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Solving Fuzzy Attribute Quality Control Charts with proposed Ranking Function

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Abstract

The attribute quality control charts are one of the main useful tools to use in control of quality product in companies. In this paper utilizing the statistical procedures to find the attribute quality control charts for through fuzzified the real data which we got it from Baghdad Soft Drink Company in Iraq, by using triangular membership function to obtain the fuzzy numbers then employing the proposed ranking function to transform to traditional sample. Then, compare between crisp and fuzzy attribute quality control.

Keywords: Quality Control, Fuzzy Set Theory, Attributes Control Charts, Ranking function.

1.Introduction

Statistical procedures are one of the main useful tools in supervision the production process by using control charts and sample inspection plans and these two procedures depend on the random variations which happen in terminology then determine the production process is subject to specifications of quality control or not, since the units produced differ in quality, if these imbalances and deviations are minor, the production is acceptable but if these differences and deviations exceed certain limits. The production is not acceptable.

Statistical methods were used in the field of quality control for the first time in 1924 by researcher (Shewart). The company makes the objective in manufacturing the items which vacant from defective and congruent to specifications, then utilizing the control charts to esteem the statistical techniques to control the quality production.

Control Chart is one of the most common statistical methods used in terms of monitoring the changes that occur during the stages of product process, it is determined by whether the process is statistically accurate by the observation recorded from the samples drawn.

The data was taken from one of the important production plants is a laboratory for the production of drinking water bottles to detect the specifications and efficiency of production

accurately and quickly, the results showed that fuzzy control chart are more accurate and economically faster in controlling the quality of production, leading to the detection of defective units during the production process, which helps to detect error quickly.
Zadeh is the first from discover fuzzy set theory in (1965)[1]. Bradshaw (1983), for the first time used fuzzy sets as a basis for the explanation of the measurement of conformity of each product units with the specifications[2]. T. Raz and J. Wang (1990) have attempted to extend the use of control charts to allow for linguistic variables[3]. Ray Cheng et al (1995), proposed economic statistical np-control chart design[4]. F.Franceschine and D.Romano (1999), proposed a method for the online control of qualitative of the product/service using control charts for linguistic variables[5]. K.Latva-Kayra (2001), proposed EWMA and CUSUM with fuzzy control limits and their fuzzy combination is used[6]. M.Gulbayand C.Kahraman(2006),the direct fuzzy approach to fuzzy control charts without any distrotion, and fuzzy abnormal pattern rules based on the probabilities of fuzzy events is proposed[7]. Chih-Hsuan Wang-Way Kuo (2007), multiresolution relied on robust fuzzy clustering approach[8]. H.Moheb Alizadeh, A.R.Arshadi Khamseh and S.M.T Fatemi Ghomi (2010), developed multivariate variable control charts in fuzzy mode[9]. Osman Taylan, and Ibrahim A.Darrab (2012), describe the use of artificial intelligence (AI) methods such as fuzzy logic and neural networks in quality control and improvement[10]. Mohammad Hossein and IR (2014), provide a literature review of the control chart under a fuzzy environment with proposing several classifications and analyzes[11]. P.Fernández and other IR (2015), the use of fuzzy control charts becomes inevitable when statistical data considered are vague or affected by uncertainty[12]. M.Hadi and M.Mahmoudzadeh (2017), presented the fuzzy statistical process control development for attribute quality control chart by using Monte Carlo simulation method[13]. The aim of the study is applying the crisp control chart and fuzzy control chart for real data by utilizing of triangular membership function. Then employing the proposed ranking function to find the attribute quality control when (w=0.2 , λ=0.5) and (w=0.6 , λ=0.9). This paper is organized as follows. In section 2, showing the attribute control chart technique. In section 3, showing the fuzzy set theory. In section 4, introducing the new method of ranking function. In section 5, introduction the application of real data. In section 6, numerical results are shown. In section 7, conclusions are given.

