii) (H,P) is soft (1,2)*-ω-closed iff $(1,2)^*\text{-}\omega cl(H,P)=(H,P)$.

(iv) If $(H,P)\tilde{\subseteq}(K,P)$, then $(1,2)^*\text{-}\omega cl(H,P)\tilde{\subseteq}(1,2)^*\text{-}\omega cl(K,P)$.

Definitions (1.19) [4]: A soft subset (H,P) of a soft bitopological space $(U,\tilde{\tau}_1,\tilde{\tau}_2,P)$ is called:

(i) Soft (1,2)*-α-ω-open if $(H,P)\tilde{\subseteq}(1,2)^*\text{-}\omega int(\tilde{\tau}_1\tilde{\tau}_2 cl((1,2)^*\text{-}\omega int(H,P)))$.

(ii) Soft (1,2)*-pre-ω-open if $(H,P)\tilde{\subseteq}(1,2)^*\text{-}\omega int(\tilde{\tau}_1\tilde{\tau}_2 cl(H,P))$.

(iii) Soft (1,2)*-b-ω-open if $(H,P)\tilde{\subseteq}(1,2)^*\text{-}\omega int(\tilde{\tau}_1\tilde{\tau}_2 cl(H,P))\tilde{\cup}\tilde{\tau}_1\tilde{\tau}_2 cl((1,2)^*\text{-}\omega int(H,P))$.

(iv) Soft (1,2)*-β-ω-open if $(H,P)\tilde{\subseteq}\tilde{\tau}_1\tilde{\tau}_2 cl((1,2)^*\text{-}\omega int(\tilde{\tau}_1\tilde{\tau}_2 cl(H,P)))$.

Proposition (1.20) [4]: If $(U,\tilde{\tau}_1,\tilde{\tau}_2,P)$ is a soft bitopological space, then the following hold:

(i) Every soft $\tilde{\tau}_1\tilde{\tau}_2$-open set is soft (1,2)*-ω-open.

(ii) Every soft (1,2)*-ω-open set is soft (1,2)*-α-ω-open.

(iii) Every soft (1,2)*-α-ω-open set is soft (1,2)*-pre-ω-open.

(iv) Every soft (1,2)*-pre-ω-open set is soft (1,2)*-b-ω-open.

(v) Every soft (1,2)*-b-ω-open set is soft (1,2)*-β-ω-open.

Definition (1.21) [4]: Let $(U,\tilde{\tau}_1,\tilde{\tau}_2,P)$ be a soft bitopological space and $(H,P)\tilde{\subseteq}\tilde{U}$. Then the soft (1,2)*-α-ω-closure (resp. soft (1,2)*-pre-ω-closure, soft (1,2)*-b-ω-closure,soft (1,2)*-β-ω-closure) of (H,P), denoted by (1,2)*-α-$\omega cl(H,P)$ (resp. (1,2)*-pre-$\omega cl(H,P)$, (1,2)*-b-$\omega cl(H,P)$, (1,2)*-β-$\omega cl(H,P)$) is the intersection of all soft (1,2)*-α-ω-closed (resp. soft (1,2)*-pre-ω-closed, soft (1,2)*-b-ω-closed, soft (1,2)*-β-ω-closed) sets in $\tilde{U}$ which contains (H,P).

Definition (1.22) [4]: A soft subset (N,P) of a soft bitopological space $(U,\tilde{\tau}_1,\tilde{\tau}_2,P)$ is called a soft (1,2)*-ω-neighborhood (resp. soft (1,2)*-α-ω-neighborhood, soft (1,2)*-pre-ω-neighborhood, soft (1,2)*-b-ω-neighborhood, soft (1,2)*-β-ω-neighborhood) of a soft point $\tilde{x}$ in $\tilde{U}$ if there exists a soft (1,2)*-ω-open (resp. soft (1,2)*-α-ω-open, soft (1,2)*-pre-ω-open, soft (1,2)*-b-ω-open, soft (1,2)*-β-ω-open) set (H,P) in $\tilde{U}$ such that $\tilde{x}\tilde{\in}(H,P)\tilde{\subseteq}(N,P)$.

Definition (1.23)[8]: Let $(U,\tilde{\tau}_1,\tilde{\tau}_2,P)$ be a soft bitopological space over U and $\varphi\neq Y\subseteq U$. Then $\tilde{\tau}_{1\tilde{Y}}=\{(M,P)\tilde{\cap}\tilde{Y}:(M,P)\tilde{\in}\tilde{\tau}_1\}$ and $\tilde{\tau}_{2\tilde{Y}}=\{(N,P)\tilde{\cap}\tilde{Y}:(N,P)\tilde{\in}\tilde{\tau}_2\}$ are called the relative soft topologies on $\tilde{Y}$ and $(Y,\tilde{\tau}_{1\tilde{Y}},\tilde{\tau}_{2\tilde{Y}},P)$ is called the relative soft bitopological space of $(U,\tilde{\tau}_1,\tilde{\tau}_2,P)$.

2. Soft (1,2)*-Omega Separation Axioms and Weak Soft (1,2)*-Omega Separation Axioms

Now, we introduce and study new types of soft separation axioms in soft bitopological spaces called soft (1,2)*-ω-separation axioms and weak soft (1,2)*-ω-separation axioms such as soft (1,2)*-ω-$\tilde{T}_i$-spaces, soft (1,2)*-α-ω-$\tilde{T}_i$-spaces, soft (1,2)*-pre-ω-$\tilde{T}_i$-spaces, soft

(1,2)*-b-ω-$\tilde{T}_i$-spaces, and soft (1,2)*-β-ω-$\tilde{T}_i$-spaces, for $i = 0, \frac{1}{2}, 1, 2$. The fundamental properties and equivalent definitions of these types of soft separation axioms also, have been studied.

Definitions (2.1): A soft bitopological space $(U, \tilde{\tau}_1, \tilde{\tau}_2, P)$ is called a soft (1,2)*-ω-$\tilde{T}_0$-space (resp. soft (1,2)*-α-ω-$\tilde{T}_0$-space, soft (1,2)*-pre-ω-$\tilde{T}_0$-space, soft (1,2)*-b-ω-$\tilde{T}_0$-space, soft (1,2)*-β-ω-$\tilde{T}_0$-space) if for any two distinct soft points $\tilde{x}$ and $\tilde{y}$ of $\tilde{U}$, there exists a soft (1,2)*-ω-open (resp. soft (1,2)*-α-ω-open, soft (1,2)*-pre-ω-open, soft (1,2)*-b-ω-open, soft (1,2)*-β-ω-open) set in $\tilde{U}$ containing one of the soft points but not the other.

Proposition (2.2): Every soft (1,2)*-$\tilde{T}_0$-space is a soft (1,2)*-ω-$\tilde{T}_0$-space (resp. soft (1,2)*-α-ω-$\tilde{T}_0$-space, soft (1,2)*-pre-ω-$\tilde{T}_0$-space, soft (1,2)*-b-ω-$\tilde{T}_0$-space, soft (1,2)*-β-ω-$\tilde{T}_0$-space).

Proof: It is obvious.

Remark (2.3): The converse of proposition (2.2) is not true in general we can see by the following example:

Example (2.4): Let $U = \{a, b, c\}$ and $P = \{p_1, p_2\}$ and let $\tilde{\tau}_1 = \{\tilde{U}, \tilde{\varphi}, (H_1, P)\}$ and $\tilde{\tau}_2 = \{\tilde{U}, \tilde{\varphi}, (H_2, P)\}$ be soft topologies over U, where $(H_1, P) = \{(p_1, \{a\}), (p_2, \{a\})\}$, $(H_2, P) = \{(p_1, \{b\}), (p_2, \{b\})\}$ and $(H_3, P) = \{(p_1, \{a, b\}), (p_2, \{a, b\})\}$. The soft sets in $\{\tilde{U}, \tilde{\varphi}, (H_1, P), (H_2, P), (H_3, P)\}$ are soft $\tilde{\tau}_1\tilde{\tau}_2$-open. Thus $(U, \tilde{\tau}_1, \tilde{\tau}_2, P)$ is a soft (1,2)*-ω-$\tilde{T}_0$-space (resp. soft (1,2)*-α-ω-$\tilde{T}_0$-space, soft (1,2)*-pre-ω-$\tilde{T}_0$-space, soft (1,2)*-b-ω-$\tilde{T}_0$-space, soft (1,2)*-β-ω-$\tilde{T}_0$-space), but is not soft (1,2)*-$\tilde{T}_0$-space, since $(p_1, \{a\}) = \tilde{x} \neq \tilde{y} = (p_2, \{a\})$, but there exists no soft $\tilde{\tau}_1\tilde{\tau}_2$-open set containing one of the soft points but not the other.

Theorem (2.٥): A soft bitopological space $(U, \tilde{\tau}_1, \tilde{\tau}_2, P)$ is a soft (1,2)*-ω-$\tilde{T}_0$-space (resp. soft (1,2)*-α-ω-$\tilde{T}_0$-space, soft (1,2)*-pre-ω-$\tilde{T}_0$-space, soft (1,2)*-b-ω-$\tilde{T}_0$-space, soft (1,2)*-β-ω-$\tilde{T}_0$-space) if and only if (1,2)*-$\omega cl(\{\tilde{x}\}) \neq$ (1,2)*-$\omega cl(\{\tilde{y}\})$ (resp. (1,2)*-α-$\omega cl(\{\tilde{x}\}) \neq$ (1,2)*-α-$\omega cl(\{\tilde{y}\})$, (1,2)*-pre-$\omega cl(\{\tilde{x}\}) \neq$ (1,2)*-pre-$\omega cl(\{\tilde{y}\})$, (1,2)*-b-$\omega cl(\{\tilde{x}\}) \neq$ (1,2)*-b-$\omega cl(\{\tilde{y}\})$, (1,2)*-β-$\omega cl(\{\tilde{x}\}) \neq$ (1,2)*-β-$\omega cl(\{\tilde{y}\})$) for any two distinct soft points $\tilde{x}$ and $\tilde{y}$ of $\tilde{U}$.

