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A New Approach to Solving Linear Fractional Programming Problem with Rough Interval Coefficients in the Objective Function

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Abstract

This paper presents a linear fractional programming problem (LFPP) with rough interval coefficients (RICs) in the objective function. It shows that the LFPP with RICs in the objective function can be converted into a linear programming problem (LPP) with RICs by using the variable transformations. To solve this problem, we will make two LPP with interval coefficients (ICs). Next, those four LPPs can be constructed under these assumptions; the LPPs can be solved by the classical simplex method and used with MS Excel Solver. There is also argumentation about solving this type of linear fractional optimization programming problem. The derived theory can be applied to several numerical examples with its details, but we show only two examples for promising.

Keywords: Linear Fractional Programming, Linear Programming, Rough Interval Function, Rough Interval Coefficients, Interval Coefficients.

1. Introduction

The essential optimization is nonlinear programming; the linear fractional programming problem is one of them. The coefficient on the system can commonly not be established exactly in various linear programming problems. The interval technique, in which unknown coefficients are converted into intervals, is one way to address this programming problem. Because many real-world problems are expressed as fractional functions, fractional programming has gained much control. These issues frequently arise in situations involving actual capital return on investment versus required capital. The objective function of a linear fractional programming problem is highly valuable in construction, such as economic and company organization. Charnes and Cooper have presented several solutions to this problem.

Charnes and Cooper's method [1]can convert a fractional programming problem into a linear programming problem, which is known as linear fractional programming. [2] the notion of a rough interval will be applied to model dual uncertain information of different parameters. The related solution method for rough interval fuzzy linear programming problems with dual uncertain solutions will be described.

The rough set (RS) theory's main principle is that each ambiguous notion is substituted by a couple of exact conceptions known as the lower and upper approximations of the vague idea. For an ambiguous idea RS, a lower approximation includes complete items that surely belong to the model RS, and an upper approximation comprises all items that possibly belong to the model RS. In other words, the concept's lower approximation is the union of all fundamental concepts contained in it. The concept's upper approximation is the union of all fundamental concepts with a non-empty set intersection. Pawlak[3] introduced the rough set theory in 1982 as a strategy for managing ambiguity and uncertainty together.

Many techniques for solving fractional programming difficulties have emerged in recent decades. Some works on rough programming have been developed in the last decade. [4] and [5] developed a new sort of rough programming recently, where they established two solution concepts: surely optimum solution and possibly optimal solution. [6] proposed rough intervals (RI) to transact with partly uncertain or ill-defined relevant variables. The rough set ideas are adapted to represent constant variables. It is worth noting that, at first, rough sets could only handle discrete objects and couldn't express continuous values. Rough intervals are a subset of RSs. It satisfies all of the rough set's attributes and key ideas, such as the notions of upper and lower approximation[7]. [8] introduced a method to solve LFPP with ICs in the objective function by variable transformation. The initial problem is converted into a nonlinear programming problem, which is finally transformed into an LPP. [7] presented a solution of the fully rough interval coefficients of LPP. They constructed two-interval coefficients of LPP and found some new solutions. [9] suggested a new method to solve the rough interval linear fractional programming problem and introduced two possible kinds of formula transformation with its proof. They obtained optimal values and solutions for the initial LFPP with RICs. Khalifa[10] defined a new approach to solving LFPP with RICs in the objective function when the problem can be transformed into a series of LPPs with RI under some assumptions. We can be used our approach of multi-objective linear fractional programming problem or multi-objective linear programming problem with RICs, when maybe changed the coefficients of examples from[11],[12], and [13] into RICs.

In this paper, the concentration of our argumentation and investigation promotes an approach to finding surely and possibly optimal solutions to a linear fractional programming problem with rough interval coefficients in the objective function.

The remainder of the paper is laid out as follows. Section 2 presents some fundamental understanding of rough intervals. The formulation and derivation of LFPP with RICs are described in section 3. Two numerical examples are put to the test in Section 4. Finally, in section 5, there are some concluding observations.

2. Definitions

Definition: 2.1. Let E denotes a compact set of real numbers. A rough interval $A \subseteq E$ is defined

$A = (A_*, A^*)$, where $A_* \subseteq A^*, A_*$ and $A^*$are conventional lower and upper approximation intervals of$A$, respectively.

$A_* = [a_*^l, a_*^u]$and$A^* = [a^{*l}, a^{*u}]$. Where a_*^l, a_*^u, a^{*l} and a^{*u} are real numbers.

Note that the intervals A_* and A^* are not the complement each other.