2.Attributes Control Charts

In some cases, production units can be divided into two types defective and invalid production units and non-defective and valid production units, this means that the units produced are described by specific properties or characteristics. If assume the withdrawal of n_i from random samples of equal sizes from a specific production process during regular interval and which distributes Binomial Distribution and assume that the defective ratios in production are (p) and the non-defective ratios are (1-p) then the ratios of defective values are between the two limits ($\bar{p} \mp \sqrt{\frac{p(1-p)}{n}}$) where as $\bar{p} = \frac{\sum p_i}{n}$ is the defective rate of proportions, the control chart is represented by three parallel lines:

The middle limit of attribute control is:

CL=$\bar{P} = \frac{\sum p_i}{n}$ (1)

The upper limit of attribute control is:

UCL= $(\bar{p} + \sqrt{\frac{p(1-p)}{n}})$ (2)

The lower limit of attribute control is:

LCL= $(\bar{p} - \sqrt{\frac{p(1-p)}{n}})$ (3)

Where the proportion of defective is:

P=$\frac{\text{defective}}{\text{production}}$ (4)

If one or more defective proportions are outside the upper and lower control limits, then the production process is outside the control limit. Otherwise, the production process is under control.

3-Fuzzy set: [14**]** Let X be the universal set. A fuzzy set in X is a set of ordered pairs, A={(x,$\mu_A(x)); x \epsilon X\}$, where $\mu_A : X \rightarrow [0,1]$ is called the membership set.

α-Cats of a Fuzzy Set:[14] The crisp set that contains all the elements of X that have non-zero membership grades in a fuzzy set A is called the support of the set A, denoted by Supp(A). i.e.,Supp(A)={x ϵX :A(x) $\geq 0\}$.

The membership function that we use it is as following

$$\mu_A(x) = \begin{cases} \frac{\lambda(x-a)}{(b-a} & a \leq x \leq b \\ \lambda & x = b \\ \frac{\lambda(c-x)}{(c-b)} & b \leq x \leq c \end{cases} \quad (5)$$

4-Ranking function: [8] Let Ã, $\tilde{B}$ ϵ E, define is ranking of A, B by R(.) on E, i.e

R(Ã) > R($\tilde{B}$) ↔ Ã > $\tilde{B}$

R(Ã) < R($\tilde{B}$) ↔ Ã < $\tilde{B}$

R(Ã)= R($\tilde{B}$) ↔ Ã ≈ $\tilde{B}$

Considure that the triangular fuzzy numbers represented by Ã= (a, b, c), where b is the all sample, a is the lift width and c is the right width.

Now, presented the arbitrary fuzzy numbers Ã(α) by an ordered pair of function $[\tilde{A}^L(\alpha), \tilde{A}^U(\alpha)]$, where $\tilde{A}^L(\alpha)$ is a bounded left continuous nondecreasing function over [0,1] and $\tilde{A}^U(\alpha)$ is a bounded left continuous nonincreasing function over [0,1] , $\tilde{A}^L(\alpha) \leq \tilde{A}^U(\alpha)$.

Where $\tilde{A}^L(\alpha) = inf\{x|\tilde{A}(x) \geq \alpha\}$ and $\tilde{A}^U(\alpha) = sup\{x|\tilde{A}(x) \geq \alpha\}$.

Now, the utilizing the triangular membership function to find the new ranking function which is as follows:

$\alpha = \frac{\lambda(x-a)}{(b-a)}$ α=$\frac{\lambda(c-x)}{(c-b)}$

$x = \frac{\alpha}{\lambda}(b-a) + a$ $x = c - \frac{\alpha}{\lambda}((c-b)$

$\tilde{A}^L(\alpha) = a + \frac{\alpha}{\lambda}(b-a)$ $\tilde{A}^u(\alpha) = c - \frac{\alpha}{\lambda}(c-b)$