Proof: Let $\tilde{x}, \tilde{y} \tilde{\in} \tilde{U}$ such that $\tilde{x} \neq \tilde{y}$. Since $(U, \tilde{\tau}_1, \tilde{\tau}_2, P)$ is a soft (1,2)*-ω-$\tilde{T}_0$-space, then there exists a soft (1,2)*-ω-open set (H, P) containing $\tilde{x}$, but not $\tilde{y}$. Therefore $\tilde{U} - (H, P)$ is a soft (1,2)*-ω-closed set containing $\tilde{y}$, but not $\tilde{x}$. Hence (1,2)*-$\omega cl(\{\tilde{y}\}) \tilde{\subseteq} \tilde{U} - (H, P)$. Since $\tilde{x} \tilde{\notin} \tilde{U} - (H, P)$, this implies that $\tilde{x} \tilde{\notin}$ (1,2)*-$\omega cl(\{\tilde{y}\})$. So we get, $\tilde{x} \tilde{\in}$ (1,2)*-$\omega cl(\{\tilde{x}\})$, but $\tilde{x} \tilde{\notin}$ (1,2)*-$\omega cl(\{\tilde{y}\})$. Thus (1,2)*-$\omega cl(\{\tilde{x}\}) \neq$ (1,2)*-$\omega cl(\{\tilde{y}\})$.

Conversely, to prove that $(U, \tilde{\tau}_1, \tilde{\tau}_2, P)$ is a soft (1,2)*-ω-$\tilde{T}_0$-space. Let $\tilde{x}, \tilde{y} \tilde{\in} \tilde{U}$ such that $\tilde{x} \neq \tilde{y}$. Since (1,2)*-$\omega cl(\{\tilde{x}\}) \neq$ (1,2)*-$\omega cl(\{\tilde{y}\})$, then there exists $\tilde{z} \tilde{\in} \tilde{U}$ such that $\tilde{z} \tilde{\in}$ (1,2)*-$\omega cl(\{\tilde{x}\})$, but $\tilde{z} \tilde{\notin}$ (1,2)*-$\omega cl(\{\tilde{y}\})$. Suppose $\tilde{z} \tilde{\in}$ (1,2)*-$\omega cl(\{\tilde{x}\})$, to show that $\tilde{x} \tilde{\notin}$ (1,2)*-$\omega cl(\{\tilde{y}\})$. If $\tilde{x} \tilde{\in}$ (1,2)*-$\omega cl(\{\tilde{y}\}) \Rightarrow \{\tilde{x}\} \tilde{\subseteq}$ (1,2)*-$\omega cl(\{\tilde{y}\}) \Rightarrow$ (1,2)*-$\omega cl(\{\tilde{x}\}) \tilde{\subseteq}$ (1,2)*-$\omega cl($(1,2)*-$\omega cl(\{\tilde{y}\}))$ = (1,2)*-$\omega cl(\{\tilde{y}\})$. Since $\tilde{z} \tilde{\in}$ (1,2)*-$\omega cl(\{\tilde{x}\}) \Rightarrow \tilde{z} \tilde{\in}$ (1,2)*-$\omega cl(\{\tilde{y}\})$ which is a contradiction. Thus $\tilde{x} \tilde{\notin}$ (1,2)*-$\omega cl(\{\tilde{y}\}) \Rightarrow \tilde{x} \tilde{\in} \tilde{U}$ - (1,2)*-$\omega cl(\{\tilde{y}\})$, but (1,2)*-$\omega cl(\{\tilde{y}\})$ is soft (1,2)*-ω-closed, so $\tilde{U}$ - (1,2)*-$\omega cl(\{\tilde{y}\})$ is soft (1,2)*-ω-open which contains $\tilde{x}$, but not $\tilde{y}$. Therefore $(U, \tilde{\tau}_1, \tilde{\tau}_2, P)$ is a soft (1,2)*-ω-$\tilde{T}_0$-space.

Similarly, we can prove other cases.

Theorem (2.6): Every soft bitopological space $(U,\tilde{\tau}_1,\tilde{\tau}_2,P)$ is a soft (1,2)*-ω-$\tilde{T}_0$-space (resp. soft (1,2)*-α-ω-$\tilde{T}_0$-space, soft (1,2)*-pre-ω-$\tilde{T}_0$-space, soft (1,2)*-b-ω-$\tilde{T}_0$-space, soft (1,2)*-β-ω-$\tilde{T}_0$-space).

Proof: Let $(U,\tilde{\tau}_1,\tilde{\tau}_2,P)$ be any soft bitopological space and $\tilde{x},\tilde{y}\,\tilde{\in}\,\tilde{U}$ such that $\tilde{x}\neq\tilde{y}$. Since $\tilde{U}-\{\tilde{y}\}$ is a soft (1,2)*-ω-open (resp. soft (1,2)*-α-ω-open, soft (1,2)*-pre-ω-open, soft (1,2)*-b-ω-open, soft (1,2)*-β-ω-open) subset of $\tilde{U}$ containing $\tilde{x}$, but not $\tilde{y}$. Therefore $(U,\tilde{\tau}_1,\tilde{\tau}_2,P)$ is a soft (1,2)*-ω-$\tilde{T}_0$-space (resp. soft (1,2)*-α-ω-$\tilde{T}_0$-space, soft (1,2)*-pre-ω-$\tilde{T}_0$-space, soft (1,2)*-b-ω-$\tilde{T}_0$-space, soft (1,2)*-β-ω-$\tilde{T}_0$-space).

Corollary (2.7): Every soft subspace of a soft (1,2)*-ω-$\tilde{T}_0$-space (resp. soft (1,2)*-α-ω-$\tilde{T}_0$-space, soft (1,2)*-pre-ω-$\tilde{T}_0$-space, soft (1,2)*-b-ω-$\tilde{T}_0$-space, soft (1,2)*-β-ω-$\tilde{T}_0$-space) is a soft (1,2)*-ω-$\tilde{T}_0$-space (resp. soft (1,2)*-α-ω-$\tilde{T}_0$-space, soft (1,2)*-pre-ω-$\tilde{T}_0$-space, soft (1,2)*-b-ω-$\tilde{T}_0$-space, soft (1,2)*-β-ω-$\tilde{T}_0$-space).

Proof: It is obvious.

Proposition (2.8): If $(U,\tilde{\tau}_1,P)$ or $(U,\tilde{\tau}_2,P)$ is a soft $\tilde{T}_0$-space, then $(U,\tilde{\tau}_1,\tilde{\tau}_2,P)$ is a soft (1,2)*-ω-$\tilde{T}_0$-space (resp. soft (1,2)*-α-ω-$\tilde{T}_0$-space, soft (1,2)*-pre-ω-$\tilde{T}_0$-space, soft (1,2)*-b-ω-$\tilde{T}_0$-space, soft (1,2)*-β-ω-$\tilde{T}_0$-space).

Proof: It follows from the fact $\tilde{\tau}_i\,\tilde{\subseteq}$ soft $\tilde{\tau}_1\tilde{\tau}_2$-open sets in $\tilde{U}$, $i=1,2$ and proposition (2.2).

Remark (2.9): The converse of proposition (2.8) is not true in general in example (2.4), $(U,\tilde{\tau}_1,\tilde{\tau}_2,P)$ is a soft (1,2)*-ω-$\tilde{T}_0$-space (resp. soft (1,2)*-α-ω-$\tilde{T}_0$-space, soft (1,2)*-pre-ω-$\tilde{T}_0$-space, soft (1,2)*-b-ω-$\tilde{T}_0$-space, soft (1,2)*-β-ω-$\tilde{T}_0$-space), but both $(U,\tilde{\tau}_1,P)$ and $(U,\tilde{\tau}_2,P)$ are not soft $\tilde{T}_0$-space.

Definition (2.10): A soft bitopological space $(U,\tilde{\tau}_1,\tilde{\tau}_2,P)$ is called a soft (1,2)*-ω-$\tilde{T}_{1/2}$-space (resp. soft (1,2)*-α-ω-$\tilde{T}_{1/2}$-space, soft (1,2)*-pre-ω-$\tilde{T}_{1/2}$-space, soft (1,2)*-b-ω-$\tilde{T}_{1/2}$-space, soft (1,2)*-β-ω-$\tilde{T}_{1/2}$-space) if every soft singleton set in $\tilde{U}$ is either soft (1,2)*-ω-open (resp. soft (1,2)*-α-ω-open, soft (1,2)*-pre-ω-open, soft (1,2)*-b-ω-open, soft (1,2)*-β-ω-open) or soft (1,2)*-ω-closed (resp. soft (1,2)*-α-ω-closed, soft (1,2)*-pre-ω-closed, soft (1,2)*-b-ω-closed, soft (1,2)*-β-ω-closed).

Proposition (2.11): Every soft (1,2)*-$\tilde{T}_{1/2}$-space is a soft (1,2)*-ω-$\tilde{T}_{1/2}$-space (resp. soft (1,2)*-α-ω-$\tilde{T}_{1/2}$-space, soft (1,2)*-pre-ω-$\tilde{T}_{1/2}$-space, soft (1,2)*-b-ω-$\tilde{T}_{1/2}$-space, soft (1,2)*-β-ω-$\tilde{T}_{1/2}$-space).