The arithmetic operations on rough intervals are based on interval arithmetic. Some of these mathematical procedures will be explained as follows:

Let $A = ([a_*^l, a_*^u], [a^{*l}, a^{*u}])$ and $B = ([b_*^l, b_*^u], [b^{*l}, b^{*u}])$ be two rough intervals. Then we have:

$A + B = ([a_*^l + b_*^l, a_*^u + b_*^u], [a^{*l} + b^{*l}, a^{*u} + b^{*u}])$

$A - B = ([a_*^l - b_*^u, a_*^u - b_*^l], [a^{*l} - b^{*u}, a^{*u} - b^{*l}])$

$A \times B = ([a_*^l \times b_*^l, a_*^u \times b_*^u], [a^{*l} \times b^{*l}, a^{*u} \times b^{*u}])$

$A \div B = ([a_*^l \div b_*^u, a_*^u \div b_*^l], [a^{*l} \div b^{*u}, a^{*u} \div b^{*l}])$

Definition: 2.2. Function $f: R^n \to A$ is called a rough interval function with $f(x) = \big(f_*(x), f^*(x)\big)$, where for every$x \in R^n$, $f_*(x), f^*(x)$ are lower and upper approximation interval valued functions.

Definition: 2.3. A feasible point $x^* \in S$ is said to be an optimal solution of optimization problem LFPP with RICs, if there does not exist $x \in S$, such that$f(x^*) \leq f(x)$. Where $S = \{x \in R^n: Ax \leq b, x \geq 0\}$

3. Formulation of the problem

The generally extended form of a LFPP with RICs in the objective function is as follows:

$$\left.\begin{aligned} & Max.Z = \frac{\sum_{i=1}^{k}\{(A_{*i}, A_i^*)x_i + (A_{*k+1}, A_{k+1}^*)\}}{\sum_{i=1}^{k}\{(B_{*i}, B_i^*)x_i + (B_{*k+1}, B_{k+1}^*)\}} \\ & \text{Subject to:} \\ & \sum_{i=1}^{k} A_i x_i \leq b \\ & \text{For each } x_i \geq 0, i = 1,2, \dots, k. \end{aligned}\right\} \quad (1)$$

Where A_i, $i = 1,2, \dots, k$, and b both are m-dimensional constant column vectors.

Assume that $\sum_{i=1}^{k}(B_{*i},B_i^*)x_i+(B_{*k+1},B_{k+1}^*)\ >0$, for all $x_i\in S, i=1,2,\ldots,k$, where S is the compact feasible region of the problem (1).

In the deriving theory for solving the problem (1), we introduce the variable, $t=\frac{1}{\sum_{i=1}^{k}(B_{*i},B_i^*)x_i+(B_{*k+1},B_{k+1}^*)}$, to transform LFP problem with RICs in the objective function into a rough interval linear programming problem by the method of Charnes and Cooper[1], and then we have:

$$Max.Z=\sum_{i=1}^{k}(A_{*i},A_i^*)x_it+(A_{*k+1},A_{k+1}^*)t$$

Subject to:

$$\sum_{i=1}^{k}(B_{*i},B_i^*)x_it+(B_{*k+1},B_{k+1}^*)t=1 \qquad (2)$$

$$\sum_{i=1}^{k}A_ix_it-bt\le 0$$

$$x_i,t\ge 0, i=1,2,\ldots,k.$$

By introducing variable $u_i=x_it, i=1,2,\ldots,k$, problem (2) is transformed into the following equivalent problem:.

$$Max.Z=\sum_{i=1}^{k}(A_{*i},A_i^*)u_i+(A_{*k+1},A_{k+1}^*)t$$

Subject to:

$$\sum_{i=1}^{k}(B_{*i},B_i^*)u_i+(B_{*k+1},B_{k+1}^*)t=1 \qquad (3)$$

$$\sum_{i=1}^{k}A_iu_i-bt\le 0$$

$$u_i,t\ge 0, i=1,2,\ldots,k.$$

By Hamzehee et al. [7], we will build two linear programming with interval coefficients. One of these problems is linear programming, where all of it is coefficients are lower approximate to rough intervals. The other is linear programming where all of its coefficients are upper approximate of rough intervals. To solve problem (3), the following two linear programming with interval coefficients are discussed in the sequel.

$$Max.Z=\sum_{i=1}^{k}A_{*i}u_i+A_{*i+k}t$$

Subject to:

$$\sum_{i=1}^{k}B_{*i}u_i+B_{*i+k}t=1 \qquad (4)$$

$$\sum_{i=1}^{k}A_iu_i-bt\le 0$$

$$u_i,t\ge 0, i=1,2,\ldots,k.$$

And

$$Max.Z = \sum_{i=1}^{k} A_i^* u_i + A_{i+k}^* t$$

Subject to:

$$\sum_{i=1}^{k} B_i^* u_i + B_{i+k}^* t = 1$$

$$\sum_{i=1}^{k} A_i u_i - bt \leq 0$$

$$u_i, t \geq 0, i = 1,2, \dots, k. \qquad (5)$$

In the beginning, we can derive problem (4).