$$R(\tilde{A}) = \frac{\int_w^\lambda [\tilde{A}^L(\alpha) + \tilde{A}^u(\alpha)]\, d\alpha}{\int_w^\lambda \alpha\, d\alpha} = \frac{\int_w^\lambda \left[a + \frac{\alpha}{\lambda}(b-a) + c - \frac{\alpha}{\lambda}(c-b)\right] d\alpha}{\int_w^\lambda \alpha\, d\alpha}$$

$$= \frac{\int_w^\lambda \left[(a+c) + \frac{\alpha}{\lambda}(2b-a-c)\right] d\alpha}{\int_w^\lambda \alpha\, d\alpha} = \frac{[(a+b)\alpha + \frac{\alpha^2}{2\lambda}(2b-a-c)]\Big|_w^\lambda}{\frac{\alpha^2}{2}\Big|_w^\lambda}$$

$$=\frac{(a+c)(\lambda-w)+\frac{(\lambda^2-w^2)}{2\lambda}(2b-a-c)}{\frac{\lambda^2-w^2}{2}}=\frac{(a+c)(\lambda-w)+\frac{(\lambda-w)(\lambda+w)}{2\lambda}(2b-a-c)}{\frac{(\lambda-w)(\lambda+w)}{2}}$$

$$=\frac{2\lambda(a+b)+(\lambda+w)(2b-a-c)}{2\lambda}*\frac{2}{(\lambda+w)}=\frac{2\lambda(a+b)+(\lambda+w)(2b-a-c)}{\lambda(\lambda+w)}$$

$$=\frac{\lambda(2b+a+c)+w(2b-a-c)}{\lambda(\lambda+w)} \qquad (6)$$

5.Application

Baghdad Soft Drinks Company is one of the most important companies operating in the province of Baghdad /Zaafaraniya and the establishment of this company dates back to the 1960s as one of the establishments of the Ministry of Industry and Minerals where issued a founding license from the Directorate General of Industrial Development no. 474 in 1961/12/21. However, it became a mixed company in 1989 in accordance with companies Law no. 36 0f 1983 and procedures of establishing the company were completed by issuing the certificate of incorporation under the decision of the Registrar of companies in the Ministry of Commerce no. (m.s/3315) in 1989/3/23.

The company has 5 factories, each of which contains production lines:

a) Degla factory: consists of four production lines.
b) Euphrates factory: consists of four production lines.
c) Shatt al-Arab factory: consists of two production lines.
d) Rafidain factory: consist of one production line.
e) Aquafina factory: consists of two production lines.

In addition to three factories to manufacture carbon dioxide gas.

The company is licensed to produce soft drinks from Pepsi Co. International and latter takes samples from markets and is examined to assess the quality of production and the company is adjacent to distribute its products in central and southern Iraq.

6.Numerical Results

The samples that we get it from Baghdad Soft Drinks Company are as follows:

Table 1. contain defective and production from company

n	defective	production	n	defective	production	n	defective	production
1	2613	1035510	11	3953	1205130	21	224	179970
2	3337	1161630	12	910	809400	22	920	590268
3	3087	1118460	13	1932	1064520	23	778	1007520
4	2650	1104840	14	452	366990	24	512	406200
5	1974	896250	15	947	654330	25	263	528000
6	2566	1150950	16	3544	1115640	26	896	1204500
7	3417	1138590	17	1632	1120320	27	731	749592
8	1405	772566	18	845	769740	28	444	808500
9	1837	982566	19	1562	1054260	29	2054	1118700
10	2539	1121220	20	1602	1060890	30	312	314130

Now, applying attributes control charts in all samples of **Table (1)**

First, find (p) in equation (4).