Proof: It is obvious.

Remark (2.12): The converse of proposition (2.11) is not true in general. In example (2.4) $(U,\tilde{\tau}_1,\tilde{\tau}_2,P)$ is a soft (1,2)*-ω-$\tilde{T}_{1/2}$-space (resp. soft (1,2)*-α-ω-$\tilde{T}_{1/2}$-space, soft (1,2)*-pre-ω-$\tilde{T}_{1/2}$-space, soft (1,2)*-b-ω-$\tilde{T}_{1/2}$-space, soft (1,2)*-β-ω-$\tilde{T}_{1/2}$-space), but is not soft (1,2)*-$\tilde{T}_{1/2}$-space.

Proposition (2.13): Every soft (1,2)*-$\tilde{T}_{1/2}$-space (resp. soft (1,2)*-ω-$\tilde{T}_{1/2}$-space, soft (1,2)*-α-ω-$\tilde{T}_{1/2}$-space, soft (1,2)*-pre-ω-$\tilde{T}_{1/2}$-space, soft (1,2)*-b-ω-$\tilde{T}_{1/2}$-space, soft (1,2)*-β-ω-$\tilde{T}_{1/2}$-space) is a soft (1,2)*-$\tilde{T}_0$-space (resp. soft (1,2)*-ω-$\tilde{T}_0$-space, soft (1,2)*-α-ω-$\tilde{T}_0$-space, soft (1,2)*-pre-ω-$\tilde{T}_0$-space, soft (1,2)*-b-ω-$\tilde{T}_0$-space, soft (1,2)*-β-ω-$\tilde{T}_0$-space).

Proof: Let $(U,\tilde{\tau}_1,\tilde{\tau}_2,P)$ be a soft (1,2)*-ω-$\tilde{T}_{1/2}$-space and let $\tilde{x},\tilde{y}\tilde{\in}\tilde{U}$ such that $\tilde{x}\neq\tilde{y}$. If $\{\tilde{x}\}$ is soft (1,2)*-ω-open, then $\{\tilde{x}\}$ is a soft (1,2)*-ω-open set containing $\tilde{x}$, but not $\tilde{y}$ and if $\{\tilde{x}\}$ is soft (1,2)*-ω-closed, then $\tilde{U}-\{\tilde{x}\}$ is a soft (1,2)*-ω-open set containing $\tilde{y}$, but not $\tilde{x}$. Therefore $(U,\tilde{\tau}_1,\tilde{\tau}_2,P)$ is a soft (1,2)*-ω-$\tilde{T}_0$-space. Similarly, we can prove that other cases.

Remark (2.14): The soft (1,2)*-$\tilde{T}_0$-space may not be soft (1,2)*-$\tilde{T}_{1/2}$-space in general we can see in the following example:

Example (2.15): Let $U=\{a,b\}$ and $P=\{p_1,p_2\}$ and let $\tilde{\tau}_1=\{\tilde{U},\tilde{\varphi},(H_1,P)\}$ and $\tilde{\tau}_2=\{\tilde{U},\tilde{\varphi},(H_2,P)\}$ be soft topologies over U, where $(H_1,P)=\{(p_1,\{a\}),(p_2,\{b\})\}$, $(H_2,P)=\{(p_1,\{b\}),(p_2,\{b\})\}$ and $(H_3,P)=\{(p_1,\{a,b\}),(p_2,\{b\})\}$. The soft sets in $\{\tilde{U},\tilde{\varphi},(H_1,P),(H_2,P),(H_3,P)\}$ are soft $\tilde{\tau}_1\tilde{\tau}_2$-open sets. Thus $(U,\tilde{\tau}_1,\tilde{\tau}_2,P)$ is a soft (1,2)*-$\tilde{T}_0$-space, but is not soft (1,2)*-$\tilde{T}_{1/2}$-space, since $\{(p_1,\{a\})\}=\{\tilde{x}\}$ is not soft $\tilde{\tau}_1\tilde{\tau}_2$-open and is not soft $\tilde{\tau}_1\tilde{\tau}_2$-closed.

Theorem (2.16): Every soft bitopological space is a soft (1,2)*-ω-$\tilde{T}_{1/2}$-space (resp. soft (1,2)*-α-ω-$\tilde{T}_{1/2}$-space, soft (1,2)*-pre-ω-$\tilde{T}_{1/2}$-space, soft (1,2)*-b-ω-$\tilde{T}_{1/2}$-space, soft (1,2)*-β-ω-$\tilde{T}_{1/2}$-space).

Proof: Let $(U,\tilde{\tau}_1,\tilde{\tau}_2,P)$ be any soft bitopological space and $\tilde{x}\tilde{\in}\tilde{U}$. Since $\tilde{U}-\{\tilde{x}\}$ is a soft (1,2)*-ω-open (resp. soft (1,2)*-α-ω-open, soft (1,2)*-pre-ω-open, soft (1,2)*-b-ω-open, soft (1,2)*-β-ω-open) subset of $\tilde{U}$, then $\{\tilde{x}\}$ is a soft (1,2)*-ω-closed (resp. soft (1,2)*-α-ω-closed, soft (1,2)*-pre-ω-closed, soft (1,2)*-b-ω-closed, soft (1,2)*-β-ω-closed) subset of $\tilde{U}$. Therefore $(U,\tilde{\tau}_1,\tilde{\tau}_2,P)$ is a soft (1,2)*-ω-$\tilde{T}_{1/2}$-space (resp. soft (1,2)*-α-ω-$\tilde{T}_{1/2}$-space, soft (1,2)*-pre-ω-$\tilde{T}_{1/2}$-space, soft (1,2)*-b-ω-$\tilde{T}_{1/2}$-space, soft (1,2)*-β-ω-$\tilde{T}_{1/2}$-space).

Corollary (2.17): A soft bitopological space $(U,\tilde{\tau}_1,\tilde{\tau}_2,P)$ is a soft (1,2)*-ω-$\tilde{T}_{1/2}$-space (resp. soft (1,2)*-α-ω-$\tilde{T}_{1/2}$-space, soft (1,2)*-pre-ω-$\tilde{T}_{1/2}$-space, soft (1,2)*-b-ω-$\tilde{T}_{1/2}$-space, soft (1,2)*-β-ω-$\tilde{T}_{1/2}$-space) iff it is a soft (1,2)*-ω-$\tilde{T}_0$-space (resp. soft (1,2)*-α-ω-$\tilde{T}_0$-space, soft (1,2)*-pre-ω-$\tilde{T}_0$-space, soft (1,2)*-b-ω-$\tilde{T}_0$-space, soft (1,2)*-β-ω-$\tilde{T}_0$-space).

Proof: It follows that from the proposition (2.13) and theorem (2.16).

Corollary (2.18): Every soft subspace of a soft (1,2)*-ω-$\tilde{T}_{1/2}$-space (resp. soft (1,2)*-α-ω-$\tilde{T}_{1/2}$-space, soft (1,2)*-pre-ω-$\tilde{T}_{1/2}$-space, soft (1,2)*-b-ω-$\tilde{T}_{1/2}$-space, soft (1,2)*-β-ω-$\tilde{T}_{1/2}$-space) is a soft (1,2)*-ω-$\tilde{T}_{1/2}$-space (resp. soft (1,2)*-α-ω-$\tilde{T}_{1/2}$-space, soft (1,2)*-pre-ω-$\tilde{T}_{1/2}$-space, soft (1,2)*-b-ω-$\tilde{T}_{1/2}$-space, soft (1,2)*-β-ω-$\tilde{T}_{1/2}$-space).

Proof: It follows that from the theorem (2.16).

Proposition (2.19): If $(U,\tilde{\tau}_1,P)$ or $(U,\tilde{\tau}_2,P)$ is a soft $\tilde{T}_{1/2}$-space, then $(U,\tilde{\tau}_1,\tilde{\tau}_2,P)$ is a soft (1,2)*-ω-$\tilde{T}_{1/2}$-space (resp. soft (1,2)*-α-ω-$\tilde{T}_{1/2}$-space, soft (1,2)*-pre-ω-$\tilde{T}_{1/2}$-space, soft (1,2)*-b-ω-$\tilde{T}_{1/2}$-space, soft (1,2)*-β-ω-$\tilde{T}_{1/2}$-space).

Proof: It follows from the fact $\tilde{\tau}_i \tilde{\subseteq}$ soft $\tilde{\tau}_1\tilde{\tau}_2$-open sets in $\tilde{U}$, $i=1,2$ and proposition (2.11).

Remark (2.20): The converse of proposition (2.19) is not true in general in example (2.4) $(U,\tilde{\tau}_1,\tilde{\tau}_2,P)$ is a soft (1,2)*-ω-$\tilde{T}_{1/2}$-space (resp. soft (1,2)*-α-ω-$\tilde{T}_{1/2}$-space, soft (1,2)*-pre-ω-$\tilde{T}_{1/2}$-space, soft (1,2)*-b-ω-$\tilde{T}_{1/2}$-space, soft (1,2)*-β-ω-$\tilde{T}_{1/2}$-space), but both $(U,\tilde{\tau}_1,P)$ and $(U,\tilde{\tau}_2,P)$ are not soft (1,2)*-$\tilde{T}_{1/2}$-space.

