The Construction of (k,3)-Arcs in PG(2,9) by Using Geometrical Method

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Abstract

In this work, we construct projectively distinct (k,3)-arcs in the projective plane PG(2,9) by applying a geometrical method. The cubic curves have been been constructed by using the general equation of the cubic.

We found that there are complete (13,3)-arcs, complete (15,3)-arcs and we found that the only (16,3)-arcs lead to maximum completeness.

Keywords: arcs, secant, Projective , Plane

Introduction

A (k,n)-arc K in PG(2,q) is a set of K points, s.t. some n, but no n + 1 of them are collinear. A (k,n)-arc K is complete if there is no (k+1,n)-arc containing it, [1].

A line m of the plane containing exactly 3 points of K is called a trisecant of K.

Let K be a (k,3)-arc. The points of PG(2,9)\K which are not on any trisecant of K will be called the points of index zero for K. The (k,3)-arc is complete iff there is no point of index zero for it, so every point of the plane lies on a trisecant of K, [2].

The projective plane PG(2,9) contains 91 points, 91 lines, every line contain 10 points, and every point is on 10 lines, [3]. Let P_i and L_i, i = 1, …, 91, be the points and lines of PG(2,9), respectively.

Any point can be represented by a triple $(x_0,x_1,x_2) \neq (0,0,0)$, x_0,x_1,x_2 are in GF(9) and two triples (x_0,x_1,x_2) and (y_0,y_1,y_2) represent the same point if there exists λ in GF(9)\{0} s.t. $x_i = \lambda y_i$, i = 0, 1, 2.

Also any line in PG(2,9) can be represented by a triple $[x_0,x_1,x_2] \neq [0,0,0]$, x_0,x_1,x_2 are in GF(9). Two triples $[x_0,x_1,x_2]$ and $[y_0,y_1,y_2]$ represent the same line if there exists λ in GF(9)\{0} s.t. $x_i = \lambda y_i$, i = 0, 1, 2.

The addition and multiplication's operation of GF(9), [3] are given in the tables.

The point (x_0,x_1,x_2) is on the line $[y_0,y_1,y_2]$ iff $x_0y_0 + x_1y_1 + x_2y_2 = 0$, [2].

Let i stands for the point P_i.

All the points and lines of PG(2,9) are given in the table.

The Construction of (k,3)-Arcs for k ≥ 7: [3]

We construct projectively distinct (K,3)-arcs in projective plane PG(2,9) by using a geometrical method. This method having the following steps:

The general equation of the cubic is:

$$F = \alpha_1 x_0^3 + \alpha_2 x_0^2 x_1 + \alpha_3 x_0^2 x_2 + \alpha_4 x_0 x_1^2 + \alpha_5 x_0 x_1 x_2 + \alpha_6 x_0 x_2^2 + \alpha_7 x_1^3 + \alpha_8 x_1^2 x_2 + \alpha_9 x_1 x_2^2 + \alpha_{10} x_2^3 = 0 \quad \ldots(1)$$

By substituting the points 1,2,11,12,20 of the (7,3)-arc which is a quadrangle whose vertices the points 1, 2, 11, 21 and the diagonal points 3, 12,20, we get:

1 = (1,0,0) $\Rightarrow \alpha_1 = 0$
2 = (0,1,0) $\Rightarrow \alpha_7 = 0$
11 = (0,0,1) $\Rightarrow \alpha_{10} = 0$
3 = (1,1,0) $\Rightarrow \alpha_2 + \alpha_4 = 0$
12 = (1,0,1) $\Rightarrow \alpha_3 + \alpha_6 = 0$
20 = (0,1,1) $\Rightarrow \alpha_8 + \alpha_9 = 0$
21 = (1,1,1) $\Rightarrow \alpha_5 = 0$

So (1) becomes:

$$\alpha_4(x_0x_1^2 - x_0^2x_1) + \alpha_6(x_0x_2^2 - x_0^2x_2) + \alpha_9(x_1x_2^2 - x_1^2x_2) = 0 \quad \ldots(2)$$

If $\alpha_4 = 0$, then $\alpha_6(x_0x_2^2 - x_0^2x_2) + \alpha_9(x_1x_2^2 - x_1^2x_2) = 0$

$$x_2\{\alpha_6 x_0(x_2 - x_0) + \alpha_9 x_1(x_2 - x_1)\} = 0$$

is reducible, therefore $\alpha_4 \neq 0$. So $\alpha_6 \neq 0$ and $\alpha_9 \neq 0$. Dividing equation (2) by α_4, we get:

$$x_0x_1^2 - x_0^2x_1 + \alpha(x_0x_2^2 - x_0^2x_2) + \beta(x_1x_2^2 - x_1^2x_2) = 0$$

where: $\alpha = \frac{\alpha_6}{\alpha_4} \neq 0, \beta = \frac{\alpha_9}{\alpha_4} \neq 0$.