$P_1=\frac{2613}{1035510}=0.002523$ and so that

Table 2. for the value (P)

n	p	n	p	n	p
1	0.002523394	11	0.003280144	21	0.001244652
2	0.002872688	12	0.00112429	22	0.001558614
3	0.002760045	13	0.001814902	23	0.000772193
4	0.002398537	14	0.001231641	24	0.001260463
5	0.00220251	15	0.001447282	25	0.000498106
6	0.002229463	16	0.003176652	26	0.000743877
7	0.00300108	17	0.001456727	27	0.000975197
8	0.001818615	18	0.001097773	28	0.000549165
9	0.001869595	19	0.001481608	29	0.00183606
10	0.002264498	20	0.001510053	30	0.000993219

Second compute the middle limit of attribute control in equation (1)

CL= $\bar{P} = \frac{\sum_1^{30} P_i}{30} = \frac{0.051993}{30} = 0.001733$

∗After that, compute the upper limit of attribute control in equation (2)

UCL=$\bar{P} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$ =0.024515

Now, compute the lower limit of attribute control in equation (3)

LCL=$\bar{P} - 3 * \sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = -0.02105$

Finally, drawing the charts of quality control with depend on the attributes control charts

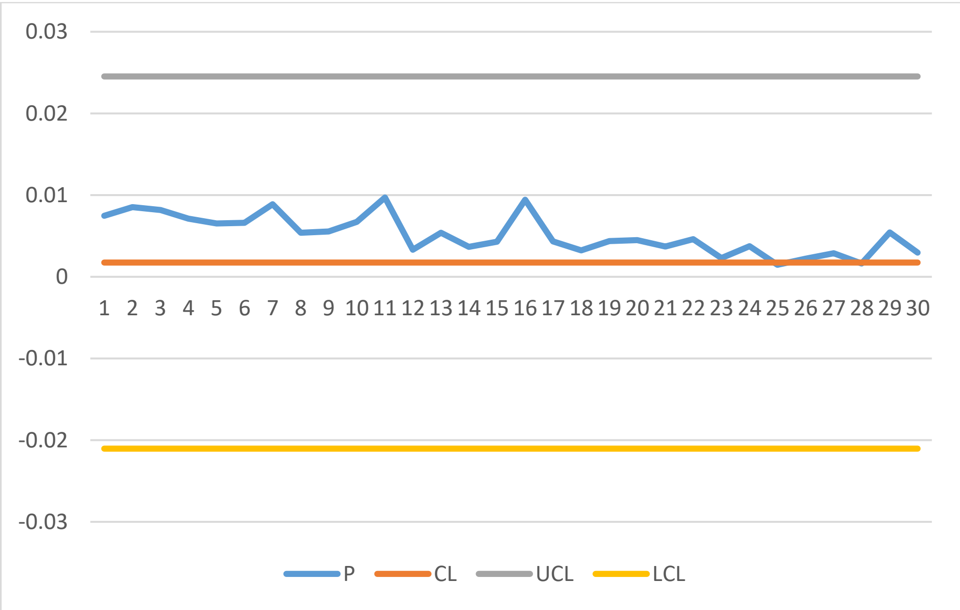

Figure 1. Contain the charts to the original values

Then the fuzzified the data to make it vague numbers by using the membership function in equation (5) which are as follows:

$(a, b, c) = (all\ samples - \bar{x}, all\ samples\ , all\ samples + \bar{x})$ (7)

Therefore, applying the new ranking function in equation (7) to transform the fuzzy numbers to crisp numbers but the new ranking function depend upon w, λ∈[0,1].

Now, computing a new ranking function by utilizing the values of w, λ.

The values of the w, λ are λ=0.5, w=0.2 .

Then using equation (6) to find the ranking function

Table 3. Fuzzy ranking function of defective samples

n	defective	n	defective	n	defective
1	14931.42857	11	22588.57143	21	1280
2	19068.57143	12	5200	22	5257.142857
3	17640	13	11040	23	4445.714286
4	15142.85714	14	2582.857143	24	2925.714286
5	11280	15	5411.428571	25	1502.857143
6	14662.85714	16	20251.42857	26	5120
7	19525.71429	17	9325.714286	27	4177.142857
8	8028.571429	18	4828.571429	28	2537.142857
9	10497.14286	19	8925.714286	29	11737.14286
10	14508.57143	20	9154.285714	30	1782.857143

Now, applying attributes control charts in all samples when λ=0.5, w=0.2

First, find (p) in equation (4).