Definition (2.21): A soft bitopological space $(U,\tilde{\tau}_1,\tilde{\tau}_2,P)$ is called a soft (1,2)*-ω-$\tilde{T}_1$-space (resp. soft (1,2)*-α-ω-$\tilde{T}_1$-space, soft (1,2)*-pre-ω-$\tilde{T}_1$-space, soft (1,2)*-b-ω-$\tilde{T}_1$-space, soft (1,2)*-β-ω-$\tilde{T}_1$-space) if for any two distinct soft points $\tilde{x}$ and $\tilde{y}$ of $\tilde{U}$, there exists a soft (1,2)*-ω-open (resp. soft (1,2)*-α-ω-open, soft (1,2)*-pre-ω-open, soft (1,2)*-b-ω-open, soft (1,2)*-β-ω-open) set in $\tilde{U}$ containing $\tilde{x}$ but not $\tilde{y}$ and a soft (1,2)*-ω-open (resp. soft (1,2)*-α-ω-open, soft (1,2)*-pre-ω-open, soft (1,2)*-b-ω-open, soft (1,2)*-β-ω-open) set in $\tilde{U}$ containing $\tilde{y}$ but not $\tilde{x}$.

Proposition (2.22): Every soft (1,2)*-ω-$\tilde{T}_1$-space (resp. soft (1,2)*-$\tilde{T}_1$-space, soft (1,2)*-α-ω-$\tilde{T}_1$-space, soft (1,2)*-pre-ω-$\tilde{T}_1$-space, soft (1,2)*-b-ω-$\tilde{T}_1$-space, soft (1,2)*-β-ω-$\tilde{T}_1$-space) is a soft (1,2)*-ω-$\tilde{T}_{1/2}$-space (resp. soft (1,2)*-$\tilde{T}_{1/2}$-space, soft (1,2)*-α-ω-$\tilde{T}_{1/2}$-space, soft (1,2)*-pre-ω-$\tilde{T}_{1/2}$-space, soft (1,2)*-b-ω-$\tilde{T}_{1/2}$-space, soft (1,2)*-β-ω-$\tilde{T}_{1/2}$-space).

Remark (2.23): The soft (1,2)*-$\tilde{T}_{1/2}$-space may not be soft (1,2)*-$\tilde{T}_1$-space in general we can see in the following example:

Example (2.24): Let $U=\{a,b\}$ and $P=\{p\}$ and let $\tilde{\tau}_1=\{\tilde{U},\tilde{\varphi},(H,P)\}$ and $\tilde{\tau}_2=\{\tilde{U},\tilde{\varphi}\}$ be soft topologies over U, where $(H,P)=\{(p,\{a\})\}$. The soft sets in $\{\tilde{U},\tilde{\varphi},(H,P)\}$ are soft $\tilde{\tau}_1\tilde{\tau}_2$-open sets. Thus $(U,\tilde{\tau}_1,\tilde{\tau}_2,P)$ is a soft (1,2)*-$\tilde{T}_{1/2}$-space, but is not soft (1,2)*-$\tilde{T}_1$-space, since $(p,\{a\})=\tilde{x}\neq\tilde{y}=(p,\{b\})$, but there exists no soft $\tilde{\tau}_1\tilde{\tau}_2$-open set containing $\tilde{y}$, but not containing $\tilde{x}$.

Proposition (2.25): Every soft (1,2)*-$\tilde{T}_1$-space is a soft (1,2)*-ω-$\tilde{T}_1$-space (resp. soft (1,2)*-α-ω-$\tilde{T}_1$-space, soft (1,2)*-pre-ω-$\tilde{T}_1$-space, soft (1,2)*-b-ω-$\tilde{T}_1$-space, soft (1,2)*-β-ω-$\tilde{T}_1$-space).

Proof: It is obvious.

Remark (2.26): The converse of proposition (2.25) is not true in general. We see that in the following example:

Example (2.27): Let $U=\{a,b\}$ and $P=\{p_1,p_2\}$ and let $\tilde{\tau}_1=\{\tilde{U},\tilde{\varphi},(H_1,P)\}$ and $\tilde{\tau}_2=\{\tilde{U},\tilde{\varphi},(H_2,P)\}$ be soft topologies over U, where $(H_1,P)=\{(p_1,\{a\}),(p_2,\{b\})\}$ and $(H_2,P)=\{(p_1,\{b\}),(p_2,\{a\})\}$. The soft sets in $\{\tilde{U},\tilde{\varphi},(H_1,P),(H_2,P)\}$ are soft $\tilde{\tau}_1\tilde{\tau}_2$-open. Thus $(U,\tilde{\tau}_1,\tilde{\tau}_2,P)$ is a soft (1,2)*-ω-$\tilde{T}_1$-space (resp. soft (1,2)*-α-ω-$\tilde{T}_1$-space, soft (1,2)*-pre-ω-$\tilde{T}_1$-space, soft (1,2)*-b-ω-$\tilde{T}_1$-space, soft (1,2)*-β-ω-$\tilde{T}_1$-space), but is not soft (1,2)*-$\tilde{T}_1$-space.

Theorem (2.28): In a soft bitopological space $(U,\tilde{\tau}_1,\tilde{\tau}_2,P)$ the following statements are equivalent.

(i) $(U,\tilde{\tau}_1,\tilde{\tau}_2,P)$ is a soft (1,2)*-ω-$\tilde{T}_1$-space (resp. soft (1,2)*-α-ω-$\tilde{T}_1$-space, soft (1,2)*-pre-ω-$\tilde{T}_1$-space, soft (1,2)*-b-ω-$\tilde{T}_1$-space, soft (1,2)*-β-ω-$\tilde{T}_1$-space)

(ii) For each $\tilde{x}\tilde{\in}\tilde{U}$, $\{\tilde{x}\}$ is a soft (1,2)*-ω-closed (resp. soft (1,2)*-α-ω-closed, soft (1,2)*-pre-ω-closed, soft (1,2)*-b-ω-closed, soft (1,2)*-β-ω-closed) set in $\tilde{U}$.

(iii) Every soft subset of $\tilde{U}$ is the intersection of all soft (1,2)*-ω-open (resp. soft (1,2)*-α-ω-open, soft (1,2)*-pre-ω-open, soft (1,2)*-b-ω-open, soft (1,2)*-β-ω-open) sets containing it.

(iv) The intersection of all soft (1,2)*-ω-open (resp. soft (1,2)*-α-ω-open, soft (1,2)*-pre-ω-open, soft (1,2)*-b-ω-open, soft (1,2)*-β-ω-open) sets containing the soft point $\tilde{x}\tilde{\in}\tilde{U}$ is $\{\tilde{x}\}$.

Proof: $(i)\Rightarrow(ii)$. Let $\tilde{x}\tilde{\in}\tilde{U}$. To prove that $\{\tilde{x}\}$ is soft (1,2)*-ω-closed in $\tilde{U}$. Let $\tilde{y}\tilde{\notin}\{\tilde{x}\}$ $\Rightarrow$ $\tilde{x}\neq\tilde{y}$. Since $(U,\tilde{\tau}_1,\tilde{\tau}_2,P)$ is a soft (1,2)*-ω-$\tilde{T}_1$-space, then there is a soft (1,2)*-ω-open set (H,P) in $\tilde{U}$ such that $\tilde{y}\tilde{\in}(H,P)$, but $\tilde{x}\tilde{\notin}(H,P)$ $\Rightarrow$ $\{\tilde{x}\}\tilde{\cap}(H,P)=\tilde{\varphi}$ $\Rightarrow$ $\{\tilde{x}\}\tilde{\subseteq}(H,P)^c$ $\Rightarrow$ $(1,2)^*\text{-}\omega cl(\{\tilde{x}\})\tilde{\subseteq}(1,2)^*\text{-}\omega cl((H,P)^c)=(H,P)^c$. Since $\tilde{y}\tilde{\notin}(H,P)^c$ $\Rightarrow$ $\tilde{y}\tilde{\notin}(1,2)^*\text{-}\omega cl(\{\tilde{x}\})$ $\Rightarrow$ $(1,2)^*\text{-}\omega cl(\{\tilde{x}\})=\{\tilde{x}\}$. Therefore $\{\tilde{x}\}$ is a soft (1,2)*-ω-closed set in $\tilde{U}$.

$(ii)\Rightarrow(iii)$. Let $(H,P)\tilde{\subset}\tilde{U}$ and $\tilde{y}\tilde{\notin}(H,P)$. Then $(H,P)\tilde{\subset}\{\tilde{y}\}^c$ and $\{\tilde{y}\}^c$ is soft (1,2)*-ω-open in $\tilde{U}$. Hence $(H,P)=\tilde{\bigcap}\{\{\tilde{y}\}^c:\tilde{y}\tilde{\in}(H,P)^c\}$ is the intersection of all soft (1,2)*-ω-open sets containing (H,P).

$(iii)\Rightarrow(iv)$. Obvious.

$(iv)\Rightarrow(i)$. Let $\tilde{x},\tilde{y}\tilde{\in}\tilde{X}$, $\tilde{x}\neq\tilde{y}$. By our assumption there exist at least a soft (1,2)*-ω-open set containing $\tilde{x}$, but not $\tilde{y}$ and also a soft (1,2)*-ω-open set containing $\tilde{y}$, but not $\tilde{x}$. Therefore $(U,\tilde{\tau}_1,\tilde{\tau}_2,P)$ is a soft (1,2)*-ω-$\tilde{T}_1$-space. Similarly, we can prove that other cases.

Theorem (2.29): Every soft bitopological space is a soft (1,2)*-ω-$\tilde{T}_1$-space (resp. soft (1,2)*-α-ω-$\tilde{T}_1$-space, soft (1,2)*-pre-ω-$\tilde{T}_1$-space, soft (1,2)*-b-ω-$\tilde{T}_1$-space, soft (1,2)*-β-ω-$\tilde{T}_1$-space).