Let $x_2 = 1$, then:

$$x_0x_1^2 - x_0^2x_1 + \alpha(x_0 - x_0^2) + \beta(x_1 - x_1^2) = 0$$
$$x_0x_1(x_1 - x_0) + \alpha x_0(1 - x_0) + \beta x_1(1 - x_1) = 0$$
$$x_0x_1\{(x_1 - 1) + (1 - x_0)\} + \alpha x_0x_1(1 - x_0) + \beta x_1(1 - x_1) = 0$$
$$x_0x_1(x_1 - 1) + x_0x_1(1 - x_0) + \alpha x_0(1 - x_0) - \beta x_1(1 - x_1) = 0$$
$$x_0x_1(x_1 - 1) + x_0x_1(1 - x_0) + \alpha x_0(1 - x_0) + \beta x_1(x_1 - 1) = 0$$
$$x_1(x_1 - 1)(x_0 - \beta) + x_0(1 - x_0)(x_1 + \alpha) = 0$$
$$x_1(x_1 - 1)(x_0 - \beta) - x_0(x_0 - 1)(x_1 + \alpha) = 0 \qquad \ldots(3)$$
$$\frac{x_1(x_1 - 1)}{x_1 + \alpha} = \frac{x_0(x_0 - 1)}{x_0 - \beta}$$, where $\alpha \neq -1 = 2 \pmod{9}$, $\beta \neq 1$.

The cubic curves have been constructed by implementing equation (3) with deferent values of α and β where $\alpha, \beta = 1,2,\ldots,8$ and $\alpha, \beta \neq 0$, $\alpha \neq 2$, $\beta \neq 1$, $\alpha \neq -\beta$, because this will form a degenerated cubic.

The Set of all the Cubic Curves: [3]

1. If $\alpha = 1$, $\beta = 3$, then the cubic curve C_1 is:
$x_1(x_1 - 1)(x_0 - 3) - x_0(x_0 - 1)(x_1 + 1) = 0$
C_1 = {1,2,11,21,3,12,20,32,40,45,80,82} which is incomplete cubic, and by adding to C_1 is the point 36, which is the point of index zero for C_1, we obtain a complete (13,3)-arc.

2. If $\alpha = 1$, $\beta = 4$, then the cubic curve C_2 is:
$x_1(x_1 - 1)(x_0 - 4) - x_0(x_0 - 1)(x_1 + 1) = 0$
C_2 = {1,2,11,21,3,12,20,33,50,53,86,89}, which is incomplete cubic, and by adding to it the point 37, which is the point of index zero for C_2, we obtain a complete (13,3)-arc.

3. If $\alpha = 1$, $\beta = 5$, then the cubic curve C_3 is:
$x_1(x_1 - 1)(x_0 - 5) - x_0(x_0 - 1)(x_1 + 1) = 0$
C_3 = {1,2,11,21,3,12,20,34,68,73}, which is incomplete cubic, and by adding to it the points of index zero for C_3, which are: 32, 40, 42, 58, 79, we obtain a complete (15,3)-arc.

4. If $\alpha = 1$, $\beta = 6$, then the cubic curve C_4 is:
$x_1(x_1 - 1)(x_0 - 6) - x_0(x_0 - 1)(x_1 + 1) = 0$
C_4 = {1,2,11,21,3,12,20,35,69,72}, which is incomplete cubic, and by adding to it the points 32, 43, 54, we obtain a complete (13,3)-arc.

5. If $\alpha = 1$, $\beta = 7$, then the cubic curve C_5 is:
$x_1(x_1 - 1)(x_0 - 7) - x_0(x_0 - 1)(x_1 + 1) = 0$
C_5 = {1,2,11,21,3,12,20,36,42,43,76,77}, which is incomplete cubic, and by adding to it the point 32, we obtain a complete (13,3)-arc.

6. If $\alpha = 1$, $\beta = 8$, then the cubic curve C_6 is:
$x_1(x_1 - 1)(x_0 - 8) - x_0(x_0 - 1)(x_1 + 1) = 0$
C_6 = {1,2,11,21,3,12,20,37,52,54}, which is incomplete cubic, and by adding to it the points 32, 40, 43, 77, 78, we obtain a complete (15,3)-arc.

7. If $\alpha = 3$, $\beta = 2$, then the cubic curve C_7 is:
$x_1(x_1 - 1)(x_0 - 2) - x_0(x_0 - 1)(x_1 + 3) = 0$
C_7 = {1,2,11,21,3,12,20,52,53,67,79,80}, which is incomplete cubic, and by adding to it the point 58, we obtain a complete (13,3)-arc.