$P_1 = \frac{14931.42857}{1035510} = 0.014419396$ and so that

Table 4. For the value (P) when λ=0.5, w=0.2

n	p	n	p	n	p
1	0.014419396	11	0.01874368	21	0.007112296
2	0.016415357	12	0.006424512	22	0.008906366
3	0.015771686	13	0.010370871	23	0.004412532
4	0.013705928	14	0.00703795	24	0.007202645
5	0.012585774	15	0.008270183	25	0.00284632
6	0.012739786	16	0.018152297	26	0.004250726
7	0.01714903	17	0.008324152	27	0.005572555
8	0.010392085	18	0.00627299	28	0.003138086
9	0.010683397	19	0.008466331	29	0.01049177
10	0.012939986	20	0.008628874	30	0.005675539

Second compute the middle limit of attribute control in equation (1)

CL= $\bar{P} = \frac{\sum_{1}^{30} P_i}{30}$ = 0.009903

After that, compute the upper limit of attribute control in equation (2)

UCL=$\bar{P} + 3 * \sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$ =0.06414

Now, compute the lower limit of attribute control in equation (3)

LCL=$\bar{P} - 3 * \sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = -0.04433$

Finally, drawing the charts of quality control with depend on the attributes control charts

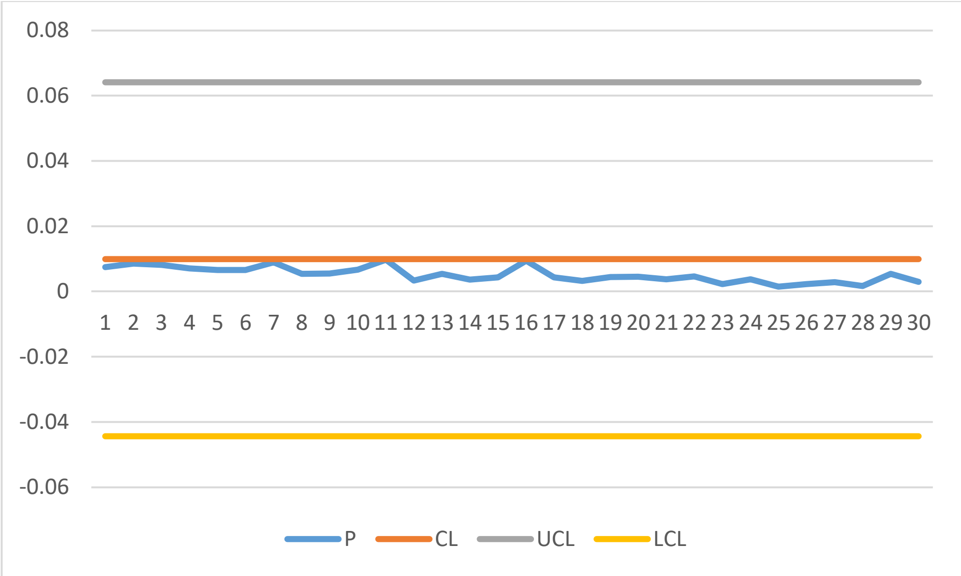

Figure 2. Contain the charts when λ=0.5, w=0.3

Now, computing a new ranking function by utilizing the values of w, λ.

The values of the w, λ are λ=0.9, w=0.6 .

Then using equation (6) to find the ranking function

Table 5. Fuzzy ranking function of defective samples

n	defective	n	Defective	n	defective
1	6968	11	10541.33333	21	597.3333333
2	8898.666667	12	2426.666667	22	2453.333333
3	8232	13	5152	23	2074.666667
4	7066.666667	14	1205.333333	24	1365.333333
5	5264	15	2525.333333	25	701.3333333
6	6842.666667	16	9450.666667	26	2389.333333
7	9112	17	4352	27	1949.333333
8	3746.666667	18	2253.333333	28	1184
9	4898.666667	19	4165.333333	29	5477.333333
10	6770.666667	20	4272	30	832

Now, applying attributes control charts in all samples when λ=0.9, w=0.6

First, find (p) in equation (4).