To solve the problem (4) an equivalent problem can be written:

$$Max.Z = [a_{*1}^l, a_{*1}^u]u_1 + [a_{*2}^l, a_{*2}^u]u_2 + \cdots + [a_{*k}^l, a_{*k}^u]u_k + [a_{*k+1}^l, a_{*k+1}^u]t$$

Subject to:

$$[b_{*1}^l, b_{*1}^u]u_1 + [b_{*2}^l, b_{*2}^u]u_2 + \cdots + [b_{*k}^l, b_{*k}^u]u_k + [b_{*k+1}^l, b_{*k+1}^u]t = 1$$

$$A_1 u_1 + A_2 u_2 + \cdots + A_k u_k - bt \leq 0$$

$$u_i, t \geq 0, i = 1,2, \dots, k.$$

The linear combination of each region interval of the problem (4) yields the following problem:

$$Max.Z = [\gamma_1 a_{*1}^l + (1-\gamma_1) a_{*1}^u]u_1 + \cdots + [\gamma_k a_{*k}^l + (1-\gamma_k) a_{*k}^u]u_k + [\gamma_{k+1} a_{*k+1}^l + (1 - \gamma_{k+1}) a_{*k+1}^u]t$$

Subject to:

$$[\delta_1 b_{*1}^l + (1-\delta_1) b_{*1}^u]u_1 + \cdots + [\delta_k b_{*k}^l + (1-\delta_k) b_{*k}^u]u_k + [\delta_{k+1} b_{*k+1}^l + (1 - \delta_{k+1}) b_{*k+1}^u]t = 1$$

$$A_1 u_1 + A_2 u_2 + \cdots + A_k u_k - bt \leq 0$$
(6)

$$u_i, t \geq 0, i = 1,2, \dots, k, \ \gamma_i, \delta_i \in [0,1], \ i = 1,2, \dots, k+1.$$

The equality constraint in problem (6) can be further reduced to:

$$[\delta_1 u_1 (b_{*1}^l - b_{*1}^u) + \cdots + \delta_k u_k (b_{*k}^l - b_{*k}^u) + \delta_{k+1} t (b_{*k+1}^l - b_{*k+1}^u)] + b_{*1}^u u_1 + \cdots + b_{*k}^u u_k + b_{*k+1}^u t = 1 \qquad (7)$$

Since $u_i, t \geq 0, i = 1,2, \dots, k, \ \delta_i \in [0,1], (b_{*i}^u - b_{*i}^l) \geq 0, \ i = 1,2, \dots, k+1.$

Therefore, (7) can be written as:

$1 \leq 1 + [\delta_1 u_1(b^u_{*1} - b^l_{*1}) + \cdots + \delta_k u_k(b^u_{*k} - b^l_{*k}) + \delta_{k+1} t(b^u_{*k+1} - b^l_{*k+1})] \leq 1 +$
$[u_1(b^u_{*1} - b^l_{*1}) + \cdots + u_k(b^u_{*k} - b^l_{*k}) + t(b^u_{*k+1} - b^l_{*k+1})]$
(8)

By combining (7) and (8), the result is

$1 \leq b^u_{*1}u_1 + \cdots + b^u_{*k}u_k + b^u_{*k+1}t \leq 1 + u_1(b^u_{*1} - b^l_{*1}) + \cdots + u_k(b^u_{*k} - b^l_{*k}) + t(b^u_{*k+1} - b^l_{*k+1})$
(9)

This is then reduced to

$b^u_{*1}u_1 + \cdots + b^u_{*k}u_k + b^u_{*k+1}t \geq 1$
(10)

And

$b^l_{*1}u_1 + \cdots + b^l_{*k}u_k + b^l_{*k+1}t \leq 1$
(11)

Therefore, using (10) and (11), the equation (6) is converted into the following equation:

$$\left.\begin{aligned}
&Max.Z = [\gamma_1 a^l_{*1} + (1-\gamma_1)a^u_{*1}]u_1 + \cdots + [\gamma_k a^l_{*k} + (1-\gamma_k)a^u_{*k}]u_k + [\gamma_{k+1}a^l_{*k+1} + (1-\gamma_{k+1})a^u_{*k+1}]t \\
&\text{Subject to:} \\
&b^u_{*1}u_1 + \cdots + b^u_{*k}u_k + b^u_{*k+1}t \geq 1 \\
&b^l_{*1}u_1 + \cdots + b^l_{*k}u_k + b^l_{*k+1}t \leq 1 \quad (12) \\
&A_1u_1 + A_2u_2 + \cdots + A_ku_k - bt \leq 0
\end{aligned}\right\} \quad (12)$$