8. If $\alpha = 3$, $\beta = 3$, then the cubic curve C_8 is:
$x_1(x_1 - 1)(x_0 - 3) - x_0(x_0 - 1)(x_1 + 3) = 0$

C_8 = {1,2,11,21,3,12,20,34,42,60,68,88}, which is incomplete cubic, and by adding to it the point 63, we obtain a complete (13,3)-arc.

9. If $\alpha = 3$, $\beta = 4$, then the cubic curve C_9 is:

$x_1(x_1 - 1)(x_0 - 4) - x_0(x_0 - 1)(x_1 + 3) = 0$

C_9 = {1,2,11,21,3,12,20,69}, which is incomplete cubic, and by adding to it the points 32, 33, 40, 43, 49, 54, 68, we obtain a complete (15,3)-arc.

10. If $\alpha = 3$, $\beta = 5$, then the cubic curve C_{10} is:

$x_1(x_1 - 1)(x_0 - 5) - x_0(x_0 - 1)(x_1 + 3) = 0$

C_{10} = {1,2,11,21,3,12,20,50,55,70,77,82}, which is incomplete cubic, and by adding to it the point 62, we obtain a complete (13,3)-arc.

11. If $\alpha = 3$, $\beta = 7$, then the cubic curve C_{11} is:

$x_1(x_1 - 1)(x_0 - 4) - x_0(x_0 - 1)(x_1 + 3) = 0$

C_{11} = {1,2,11,21,3,12,20,72}, which is incomplete cubic, and by adding to it the points 32, 33, 40, 45, 50, 67, 79, we obtain a complete (15,3)-arc.

12. If $\alpha = 3$, $\beta = 8$, then the cubic curve C_{12} is:

$x_1(x_1 - 1)(x_0 - 8) - x_0(x_0 - 1)(x_1 + 3) = 0$

C_{12} = {1,2,11,21,3,12,20,32,33,40,44,58,62,73,86,87}, which is a complete (16,3)-arc.

13. If $\alpha = 4$, $\beta = 2$, then the cubic curve C_{13} is:

$x_1(x_1 - 1)(x_0 - 2) - x_0(x_0 - 1)(x_1 + 4) = 0$

C_{13} = {1,2,11,21,3,12,20,60,64,78,82,85}, which is incomplete cubic, and by adding to it the point 49, we obtain a complete (13,3)-arc.

14. If $\alpha = 4$, $\beta = 3$, then the cubic curve C_{14} is:

$x_1(x_1 - 1)(x_0 - 3) - x_0(x_0 - 1)(x_1 + 4) = 0$

C_{14} = {1,2,11,21,3,12,20,33,43,52,69,86}, which is incomplete cubic, and by adding to it the point 54, we obtain a complete (13,3)-arc.

15. If $\alpha = 4$, $\beta = 4$, then the cubic curve C_{15} is:

$x_1(x_1 - 1)(x_0 - 4) - x_0(x_0 - 1)(x_1 + 4) = 0$

C_{15} = {1,2,11,21,3,12,20,59,62,77,80,87}, which is incomplete cubic, and by adding to it the point 55, we obtain a complete (13,3)-arc.

16. If $\alpha = 4$, $\beta = 5$, then the cubic curve C_{16} is:

$x_1(x_1 - 1)(x_0 - 5) - x_0(x_0 - 1)(x_1 + 4) = 0$

C_{16} = {1,2,11,21,3,12,20,88}, which is incomplete cubic, and by adding to it the points 32,33,40,42,50,55,58, we obtain a complete (15,3)-arc.

17. If $\alpha = 4$, $\beta = 6$, then the cubic curve:

C_{17} = {1,2,11,21,3,12,20,32,34,40,46,49,55,68,70,80}, which is a complete (14,3)-arc.

18. If $\alpha = 4$, $\beta = 7$, then the cubic curve:

C_{18} = {1,2,11,21,3,12,20,90}, which is incomplete cubic, and by adding to it the points 32, 33, 40, 42, 49, 53, 70, we obtain a complete (15,3)-arc.

19. If $\alpha = 5$, $\beta = 2$, then the cubic curve:

C_{19} = {1,2,11,21,3,12,20,36,45,50,59,76}, which is incomplete cubic, and by adding to it the point 40, we obtain a complete (13,3)-arc.

20. If $\alpha = 5$, $\beta = 3$, then the cubic curve:

C_{20} = {1,2,11,21,3,12,20,35,37,44,46,49,54,58,63,77}, which is a complete (16,3)-arc.

21. If $\alpha = 5$, $\beta = 4$, then the cubic curve:

C_{21} = {1,2,11,21,3,12,20,72,73,78,85,88}, which is incomplete cubic, and by adding to it the point 46, we obtain a complete (13,3)-arc.