Proof: Let $(U,\tilde{\tau}_1,\tilde{\tau}_2,P)$ be any soft bitopological space and $\tilde{x},\tilde{y}\tilde{\in}\tilde{U}$ such that $\tilde{x}\neq\tilde{y}$. Since $\tilde{U}-\{\tilde{x}\}$ and $\tilde{U}-\{\tilde{y}\}$ are soft (1,2)*-ω-open (resp. soft (1,2)*-α-ω-open, soft (1,2)*-pre-ω-open, soft (1,2)*-b-ω-open, soft (1,2)*-β-ω-open) sets in $\tilde{U}$ such that $\tilde{U}-\{\tilde{y}\}$ containing $\tilde{x}$, but not $\tilde{y}$ and $\tilde{U}-\{\tilde{x}\}$ containing $\tilde{y}$, but not $\tilde{x}$. Therefore $(U,\tilde{\tau}_1,\tilde{\tau}_2,P)$ is a soft (1,2)*-ω-$\tilde{T}_1$-space (resp. soft (1,2)*-α-ω-$\tilde{T}_1$-space, soft (1,2)*-pre-ω-$\tilde{T}_1$-space, soft (1,2)*-b-ω-$\tilde{T}_1$-space, soft (1,2)*-β-ω-$\tilde{T}_1$-space).

Corollary (2.30): A soft bitopological space $(U,\tilde{\tau}_1,\tilde{\tau}_2,P)$ is a soft (1,2)*-ω-$\tilde{T}_1$-space (resp. soft (1,2)*-α-ω-$\tilde{T}_1$-space, soft (1,2)*-pre-ω-$\tilde{T}_1$-space, soft (1,2)*-b-ω-$\tilde{T}_1$-space, soft (1,2)*-β-ω-$\tilde{T}_1$-space) iff it is a soft (1,2)*-ω-$\tilde{T}_{1/2}$-space (resp. soft (1,2)*-α-ω-$\tilde{T}_{1/2}$-space, soft (1,2)*-pre-ω-$\tilde{T}_{1/2}$-space, soft (1,2)*-b-ω-$\tilde{T}_{1/2}$-space, soft (1,2)*-β-ω-$\tilde{T}_{1/2}$-space).

Proof: It follows that from the proposition (2.22) and theorem (2.29).

Corollary (2.31): Every soft subspace of a soft (1,2)*-ω-$\tilde{T}_1$-space (resp. soft (1,2)*-α-ω-$\tilde{T}_1$-space, soft (1,2)*-pre-ω-$\tilde{T}_1$-space, soft (1,2)*-b-ω-$\tilde{T}_1$-space, soft (1,2)*-β-ω-$\tilde{T}_1$-space) is a soft (1,2)*-ω-$\tilde{T}_1$-space (resp. soft (1,2)*-α-ω-$\tilde{T}_1$-space, soft (1,2)*-pre-ω-$\tilde{T}_1$-space, soft (1,2)*-b-ω-$\tilde{T}_1$-space, soft (1,2)*-β-ω-$\tilde{T}_1$-space).

Proof: It follows that from the theorem (2.29).

Proposition (2.32): If $(U,\tilde{\tau}_1,P)$ or $(U,\tilde{\tau}_2,P)$ is a soft $\tilde{T}_1$-space, then $(U,\tilde{\tau}_1,\tilde{\tau}_2,P)$ is a soft (1,2)*-ω-$\tilde{T}_1$-space (resp. soft (1,2)*-α-ω-$\tilde{T}_1$-space, soft (1,2)*-pre-ω-$\tilde{T}_1$-space, soft (1,2)*-b-ω-$\tilde{T}_1$-space, soft (1,2)*-β-ω-$\tilde{T}_1$-space).

Proof: It follows from the fact $\tilde{\tau}_i \tilde{\subseteq}$ soft $\tilde{\tau}_1\tilde{\tau}_2$-open sets in $\tilde{U}$, $i = 1,2$ and proposition (2.25).

Remark (2.33): The converse of proposition (2.32) is not true in general in example (2.27) $(U,\tilde{\tau}_1,\tilde{\tau}_2,P)$ is a soft (1,2)*-ω-$\tilde{T}_1$-space (resp. soft (1,2)*-α-ω-$\tilde{T}_1$-space, soft (1,2)*-pre-ω-$\tilde{T}_1$-space, soft (1,2)*-b-ω-$\tilde{T}_1$-space, soft (1,2)*-β-ω-$\tilde{T}_1$-space), but both $(U,\tilde{\tau}_1,P)$ and $(U,\tilde{\tau}_2,P)$ are not soft $\tilde{T}_1$-space.

Definition (2.34): A soft bitopological space $(U,\tilde{\tau}_1,\tilde{\tau}_2,P)$ is called a soft (1,2)*-ω-$\tilde{T}_2$-space (resp. soft (1,2)*-α-ω-$\tilde{T}_2$-space, soft (1,2)*-pre-ω-$\tilde{T}_2$-space, soft (1,2)*-b-ω-$\tilde{T}_2$-space, soft (1,2)*-β-ω-$\tilde{T}_2$-space) if for any two distinct soft points $\tilde{x}$ and $\tilde{y}$ of $\tilde{U}$, there are two soft (1,2)*-ω-open (resp. soft (1,2)*-α-ω-open, soft (1,2)*-pre-ω-open, soft (1,2)*-b-ω-open, soft (1,2)*-β-ω-open) sets (H,P) and (K,P) in $\tilde{U}$ such that $\tilde{x}\tilde{\in}(H,P)$, $\tilde{y}\tilde{\in}(K,P)$ and $(H,P)\tilde{\cap}(K,P)=\tilde{\varphi}$.

Proposition (2.35): Every soft (1,2)*-ω-$\tilde{T}_2$-space (resp. soft (1,2)*-α-ω-$\tilde{T}_2$-space, soft (1,2)* -pre-ω-$\tilde{T}_2$-space, soft (1,2)*-b-ω-$\tilde{T}_2$-space, soft (1,2)*-β-ω-$\tilde{T}_2$-space) is a soft (1,2)*-ω-$\tilde{T}_1$-space (resp. soft (1,2)*-α-ω-$\tilde{T}_1$-space, soft (1,2)*-pre-ω-$\tilde{T}_1$-space, soft (1,2)*-b-ω-$\tilde{T}_1$-space, soft (1,2)*-β-ω-$\tilde{T}_1$-space).

Proof: Let $\tilde{x},\tilde{y}\tilde{\in}\tilde{U}$, $\tilde{x}\neq\tilde{y}$. By our assumption there are two soft (1,2)*-ω-open sets (H,P) and (K,P) in $\tilde{U}$ such that $\tilde{x}\tilde{\in}(H,P)$, $\tilde{y}\tilde{\in}(K,P)$ and $(H,P)\tilde{\cap}(K,P)=\tilde{\varphi}$. Thus (H,P) and (K,P) are soft (1,2)*-ω-open sets in $\tilde{U}$ such that (H,P) containing $\tilde{x}$, but not $\tilde{y}$ and (K,P) containing $\tilde{y}$, but not $\tilde{x}$. Therefore $(U,\tilde{\tau}_1,\tilde{\tau}_2,P)$ is a soft (1,2)*-ω-$\tilde{T}_1$-space. Similarly, we can prove that other cases.

Remark (2.36): The converse of proposition (2.35) is not true in general. We see that in the following example:

Example (2.37): Let $X=\mathfrak{R}$ and $P=\{p_1,p_2\}$ and let $\tilde{\tau}_1=\{(H,P)\tilde{\subseteq}\tilde{U}:(H,P)^c$ is finite$\}\tilde{\cup}$ $\{\tilde{\varphi}\}$ and $\tilde{\tau}_2=\{\tilde{U},\tilde{\varphi}\}$ be soft topologies over U. Thus $(U,\tilde{\tau}_1,\tilde{\tau}_2,P)$ is a soft (1,2)*-ω-$\tilde{T}_1$-space (resp. soft (1,2)*-α-ω-$\tilde{T}_1$-space, soft (1,2)*-pre-ω-$\tilde{T}_1$-space, soft (1,2)*-b-ω-$\tilde{T}_1$-space, soft (1,2)*-β-ω-$\tilde{T}_1$-space), clear that is not soft (1,2)*-β-ω-$\tilde{T}_2$-space.

Proposition (2.38):(i): Every soft (1,2)*-$\tilde{T}_2$-space is a soft (1,2)*-ω-$\tilde{T}_2$-space.

(ii) Every soft (1,2)*-ω-$\tilde{T}_2$-space is a soft (1,2)*-α-ω-$\tilde{T}_2$-space.

(iii) Every soft (1,2)*-α-ω-$\tilde{T}_2$-space is a soft (1,2)*-pre-ω-$\tilde{T}_2$-space.

(iv) Every soft (1,2)*-pre-ω-$\tilde{T}_2$-space is a soft (1,2)*-b-ω-$\tilde{T}_2$-space.

(v) Every soft (1,2)*-b-ω-$\tilde{T}_2$-space is a soft (1,2)*-β-ω-$\tilde{T}_2$-space.