$u_i, t \geq 0, i = 1,2,\dots,k,\ \gamma_i \in [0,1],\ i = 1,2,\dots,k+1.$

In addition, if we let the point $(\bar{u}_1, \bar{u}_2, \dots, \bar{u}_k, \bar{t})$ of the feasible region from the problem (12), with $\gamma_i \in [0,1], (a^l_{*i} - a^u_{*i}) \leq 0$ for$i = 1,2,\dots,k+1$. Before the objective of the problem (12) can be written as:

$[\gamma_1(a^l_{*1} - a^u_{*1})]\bar{u}_1 + \cdots + [\gamma_k(a^l_{*k} - a^u_{*k})]\bar{u}_k + [\gamma_{k+1}(a^l_{*k+1} - a^u_{*k+1})]\bar{t} + a^u_{*1}\bar{u}_1 + \cdots +$
$a^u_{*k}\bar{u}_k + a^u_{*k+1}\bar{t} \geq [(a^l_{*1} - a^u_{*1})]\bar{u}_1 + \cdots + [(a^l_{*k} - a^u_{*k})]\bar{u}_k + [(a^l_{*k+1} - a^u_{*k+1})]\bar{t} +$
$a^u_{*1}\bar{u}_1 + \cdots + a^u_{*k}\bar{u}_k + a^u_{*k+1}\bar{t}$

$= a^l_{*1}\bar{u}_1 + \cdots + a^l_{*k}\bar{u}_k + a^l_{*k+1}\bar{t}$

The right-hand side of the above equality proves that $a^l_{*1}, \dots, a^l_{*k}, a^l_{*k+1}$ is the surely lower limit of the interval coefficients in the objective function, and the same situation of the problem (12), with $\gamma_i \in [0,1], (a^u_{*i} - a^l_{*i}) \geq 0$ for$i = 1,2,\dots,k+1$. Before the objective of the problem (12) can be written as:

$[\gamma_1(a^u_{*1} - a^l_{*1})]\bar{u}_1 + \cdots + [\gamma_k(a^u_{*k} - a^l_{*k})]\bar{u}_k + [\gamma_{k+1}(a^u_{*k+1} - a^l_{*k+1})]\bar{t} + a^l_{*1}\bar{u}_1 + \cdots + a^l_{*k}\bar{u}_k + a^l_{*k+1}\bar{t} \geq [(a^u_{*1} - a^l_{*1})]\bar{u}_1 + \cdots + [(a^u_{*k} - a^l_{*k})]\bar{u}_k + [(a^u_{*k+1} - a^l_{*k+1})]\bar{t} + a^l_{*1}\bar{u}_1 + \cdots + a^l_{*k}\bar{u}_k + a^l_{*k+1}\bar{t}$

$= a^u_{*1}\bar{u}_1 + \cdots + a^u_{*k}\bar{u}_k + a^u_{*k+1}\bar{t}$

The right hand side of the above equality proves that $a^u_{*1}, \dots, a^u_{*k}, a^u_{*k+1}$ is the surely upper limit of the interval coefficient in the objective function.

Now, the surely best optimum objective function is $Z = a^u_{*1}u_1 + \cdots + a^u_{*k}u_k + a^u_{*k+1}t$ and the surely worst optimum objective function is$Z = a^l_{*1}u_1 + \cdots + a^l_{*k}u_k + a^l_{*k+1}t$.

While, to get the surely worst optimum taken of the surely lower limit of the interval coefficients in the objective function, thus the conversions formula of LFP problems with rough interval coefficients in the objective function of the surely lower limit, can be written as follows:

But in this case, assume that every constraint is smaller than or equal.

$$\left.\begin{array}{l} Max. Z = a^l_{*1}u_1 + \cdots + a^l_{*k}u_k + a^l_{*k+1}t \\ \text{Subject to:} \\ b^u_{*1}u_1 + \cdots + b^u_{*k}u_k + b^u_{*k+1}t \leq 1 \\ b^l_{*1}u_1 + \cdots + b^l_{*k}u_k + b^l_{*k+1}t \leq 1 \\ A_1u_1 + A_2u_2 + \cdots + A_ku_k - bt \leq 0 \\ u_i, t \geq 0, i = 1,2, \dots, k. \end{array}\right\} \quad (13)$$

And while, to get the surely best optimum taken of the surely upper limit of the interval coefficients in the objective function, thus the conversions formula of LFP problems with rough interval coefficients in the objective function is

$$\left.\begin{array}{l} Max. Z = a^u_{*1}u_1 + \cdots + a^u_{*k}u_k + a^u_{*k+1}t \\ \text{Subject to:} \\ b^u_{*1}u_1 + \cdots + b^u_{*k}u_k + b^u_{*k+1}t \geq 1 \\ b^l_{*1}u_1 + \cdots + b^l_{*k}u_k + b^l_{*k+1}t \leq 1 \\ A_1u_1 + A_2u_2 + \cdots + A_ku_k - bt \leq 0 \\ u_i, t \geq 0, i = 1,2, \dots, k. \end{array}\right\} \quad (14)$$