$P_1 = \frac{6968}{1035510} = 0.006729051$ and so that

Table 6. For the value (P) when λ=0.5, w=0.2

n	p	n	P	n	p
1	0.006729051	11	0.008747051	21	0.003319072
2	0.0076605	12	0.002998106	22	0.004156304
3	0.00736012	13	0.00483974	23	0.002059182
4	0.0063961	14	0.003284377	24	0.003361234
5	0.005873361	15	0.003859419	25	0.001328283
6	0.005945234	16	0.008471072	26	0.001983672
7	0.008002881	17	0.003884604	27	0.002600526
8	0.00484964	18	0.002927395	28	0.00146444
9	0.004985585	19	0.003950955	29	0.004896159
10	0.00603866	20	0.004026808	30	0.002648585

Second compute the middle limit of attribute control in equation (1)

$CL = \bar{P} = \frac{\sum_{1}^{30} P_i}{30} = 0.004621604$

After that, compute the upper limit of attribute control in equation (2)

$UCL = \bar{P} + 3 * \sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.041770943$

Now, compute the lower limit of attribute control in equation (3)

$LCL = \bar{P} - 3 * \sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = -0.032527735$

Finally, drawing the charts of quality control with depend on the attributes control charts

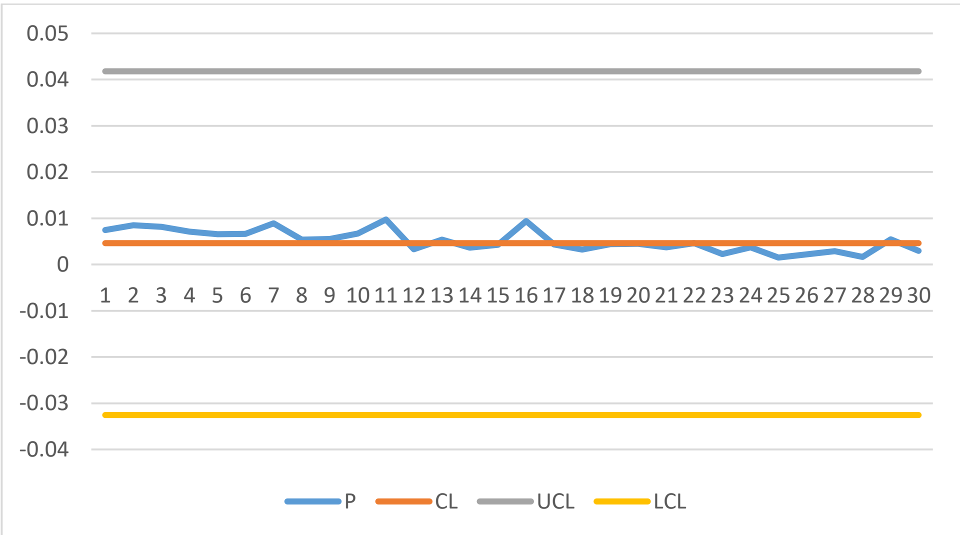

Figure 3. Contain the charts when λ=0.9, w=0.6

6.Conclusion

In beginning of employment the attribute quality control chart to calculate the proportion defective for all samples. Applying the triangular membership function to get the fuzzy numbers of defective for all samples. Then carrying out the proposed ranking function twice, the first through using (w=0.2 , λ=0.5), the second through using (w=0.6 , λ=0.9) to obtain the fuzzy number for all samples. After that, carrying out the fuzzy quality control to compute the proportion defective for all samples. Finally, comparing between crisp and fuzzy control charts for all samples of production are under control limits.

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