Remark (2.39): The converse of proposition (2.38) is not true in general as shown by the following examples:

Example (2.40): Let $U=\{a,b,c\}$ and $P=\{p_1,p_2\}$ and let $\tilde{\tau}_1=\{\tilde{U},\tilde{\varphi},(H_1,P)\}$ and $\tilde{\tau}_2=\{\tilde{U},\tilde{\varphi},(H_2,P)\}$ be soft topologies over U, where $(H_1,P)=\{(p_1,\{a,c\}),(p_2,\{a\})\}$ and $(H_2,P)=\{(p_1,\{b\}),(p_2,\{b,c\})\}$. The soft sets in $\{\tilde{U},\tilde{\varphi},(H_1,P),(H_2,P)\}$ are soft $\tilde{\tau}_1\tilde{\tau}_2$-open. Thus $(U,\tilde{\tau}_1,\tilde{\tau}_2,P)$ is a soft (1,2)*-ω-$\tilde{T}_2$-space, but is not soft (1,2)*-$\tilde{T}_2$-space. Since $(p_1,\{a\})=\tilde{x}\neq\tilde{y}=(p_2,\{a\})$, but there exists no soft $\tilde{\tau}_1\tilde{\tau}_2$-open set (K_1,P) containing $\tilde{x}$ and soft $\tilde{\tau}_1\tilde{\tau}_2$-open set (K_2,P) containing $\tilde{y}$ such that $(K_1,P)\tilde{\cap}(K_2,P)=\tilde{\varphi}$.

Example (2.41): Let $U=\mathfrak{R}$ and $P=\{p\}$ and let $\tilde{\tau}_1=\{\tilde{U},\tilde{\varphi},(H_1,P)\}$ and $\tilde{\tau}_2=\{\tilde{U},\tilde{\varphi},(H_2,P)\}$ be soft topologies over U, where $(H_1,P)=\{(p,\{-1\})\},(H_2,P)=\{(p,\{1\})\}$ and $(H_3,P)=\{(p,\{1,-1\})\}$. The soft sets in $\{\tilde{U},\tilde{\varphi},(H_1,P),(H_2,P),(H_3,P)\}$ are soft $\tilde{\tau}_1\tilde{\tau}_2$-open. Thus $(U,\tilde{\tau}_1,\tilde{\tau}_2,P)$ is a soft (1,2)*-α-ω-$\tilde{T}_2$-space, clear that is not soft (1,2)*-ω-$\tilde{T}_2$-space.

Example (2.42): Let $U=\mathfrak{R}$ and $P=\{p\}$ and let $\tilde{\tau}_1=\{\tilde{U},\tilde{\varphi},(H,P)\}$ and $\tilde{\tau}_2=\{\tilde{U},\tilde{\varphi}\}$ be soft topologies over U, where $(H,P)=\{(p,\{1\})\}$. The soft sets in $\{\tilde{U},\tilde{\varphi},(H,P)\}$ are soft $\tilde{\tau}_1\tilde{\tau}_2$-open. Thus $(U,\tilde{\tau}_1,\tilde{\tau}_2,P)$ is a soft (1,2)*-pre-ω-$\tilde{T}_2$-space, but is not soft (1,2)*-α-ω-$\tilde{T}_2$-space. Since $(p,\{2\})=\tilde{x}\neq\tilde{y}=(p,\{3\})$, but there exists no a soft (1,2)*-α-ω-open set (K_1,P) containing $\tilde{x}$ and a soft (1,2)*-α-ω-open set (K_2,P) containing $\tilde{y}$ such that $(K_1,P)\tilde{\cap}(K_2,P)=\tilde{\varphi}$.

Theorem (2.43): For a soft bitopological space $(U,\tilde{\tau}_1,\tilde{\tau}_2,P)$ the following statements are equivalent.

(i) $(U,\tilde{\tau}_1,\tilde{\tau}_2,P)$ is a soft (1,2)*-ω-$\tilde{T}_2$-space (resp. soft (1,2)*-α-ω-$\tilde{T}_2$-space, soft (1,2)*-pre-ω-$\tilde{T}_2$-space, soft (1,2)*-b-ω-$\tilde{T}_2$-space, soft (1,2)*-β-ω-$\tilde{T}_2$-space)

(ii) If $\tilde{x}\tilde{\in}\tilde{U}$, then for each $\tilde{y}\neq\tilde{x}$, there is a soft (1,2)*-ω-neighborhood (resp. soft (1,2)*-α-ω-neighborhood, soft (1,2)*-pre-ω-neighborhood, soft (1,2)*-b-ω-neighborhood, soft (1,2)*-β-ω-neighborhood) (N,P) of $\tilde{x}$ such that $\tilde{y}\tilde{\notin}(1,2)^*\text{-}\omega cl(N,P)$ (resp. $\tilde{y}\tilde{\notin}(1,2)^*\text{-}\alpha\text{-}\omega cl(N,P)$, $\tilde{y}\tilde{\notin}(1,2)^*\text{-pre-}\omega cl(N,P)$, $\tilde{y}\tilde{\notin}(1,2)^*\text{-b-}\omega cl(N,P)$, $\tilde{y}\tilde{\notin}(1,2)^*\text{-}\beta\text{-}\omega cl(N,P)$).

(iii) For each $\tilde{x}\tilde{\in}\tilde{U}$, $\tilde{\cap}\{(1,2)^*\text{-}\omega cl(N,P):(N,P)$ is a soft (1,2)*-ω-neighborhood of $\tilde{x}$ } (resp. $\tilde{\cap}\{(1,2)^*\text{-}\alpha\text{-}\omega cl(N,P):(N,P)$ is a soft (1,2)*-α-ω-neighborhood of $\tilde{x}$ }, $\tilde{\cap}\{(1,2)^*\text{-pre-}\omega cl(N,P):(N,P)$ is a soft (1,2)*-pre-ω-neighborhood of $\tilde{x}$ }, $\tilde{\cap}\{(1,2)^*\text{-b-}\omega cl(N,P):(N,P)$ is a soft (1,2)*-b-ω-neighborhood of $\tilde{x}$ }, $\tilde{\cap}\{(1,2)^*\text{-}\beta\text{-}\omega cl(N,P):(N,P)$ is a soft (1,2)*-β-ω-neighborhood of $\tilde{x}$ }) $=\{\tilde{x}\}$.

Proof: $(i)\Rightarrow(ii)$. Let $\tilde{x}\tilde{\in}\tilde{U}$. If $\tilde{y}\tilde{\in}\tilde{U}$ such that $\tilde{y}\neq\tilde{x}$, then there exists disjoint soft (1,2)*-ω-open sets (H,P) and (K,P) such that $\tilde{x}\tilde{\in}(H,P)$ and $\tilde{y}\tilde{\in}(K,P)$. Hence $\tilde{x}\tilde{\in}(H,P)\tilde{\subseteq}(K,P)^c$ which implies that $(K,P)^c$ is a soft (1,2)*-ω-neighborhood of $\tilde{x}$. Also $(K,P)^c$ is soft (1,2)*-ω-closed and $\tilde{y}\tilde{\notin}(K,P)^c$. Let $(N,P)=(K,P)^c$. Then $\tilde{y}\tilde{\notin}(1,2)^*\text{-}\omega cl(N,P)$.

$(ii)\Rightarrow(iii)$. Obvious.

$(iii)\Rightarrow(i)$. Let $\tilde{x},\tilde{y}\tilde{\in}\tilde{U}$, $\tilde{x}\neq\tilde{y}$. By hypothesis, there is at least a soft (1,2)*-ω-neighborhood (N,P) of $\tilde{x}$ such that $\tilde{y}\tilde{\notin}(1,2)^*\text{-}\omega cl(N,P)$. We have $\tilde{x}\tilde{\notin}((1,2)^*\text{-}\omega cl(N,P))^c$ which is soft (1,2)*-ω-open. Since (N,P) is a soft (1,2)*-ω-neighborhood of $\tilde{x}$, then there exists a soft (1,2)*-ω-open set (H,P) in $\tilde{U}$ such that $\tilde{x}\tilde{\in}(H,P)\tilde{\subseteq}(N,P)$ and $(H,P)\tilde{\cap}((1,2)^*\text{-}\omega cl(N,P))^c=\tilde{\varphi}$. Hence $(U,\tilde{\tau}_1,\tilde{\tau}_2,P)$ is a soft (1,2)*-ω-$\tilde{T}_2$-space. Similarly, we can prove that other cases.

Now, we need the following lemma.

Lemma (2.44): Let $(U,\tilde{\tau}_1,\tilde{\tau}_2,P)$ be a soft bitopological space and $Y \subseteq U$. If (H,P) is a soft (1,2)*-ω-open set in $\tilde{U}$, then $(H,P)\tilde{\bigcap}\tilde{Y}$ is a soft (1,2)*-ω-open set in $\tilde{Y}$.

Proof: Let (H,P) be a soft (1,2)*-ω-open set in $\tilde{U}$. To prove that $(H,P)\tilde{\bigcap}\tilde{Y}$ is a soft (1,2)*-ω-open set in $\tilde{Y}$. Let $\tilde{y}\tilde{\in}(H,P)\tilde{\bigcap}\tilde{Y} \Rightarrow \tilde{y}\tilde{\in}(H,P)$. Since (H,P) is soft (1,2)*-ω-open in $\tilde{U}$ $\Rightarrow \exists$ a soft $\tilde{\tau}_1\tilde{\tau}_2$-open set (V,P) in $\tilde{U}$ such that $\tilde{y}\tilde{\in}(V,P)$ and $(V,P)-(H,P)$ is a soft countable set. Hence $\tilde{Y}\tilde{\bigcap}(V,P)$ is a soft $\tilde{\tau}_{1Y}\tilde{\tau}_{2Y}$-open set in $\tilde{Y}$. Since $(\tilde{Y}\tilde{\bigcap}(V,P))-(\tilde{Y}\tilde{\bigcap}(H,P)) = \tilde{Y}\tilde{\bigcap}((V,P)-(H,P)) \tilde{\subseteq} ((V,P)-(H,P))$, then $(\tilde{Y}\tilde{\bigcap}(V,P))-(\tilde{Y}\tilde{\bigcap}(H,P))$ is soft countable. Thus $(H,P)\tilde{\bigcap}\tilde{Y}$ is a soft (1,2)*-ω-open set in $\tilde{Y}$.