The optimal solution $(\bar{u}_1, \bar{u}_2, \dots, \bar{u}_k, \bar{t})$ of problems (13) and (14) is the same as the optimal solution of the original problem (4) which can be easily obtained by $(x_1^*, x_2^*, \dots, x_k^*) = (\frac{\bar{u}_1}{\bar{t}}, \frac{\bar{u}_2}{\bar{t}}, \dots, \frac{\bar{u}_k}{\bar{t}})$.

For solving problem (5), using the same derivation and assumptions of problem (4), we can obtain problem (15) &(16), with possibly worst optimum, and best optimum.

$$Max.Z = a_1^{*l} u_1 + \cdots + a_k^{*l} u_k + a_{k+1}^{*l} t$$

Subject to:

$$b_1^{*u} u_1 + \cdots + b_k^{*u} u_k + b_{k+1}^{*u} t \leq 1$$

$$b_1^{*l} u_1 + \cdots + b_k^{*l} u_k + b_{k+1}^{*l} t \leq 1 \qquad (15)$$

$$A_1 u_1 + A_2 u_2 + \cdots + A_k u_k - bt \leq 0$$

$$u_i, t \geq 0, i = 1,2, \dots, k.$$

And

$$Max.Z = a_1^{*u} u_1 + \cdots + a_k^{*u} u_k + a_{k+1}^{*u} t$$

Subject to:

$$b_1^{*u} u_1 + \cdots + b_k^{*u} u_k + b_{k+1}^{*u} t \geq 1$$

$$b_1^{*l} u_1 + \cdots + b_k^{*l} u_k + b_{k+1}^{*l} t \leq 1 \qquad (16)$$

$$A_1 u_1 + A_2 u_2 + \cdots + A_k u_k - bt \leq 0$$

$$u_i, t \geq 0, i = 1,2, \dots, k.$$

Then to solve problem (13, 14, 15, 16) using the simplex routine, we obtain the optimal value and solution of the original problem (1).

Next, will be confirmed that LFP problems with RICs in the objective function according to the fact that each fixed number n can be equivalently written as the interval $[n, n]$, and also equivalently as the rough interval $([n, n], [n, n])$, we claim that LFP is a special case of LFP with ICs in the objective function and the LFP with ICs in the objective function is a special case of LFP with RICs in the objective function.

$$Max.Z = \frac{([n_1, n_1], [n_1, n_1])x_1 + ([n_2, n_2], [n_2, n_2])x_2 + \cdots + ([n_k, n_k], [n_k, n_k])x_k + ([n_{k+1}, n_{k+1}], [n_{k+1}, n_{k+1}])}{([h_1, h_1], [h_1, h_1])x_1 + ([h_2, h_2], [h_2, h_2])x_2 + \cdots + ([h_k, h_k], [h_k, h_k])x_k + ([h_{k+1}, h_{k+1}], [h_{k+1}, h_{k+1}])}$$

Subject to:
(17)

$$\sum_{i=1}^{k} A_i x_i \leq b$$

Proof:

Because the RICs in the numerator of the objective function has a similar cost. The coefficients of the objective function at its optimum surely the best, surely the worst, possibly the best, and possibly the worst are the same optimum. By Charnes-Cooper's method, problem (17) is transformed into the following problem.

$$\left.\begin{array}{l} Max. Z = n_1u_1 + n_2u_2 + \cdots + n_ku_k + n_{k+1}t \\ \text{Subject to:} \\ h_1u_1 + \cdots + h_ku_k + h_{k+1}t \geq 1 \\ h_1u_1 + \cdots + h_ku_k + h_{k+1}t \leq 1 \\ A_1u_1 + A_2u_2 + \cdots + A_ku_k - bt \leq 0 \\ u_i, t \geq 0, i = 1,2, \dots, k. \end{array}\right\} \quad (18)$$

The combination of the first two constraints causes the following problem:

$$\left.\begin{array}{l} Max. Z = n_1u_1 + n_2u_2 + \cdots + n_ku_k + n_{k+1}t \\ \text{Subject to:} \\ h_1u_1 + \cdots + h_ku_k + h_{k+1}t = 1 \\ A_1u_1 + A_2u_2 + \cdots + A_ku_k - bt \leq 0, \text{ where } u_i, t \geq 0, i = 1,2, \dots, k. \end{array}\right\} \quad (19)$$