Proposition (2.45): Every soft subspace of a soft (1,2)*-ω-$\tilde{T}_2$-space is a soft (1,2)*-ω-$\tilde{T}_2$-space.

Proof: Let $(U,\tilde{\tau}_1,\tilde{\tau}_2,P)$ be a soft (1,2)*-ω-$\tilde{T}_2$-space and $(Y,\tilde{\tau}_{1\tilde{Y}},\tilde{\tau}_{2\tilde{Y}},P)$ be a soft subspace of $(U,\tilde{\tau}_1,\tilde{\tau}_2,P)$. To prove that $(Y,\tilde{\tau}_{1\tilde{Y}},\tilde{\tau}_{2\tilde{Y}},P)$ is a soft (1,2)*-ω-$\tilde{T}_2$-space. Let $\tilde{x},\tilde{y}\tilde{\in}\tilde{Y}$ such that $\tilde{x}\neq\tilde{y}$. Since $\tilde{Y}\tilde{\subseteq}\tilde{U}$, then $\tilde{x},\tilde{y}\tilde{\in}\tilde{U}$. But $(U,\tilde{\tau}_1,\tilde{\tau}_2,P)$ is a soft (1,2)*-ω-$\tilde{T}_2$-space, then there are two soft (1,2)*-ω-open sets (H,P) and (K,P) in $\tilde{U}$ such that $\tilde{x}\tilde{\in}(H,P)$, $\tilde{y}\tilde{\in}(K,P)$ and $(H,P)\tilde{\bigcap}(K,P)=\tilde{\varphi}$. By lemma (2.44), $(H',P)=(H,P)\tilde{\bigcap}\tilde{Y}$ and $(K',P)=(K,P)\tilde{\bigcap}\tilde{Y}$ are soft (1,2)*-ω-open sets in $\tilde{Y}$ such that $\tilde{x}\tilde{\in}(H',P)$ and $\tilde{y}\tilde{\in}(K',P)$. Since $(H',P)\tilde{\bigcap}(K',P)= ((H,P)\tilde{\bigcap}\tilde{Y})\tilde{\bigcap}((K,P)\tilde{\bigcap}\tilde{Y}) = ((H,P)\tilde{\bigcap}(K,P))\tilde{\bigcap}\tilde{Y} = \tilde{\varphi}\tilde{\bigcap}\tilde{Y}=\tilde{\varphi}$. Thus $(Y,\tilde{\tau}_{1\tilde{Y}},\tilde{\tau}_{2\tilde{Y}},P)$ is a soft (1,2)*-ω-$\tilde{T}_2$-space.

Remark (2.46): Soft subspace of a soft (1,2)*-α-ω-$\tilde{T}_2$-space is not a soft (1,2)*-α-ω-$\tilde{T}_2$-space as shown in the following example:

Example (2.47): Let $U=\mathfrak{R}$ and $P=\{p\}$ and let $\tilde{\tau}_1=\{\tilde{U},\tilde{\varphi},(H_1,P)\}$ and $\tilde{\tau}_2=\{\tilde{U},\tilde{\varphi},(H_2,P)\}$ be soft topologies over U, where $(H_1,P)=\{(p,\{-1\})\}$, $(H_2,P)=\{(p,\{1\})\}$ and $(H_3,P)= \{(p,\{1,-1\})\}$. Then $(U,\tilde{\tau}_1,\tilde{\tau}_2,P)$ is a soft (1,2)*-α-ω-$\tilde{T}_2$-space. If $Y=\mathfrak{R}-\{1\}\subset U=\mathfrak{R}$, then $\tilde{\tau}_{1\tilde{Y}}=\{\tilde{Y},\tilde{\varphi},(H_1,P)\}$ and $\tilde{\tau}_{2\tilde{Y}}=\{\tilde{Y},\tilde{\varphi}\}$ are soft topologies over Y. The soft sets in $\{\tilde{Y},\tilde{\varphi},(H_1,P)\}$ are soft $\tilde{\tau}_{\tilde{Y}_1}\tilde{\tau}_{\tilde{Y}_2}$-open. Therefore $(Y,\tilde{\tau}_{1\tilde{Y}},\tilde{\tau}_{2\tilde{Y}},P)$ is not a soft (1,2)*-α-ω-$\tilde{T}_2$-space, since $(p,\{3\})=\tilde{x}\neq\tilde{y}=(p,\{4\})$, but there exists no soft (1,2)*-α-ω-open sets (K_1,P) and (K_2,P) in $\tilde{Y}$ such that $\tilde{x}\tilde{\in}(K_1,P)$, $\tilde{y}\tilde{\in}(K_2,P)$ and $(K_1,P)\tilde{\bigcap}(K_2,P)=\tilde{\varphi}$.

Proposition (2.48): If $(U,\tilde{\tau}_1,P)$ or $(U,\tilde{\tau}_2,P)$ is a soft $\tilde{T}_2$-space, then $(U,\tilde{\tau}_1,\tilde{\tau}_2,P)$ is a soft (1,2)*-ω-$\tilde{T}_2$-space (resp. soft (1,2)*-α-ω-$\tilde{T}_2$-space, soft (1,2)*-pre-ω-$\tilde{T}_2$-space, soft (1,2)*-b-ω-$\tilde{T}_2$-space, soft (1,2)*-β-ω-$\tilde{T}_2$-space).

Proof: It follows from the fact $\tilde{\tau}_i \tilde{\subseteq}$ soft $\tilde{\tau}_1\tilde{\tau}_2$-open sets in $\tilde{U}$, $i=1,2$ and proposition (2.38).

Remark (2.49): The converse of proposition (2.48) is not true in general. We see that by the following example:

Example (2.50): Let $U=\{a,b,c,d\}$ and $P=\{p_1,p_2\}$ and let $\tilde{\tau}_1=\{\tilde{U},\tilde{\varphi},(H_1,P)\}$ and $\tilde{\tau}_2=\{\tilde{U},\tilde{\varphi},(H_2,P)\}$ be soft topologies over U, where $(H_1,P)=\{(p_1,\{a,b\}),(p_2,\{a,b\})\}$ and $(H_2,P)=\{(p_1,\{a\}),(p_2,\{a\})\}$. The soft sets in $\{\tilde{U},\tilde{\varphi},(H_1,P),(H_2,P)\}$ are soft $\tilde{\tau}_1\tilde{\tau}_2$-open. Thus $(U,\tilde{\tau}_1,\tilde{\tau}_2,P)$ is a soft (1,2)*-ω-$\tilde{T}_2$-space (resp. soft (1,2)*-α-ω-$\tilde{T}_2$-space, soft (1,2)*-

pre-ω-$\tilde{T}_2$-space, soft (1,2)*-b-ω-$\tilde{T}_2$-space, soft (1,2)*-β-ω-$\tilde{T}_2$-space), but both $(U,\tilde{\tau}_1,P)$ and $(U,\tilde{\tau}_2,P)$ are not soft $\tilde{T}_2$-space.

Definition (2.51): A soft function $f:(U,\tilde{\tau}_1,\tilde{\tau}_2,P)\to(V,\tilde{\sigma}_1,\tilde{\sigma}_2,P)$ is called strongly soft (1,2)*-ω-continuous (resp. strongly soft (1,2)*-α-ω-continuous, strongly soft (1,2)*-pre-ω-continuous, strongly soft (1,2)*-b-ω-continuous, strongly soft (1,2)*-β-ω-continuous) if $f^{-1}((H,P))$ is a soft $\tilde{\tau}_1\tilde{\tau}_2$-open set in $\tilde{U}$ for each soft (1,2)*-ω-open (resp. soft (1,2)*-α-ω-open, soft (1,2)*-pre-ω-open, soft (1,2)*-b-ω-open, soft (1,2)*-β-ω-open) set (H,P) in $\tilde{V}$.

Theorem (2.52): Let $f:(U,\tilde{\tau}_1,\tilde{\tau}_2,P)\to(V,\tilde{\sigma}_1,\tilde{\sigma}_2,P)$ be a strongly soft (1,2)*-ω-continuous (resp. strongly soft (1,2)*-α-ω-continuous, strongly soft (1,2)*-pre-ω-continuous, strongly soft (1,2)*-b-ω-continuous, strongly soft (1,2)*-β-ω-continuous) injective function. If $(V,\tilde{\sigma}_1,\tilde{\sigma}_2,P)$ is a soft (1,2)*-ω-$\tilde{T}_i$-space (resp. soft (1,2)*-α-ω-$\tilde{T}_i$-space, soft (1,2)*-pre-ω-$\tilde{T}_i$-space, soft (1,2)*-b-ω-$\tilde{T}_i$-space, soft (1,2)*-β-ω-$\tilde{T}_i$-space, then $(U,\tilde{\tau}_1,\tilde{\tau}_2,P)$ is a soft (1,2)*-$\tilde{T}_i$-space, for $i=0,\frac{1}{2},1,2$.