4. Numerical Examples

Example1: Consider the following LFPP with RICs in the objective function:

$$\left.\begin{array}{l} Max. Z = \dfrac{([1,3],[0.5,4])x_1 + ([2,4],[1,4.5])x_2}{([0.5,1.5],[0.25,2])x_1 + ([0.5,1.5],[0.25,2])x_2 + ([1,3],\left[\frac{1}{3}, 3.5\right])} \\ \text{Subject to:} \\ x_1 - x_2 \geq 1, 2x_1 + 3x_2 \leq 15, x_1 \geq 3, x_1, x_1 \geq 0. \end{array}\right\} \quad (20)$$

Solution

Problem (20) is transformed into rough interval linear programming problem by Charnes &Cooper method.

We introduce $t = \frac{1}{([0.5,1.5],[0.25,2])x_1 + ([0.5,1.5],[0.25,2])x_2 + ([1,3],\left[\frac{1}{3}, 3.5\right])}$

$$Max. Z = ([1,3],[0.5,4])x_1t + ([2,4],[1,4.5])x_2t + 0t$$

Subject to:

$([0.5,1.5],[0.25,2])x_1t + ([0.5,1.5],[0.25,2])x_2t + \left([1,3],\left[\frac{1}{3},3.5\right]\right)t = 1$

$-x_1t + x_2t + t \leq 0, 2x_1t + 3x_2t - 15t \leq 0, -x_1t + 3t \leq 0,\; x_1, x_1, t \geq 0.$

Let$u_i = x_it,\, i = 1,2.$

$$\left.\begin{array}{l} Max.Z = ([1,3],[0.5,4])u_1 + ([2,4],[1,4.5])u_2 + 0t \\ \text{Subject to:} \\ ([0.5,1.5],[0.25,2])u_1 + ([0.5,1.5],[0.25,2])u_2 + \left([1,3],\left[\frac{1}{3},3.5\right]\right)t = 1 \\ -u_1 + u_2 + t \leq 0, 2u_1 + 3u_2 - 15t \leq 0, -u_1 + 3t \leq 0, u_1, u_1, t \geq 0. \end{array}\right\} \quad (21)$$

Now, by Hamzehee et al. [7] problem (21) can be changed into two linear programming with interval coefficients (22) & (23).

$$\left.\begin{array}{l} Max.Z = ([1,3])u_1 + ([2,4])u_2 + 0t \\ \text{Subject to:} \\ ([0.5,1.5])u_1 + ([0.5,1.5])u_2 + ([1,3])t = 1 \\ -u_1 + u_2 + t \leq 0, 2u_1 + 3u_2 - 15t \leq 0, -u_1 + 3t \leq 0, u_1, u_1, t \geq 0. \end{array}\right\} \quad (22)$$

$$\left.\begin{array}{l} Max.Z = ([0.5,4])u_1 + ([1,4.5])u_2 + 0t \\ \text{Subject to:} \\ ([0.25,2])u_1 + ([0.25,2])u_2 + \left(\left[\frac{1}{3},3.5\right]\right)t = 1 \end{array}\right\} \quad (23)$$

$-u_1 + u_2 + t \leq 0, 2u_1 + 3u_2 - 15t \leq 0, -u_1 + 3t \leq 0, u_1, u_1, t \geq 0.$

Problem (22) is converted to linear programming problems (24) & (25).

$$\left.\begin{array}{l} Max.Z = u_1 + 2u_2 + 0t \\ \text{Subject to:} \\ 0.5u_1 + 0.5u_2 + t \leq 1 \\ 1.5u_1 + 1.5u_2 + 3t \leq 1 \\ -u_1 + u_2 + t \leq 0, 2u_1 + 3u_2 - 15t \leq 0, -u_1 + 3t \leq 0, u_1, u_1, t \geq 0. \end{array}\right\} \quad (24)$$

$$\left.\begin{array}{l} Max.Z = 3u_1 + 4u_2 + 0t \\ \text{Subject to:} \\ 0.5u_1 + 0.5u_2 + t \leq 1 \\ 1.5u_1 + 1.5u_2 + 3t \geq 1 \\ -u_1 + u_2 + t \leq 0, 2u_1 + 3u_2 - 15t \leq 0, -u_1 + 3t \leq 0, u_1, u_1, t \geq 0. \end{array}\right\} \quad (25)$$

And problem (23) is converted to linear programming problems (26) & (27).

$$\left.\begin{array}{l} Max.Z = 0.5u_1 + u_2 + 0t \\ \text{Subject to:} \\ 0.25u_1 + 0.25u_2 + \frac{1}{3}t \leq 1 \\ 2u_1 + 2u_2 + 3.5t \leq 1 \\ -u_1 + u_2 + t \leq 0, 2u_1 + 3u_2 - 15t \leq 0, -u_1 + 3t \leq 0, u_1, u_1, t \geq 0. \end{array}\right\} \quad (26)$$

$$\left.\begin{array}{l} Max.Z = 4u_1 + 4.5u_2 + 0t \\ \text{Subject to:} \\ 0.25u_1 + 0.25u_2 + \frac{1}{3}t \leq 1 \\ 2u_1 + 2u_2 + 3.5t \geq 1 \\ -u_1 + u_2 + t \leq 0, 2u_1 + 3u_2 - 15t \leq 0, -u_1 + 3t \leq 0, u_1, u_1, t \geq 0. \end{array}\right\} \quad (27)$$

Now we can solve problems (24), (25), (26) and (27). Using the simplex routine, we obtain tht 0.7154 is surely the worst optimum, 5.17 is surely the best optimum, 0.2767 is possibly the worst optimum, and 13.85 is possibly the best optimum. The optimal solution of the original problem is (3.6, 2.6) with the objective value ([0.7154, 5.17], [0.2767, 13.85])

Example2: Consider the following LFPP with RICs in the objective function:

$$\left.\begin{array}{l} Max.Z = \dfrac{([3.5,4.5],[3,5])x_1 + ([2,3],[1,4])x_2 + ([8,10],[7,11])}{([1,1.5],[0.5,2])x_1 + ([1.5,1.75],[1,2])x_2 + ([4.5,5.5],[4,6])} \\ \text{Subject to:} \end{array}\right\} \quad (28)$$

$x_1 + 3x_2 \leq 30, -x_1 + 2x_2 \leq 5,\ x_1, x_2 \geq 0.$

Solution

Problem (28) is transformed into a rough interval linear programming problem by Charnes &Cooper method.

We introduce $t = \frac{1}{([1,1.5],[0.5,2])x_1 + ([1.5,1.75],[1,2])x_2 + ([4.5,5.5],[4,6])}$

$Max.Z = ([3.5,4.5],[3,5])x_1t + ([2,3],[1,4])x_2t + ([8,10],[7,11])t$

Subject to:

$([1,1.5],[0.5,2])x_1t + ([1.5,1.75],[1,2])x_2t + ([4.5,5.5],[4,6])t = 1$

$x_1t + 3x_2t - 30t \leq 0, -x_1t + 2x_2t - 5t \leq 0,\ x_1, x_2, t \geq 0.$

Let $u_i = x_it,\ i = 1,2.$

$$\left.\begin{array}{l} Max.Z = ([3.5,4.5],[3,5])u_1 + ([2,3],[1,4])u_2 + ([8,10],[7,11])t \\ \text{Subject to:} \\ ([1,1.5],[0.5,2])u_1 + ([1.5,1.75],[1,2])u_2 + ([4.5,5.5],[4,6])t = 1 \\ u_1 + 3u_2 - 30t \leq 0, -u_1 + 2u_2 - 5t \leq 0,\ u_1, u_1, t \geq 0. \end{array}\right\} \quad (29)$$

Now by Hamzehee et al. [7] problem (29) can be changed into two linear programming with interval coefficients (30) & (31).

$$\left.\begin{array}{l} Max.Z = ([3.5,4.5])u_1 + ([2,3])u_2 + ([8,10])t \\ \text{Subject to:} \\ ([1,1.5])u_1 + ([1.5,1.75])u_2 + ([4.5,5.5])t = 1 \\ u_1 + 3u_2 - 30t \leq 0, -u_1 + 2u_2 - 5t \leq 0,\ u_1, u_1, t \geq 0. \end{array}\right\} \quad (30)$$

$$\left.\begin{array}{l} Max.Z = ([3,5])u_1 + ([1,4])u_2 + ([7,11])t \\ \text{Subject to:} \\ ([0.5,2])u_1 + ([1,2])u_2 + ([4,6])t = 1 \\ u_1 + 3u_2 - 30t \leq 0, -u_1 + 2u_2 - 5t \leq 0,\ u_1, u_1, t \geq 0. \end{array}\right\} \quad (31)$$

Problem (30) is converted to linear programming problems (32) & (33).

$$\left.\begin{array}{l} Max.Z = 3.5u_1 + 2u_2 + 8t \\ \text{Subject to:} \end{array}\right\} \quad (32)$$

$u_1 + 1.5u_2 + 4.5t \leq 1$

$1.5u_1 + 1.75u_2 + 5.5t \leq 1$

$u_1 + 3u_2 - 30t \leq 0, -u_1 + 2u_2 - 5t \leq 0,\ u_1, u_1, t \geq 0.$

$$\left.\begin{array}{l} Max.Z = 4.5u_1 + 3u_2 + 10t \\ \text{Subject to:} \\ u_1 + 1.5u_2 + 4.5t \leq 1 \\ 1.5u_1 + 1.75u_2 + 5.5t \geq 1 \\ u_1 + 3u_2 - 30t \leq 0, -u_1 + 2u_2 - 5t \leq 0,\ u_1, u_1, t \geq 0. \end{array}\right\} \quad (33)$$