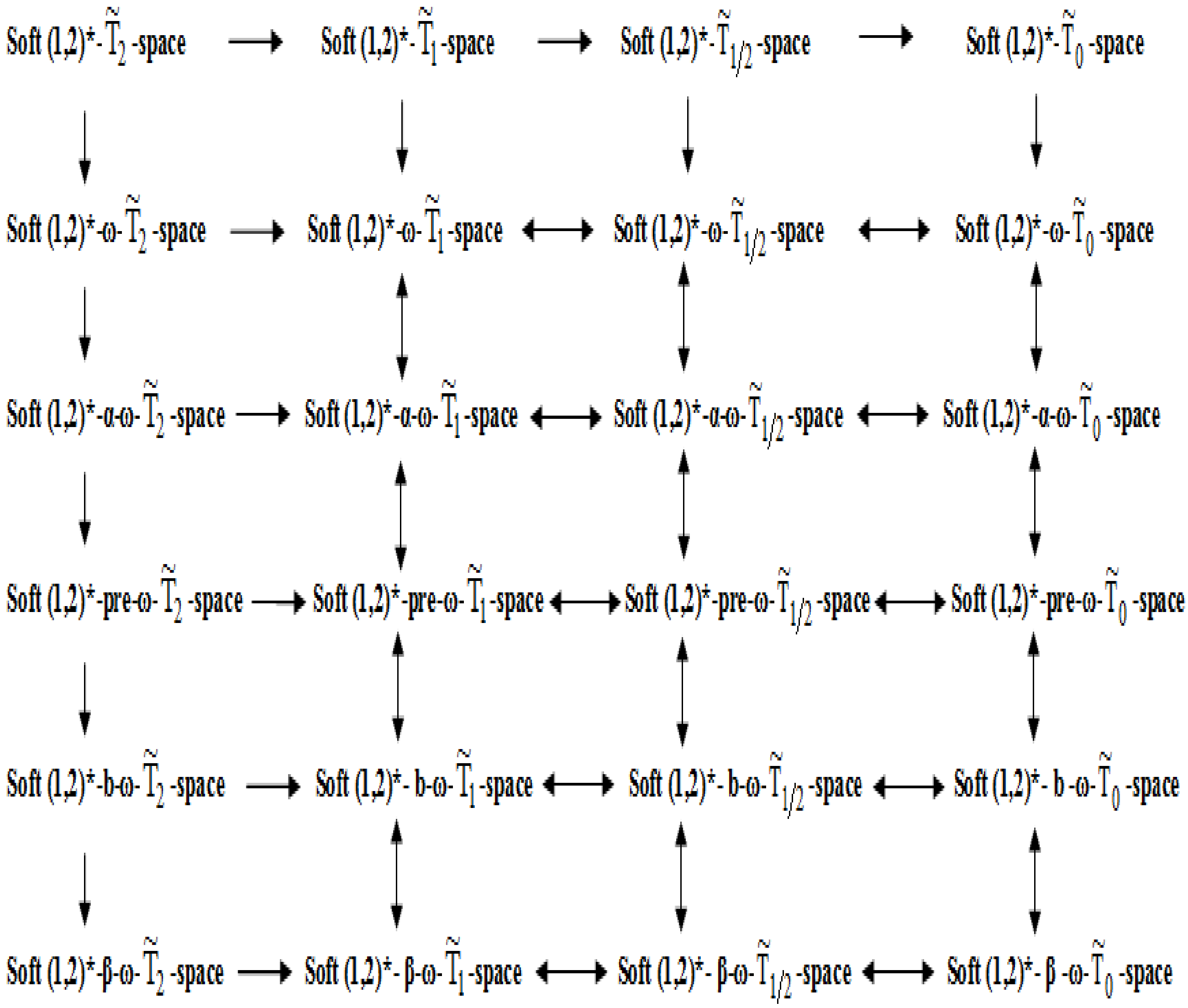
Proof: Suppose that $(V,\tilde{\sigma}_1,\tilde{\sigma}_2,P)$ is a soft (1,2)*-ω-$\tilde{T}_2$-space. Let $\tilde{x},\tilde{y}\,\tilde{\in}\,\tilde{U}$ such that $\tilde{x}\neq\tilde{y}$. Since f is injective and $(V,\tilde{\sigma}_1,\tilde{\sigma}_2,P)$ is a soft (1,2)*-ω-$\tilde{T}_2$-space, then there exists disjoint soft (1,2)*-ω-open sets (H_1,P) and (H_2,P) in $\tilde{V}$ such that $f(\tilde{x})\,\tilde{\in}\,(H_1,P)$ and $f(\tilde{y})\,\tilde{\in}\,(H_2,P)$. By definition (2.51), $f^{-1}((H_1,P))$ and $f^{-1}((H_2,P))$ are soft $\tilde{\tau}_1\tilde{\tau}_2$-open sets in $\tilde{U}$ such that $\tilde{x}\,\tilde{\in}\,f^{-1}((H_1,P)),\tilde{y}\,\tilde{\in}\,f^{-1}((H_2,P))$ and $f^{-1}((H_1,P))\,\tilde{\cap}\,f^{-1}((H_2,P))=\tilde{\varphi}$. Hence $(U,\tilde{\tau}_1,\tilde{\tau}_2,P)$ is a soft (1,2)*-$\tilde{T}_2$-space. Similarly, we can prove that other cases.

Definition (2.53): A soft function $f:(U,\tilde{\tau}_1,\tilde{\tau}_2,P)\to(V,\tilde{\sigma}_1,\tilde{\sigma}_2,P)$ is called strongly soft (1,2)*-ω-open (resp. strongly soft (1,2)*-α-ω-open, strongly soft (1,2)*-pre-ω-open, strongly soft (1,2)*-b-ω-open, strongly soft (1,2)*-β-ω-open) if $f((H,P))$ is a soft $\tilde{\sigma}_1\tilde{\sigma}_2$-open set in $\tilde{V}$ for each soft (1,2)*-ω-open (resp. soft (1,2)*-α-ω-open, soft (1,2)*-pre-ω-open, soft (1,2)*-b-ω-open, soft (1,2)*-β-ω-open) set (H,P) in $\tilde{U}$.

Theorem (2.54): Let $f:(U,\tilde{\tau}_1,\tilde{\tau}_2,P)\to(V,\tilde{\sigma}_1,\tilde{\sigma}_2,P)$ be a strongly soft (1,2)*-ω-open (resp. strongly soft (1,2)*-α-ω-open, strongly soft (1,2)*-pre-ω-open, strongly soft (1,2)*-b-ω-open, strongly soft (1,2)*-β-ω-open) bijective function. If $(U,\tilde{\tau}_1,\tilde{\tau}_2,P)$ is a soft (1,2)*-ω-$\tilde{T}_i$-space (resp. soft (1,2)*-α-ω-$\tilde{T}_i$-space, soft (1,2)*-pre-ω-$\tilde{T}_i$-space, soft (1,2)*-b-ω-$\tilde{T}_i$-space, soft (1,2)*-β-ω-$\tilde{T}_i$-space), then $(V,\tilde{\sigma}_1,\tilde{\sigma}_2,P)$ is a soft (1,2)*-$\tilde{T}_i$-space, for $i=0,\frac{1}{2},1,2$.

Proof: Suppose that $(U,\tilde{\tau}_1,\tilde{\tau}_2,P)$ is a soft (1,2)*-ω-$\tilde{T}_2$-space. Let $\tilde{y}_1,\tilde{y}_2\,\tilde{\in}\,\tilde{V}$ such that $\tilde{y}_1\neq\tilde{y}_2$. Since f is surjective, then there exists $\tilde{x}_1,\tilde{x}_2\,\tilde{\in}\,\tilde{U}$ such that $f(\tilde{x}_1)=\tilde{y}_1$ and $f(\tilde{x}_2)=\tilde{y}_2$. Since f is a function, then $\tilde{x}_1\neq\tilde{x}_2$. But $(U,\tilde{\tau}_1,\tilde{\tau}_2,P)$ is a soft (1,2)*-ω-$\tilde{T}_2$-space, then there exists disjoint soft (1,2)*-ω-open sets (H_1,P) and (H_2,P) in $\tilde{U}$ such that $\tilde{x}_1\,\tilde{\in}\,(H_1,P)$ and $\tilde{x}_2\,\tilde{\in}\,(H_2,P)$. By definition (2.53), $f((H_1,P))$ and $f((H_2,P))$ are soft $\tilde{\sigma}_1\tilde{\sigma}_2$-open sets in $\tilde{V}$ such that $f(\tilde{x}_1)\,\tilde{\in}\,f((H_1,P))$ and $f(\tilde{x}_2)\,\tilde{\in}\,f((H_2,P))$. Since f is injective, then $f((H_1,P))\,\tilde{\cap}\,f((H_2,P))=\tilde{\varphi}$. Hence $(V,\tilde{\sigma}_1,\tilde{\sigma}_2,P)$ is a soft (1,2)*-$\tilde{T}_2$-space. By the same way we can prove that other cases.

The following diagram shows the relation among soft (1,2)*-$\tilde{\mathbf{T}}_{\mathbf{i}}$-spaces, soft (1,2)*-ω-$\tilde{\mathbf{T}}_{\mathbf{i}}$-spaces, soft (1,2)*-α-ω-$\tilde{\mathbf{T}}_{\mathbf{i}}$-spaces, soft (1,2)*-pre-ω-$\tilde{\mathbf{T}}_{\mathbf{i}}$-spaces, soft (1,2)*-b-ω-$\tilde{\mathbf{T}}_{\mathbf{i}}$-spaces, and soft (1,2)*-β-ω-$\tilde{\mathbf{T}}_{\mathbf{i}}$-spaces, for $\mathbf{i}=0,\frac{1}{2},1,2$.



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