And the problem (31) is converted to linear programming problems (34) & (35).

$$\left.\begin{array}{l} Max.Z = 3u_1 + u_2 + 7t \\ \text{Subject to:} \\ 0.5u_1 + u_2 + 4t \leq 1 \\ 2u_1 + 2u_2 + 6t \leq 1 \\ u_1 + 3u_2 - 30t \leq 0, -u_1 + 2u_2 - 5t \leq 0,\ u_1, u_1, t \geq 0. \end{array}\right\} \quad (34)$$

$$\left.\begin{array}{l} Max.Z = 5u_1 + 4u_2 + 11t \\ \text{Subject to:} \\ 0.5u_1 + u_2 + 4t \leq 1 \\ 2u_1 + 2u_2 + 6t \geq 1 \\ u_1 + 3u_2 - 30t \leq 0, -u_1 + 2u_2 - 5t \leq 0,\ u_1, u_1, t \geq 0. \end{array}\right\} \quad (35)$$

Now, we can solve problems (32), (33), (34), and (35). By using the simplex routine, we obtain that 2.2376 is surely the worst optimum, 4.2028 is surely the best optimum, 1.469 is possibly the worst optimum and 8.47 is possibly the best optimum. The optimal solution of the original problem is (30, 0) with the objective value ([2.2376, 4.2028], [1.469, 8.47])

5. Conclusion

In this study, we proposed a new approach to solving a linear fractional programming problem with rough interval coefficients in the objective function. In this technique, we changed rough intervals into ordinary intervals and utilized convex combinations of points instead of intervals. The basic problem is converted into linear programming using some techniques. We configure series linear programming problems and obtain optimal solutions and values by using the simplex method. For future studies, we can try to perform this technique and notion in any fractional programming.

References

1. Charnes, A. ; Cooper, W. W. Programming with linear fractional functionals, *Naval Research logistics quarterly*, **1962,** *9, 3-4*, 181-186,
2. Lu, H.; Huang, G. ; He, L. An inexact rough-interval fuzzy linear programming method for generating conjunctive water-allocation strategies to agricultural irrigation systems, *Applied Mathematical Modelling,* **2011,** *35*, *9*, 4330-4340,.
3. Pawlak, Z. Rough sets. *International Journal of Information and Computer Scienc*e, **1982**.
4. Youness, E. A. Characterizing solutions of rough programming problems, *European Journal of Operational Research,* **2006** , *168, 3*,1019-1029,.
5. Osman, M.; Lashein, E.; Youness, E. ; Atteya, T. Mathematical programming in rough environment, *Optimisation,* **2011**, *60, 5*, 603-611.
6. Rebolledo, M. Rough intervals—enhancing intervals for qualitative modeling of technical systems, *Artificial Intelligence,* **2006**, *170, 8-9*, 667-685,.
7. Hamzehee, A.; Yaghoobi, M. A. ; Mashinchi, M. Linear programming with rough interval coefficients, *Journal of Intelligent & Fuzzy Systems,* **2014**, *26*, *3*, 1179-1189,
8. Borza, M.; Rambely, A. S. ; Saraj, M. Solving linear fractional programming problems with interval coefficients in the objective function.A new approach,*Applied Mathematical Sciences,***2012**, *6*, *69*, 3443-3452, **2012.**
9. Ammar, E. ; Muamer, M.Solving a rough interval linear fractional programming problem, *Journal: JOURNAL OF ADVANCES IN MATHEMATICS,* **2015**, *10*, *4*.
10. Khalifa, A. On solutions of linear fractional programming problems with rough-interval coefficients in the objective functions, *Journal of Fuzzy Mathematics,***2018**, *26*, *2*, 415-422,.
11. Mustafa, R. ; Sulaiman, N. A. A new Mean Deviation and Advanced Mean Deviation Techniques to Solve Multi-Objective Fractional Programming Problem Via Point-Slopes Formula, *Pakistan Journal of Statistics and Operation Research,* **2021,** 1051-1064,.
12. Sulaiman, N. A. ; Mustafa, R. B. Using harmonic mean to solve multi-objective linear programming problems, *American journal of operations Research,* **2016,** *6*, *1*, 25-30,.
13. Sulaiman, N.; Sadiq, G. ; Abdulrahim, B.Used a new transformation technique for solving multiobjective linear fractional programming problem,*IJRRAS (18),* **2014**,*2*, 122-131.