22. If $\alpha = 5$, $\beta = 5$, then the cubic curve:

C_{22} = {1,2,11,21,3,12,20,67,69,79,89,90}, which is incomplete cubic, and by adding to it the point 44, we obtain a complete (13,3)-arc.

23. If $\alpha = 5$, $\beta = 6$, then the cubic curve:

C_{23} = {1,2,11,21,3,12,20,80}, which is incomplete cubic, and by adding to it the points 32, 33,42,49,58,76,86, we obtain a complete (15,3)-arc.

24. If $\alpha = 5$, $\beta = 8$, then the cubic curve:
C_{24} = {1,2,11,21,3,12,20,82}, which is incomplete cubic, and by adding to it the points 32, 33,40,43,49,54,78, we obtain a complete (15,3)-arc.

25. If $\alpha = 6$, $\beta = 2$, then the cubic curve:
C_{25} = {1,2,11,21,3,12,20,32,40,72,77,90}, which is incomplete cubic, and by adding to it the point 76, we obtain a complete (13,3)-arc.

26. If $\alpha = 6$, $\beta = 4$, then the cubic curve:
C_{26} = {1,2,11,21,3,12,20,42}, which is incomplete cubic, and by adding to it the points 32, 33,49,53,58,68, we obtain a complete (15,3)-arc.

27. If $\alpha = 6$, $\beta = 5$, then the cubic curve:
C_{27} = {1,2,11,21,3,12,20,43}, which is incomplete cubic, and by adding to it the points 32, 33,42,49,53,58,68, we obtain a complete (15,3)-arc.

28. If $\alpha = 6$, $\beta = 6$, then the cubic curve:
C_{28} = {1,2,11,21,3,12,20,44,50,52,58,64}, which is incomplete cubic, and by adding to it the point 79, we obtain a complete (13,3)-arc.

29. If $\alpha = 6$, $\beta = 7$, then the cubic curve:
C_{29} = {1,2,11,21,3,12,20,33,34,45,67,68,78,79,85,86}, which is a complete (16,3)-arc.

30. If $\alpha = 6$, $\beta = 8$, then the cubic curve:
C_{30} = {1,2,11,21,3,12,20,46,49,53,59,60}, which is incomplete cubic, and by adding to it the point 78, we obtain a complete (13,3)-arc.

31. If $\alpha = 7$, $\beta = 2$, then the cubic curve:
C_{31} = {1,2,11,21,3,12,20,43,44,58,88,89}, which is incomplete cubic, and by adding to it the point 67, we obtain a complete (13,3)-arc.

32. If $\alpha = 7$, $\beta = 3$, then the cubic curve:
C_{32} = {1,2,11,21,3,12,20,59}, which is incomplete cubic, and by adding to it the points 32, 33,40,43,49,53,87, we obtain a complete (15,3)-arc.

33. If $\alpha = 7$, $\beta = 4$, then the cubic curve:
C_{33} = {1,2,11,21,3,12,20,36,37,54,55,60,67,70,76,79}, which is a complete (16,3)-arc.

34. If $\alpha = 7$, $\beta = 6$, then the cubic curve:
C_{34} = {1,2,11,21,3,12,20,42,62,87,90}, which is incomplete cubic, and by adding to it the point 70, we obtain a complete (13,3)-arc.

35. If $\alpha = 7$, $\beta = 7$, then the cubic curve:
C_{35} = {1,2,11,21,3,12,20,35,53,63,73,82}, which is incomplete cubic, and by adding to it the point 68, we obtain a complete (13,3)-arc.

36. If $\alpha = 7$, $\beta = 8$, then the cubic curve:

C_{36} = {1,2,11,21,3,12,20,64}, which is incomplete cubic, and by adding to it the points 32, 33, 42, 49, 50, 58, we obtain a complete (15,3)-arc.

37. If $\alpha = 8$, $\beta = 2$, then the cubic curve:
C_{37} = {1,2,11,21,3,12,20,42,46,49,69,73}, which is incomplete cubic, and by adding to it the point 85, we obtain a complete (13,3)-arc.

38. If $\alpha = 8$, $\beta = 3$, then the cubic curve:
C_{38} = {1,2,11,21,3,12,20,50}, which is incomplete cubic, and by adding to it the points 32, 33, 40, 42, 49, 63, 79, we obtain a complete (15,3)-arc.

39. If $\alpha = 8$, $\beta = 5$, then the cubic curve:
C_{39} = {1,2,11,21,3,12,20,35,36,52,62,63,76,78,85,87}, which is a complete (16,3)-arc.

40. If $\alpha = 8$, $\beta = 6$, then the cubic curve:
C_{40} = {1,2,11,21,3,12,20,53}, which is incomplete cubic, and by adding to it the points 32, 33, 40, 42, 49, 70, 73, we obtain a complete (15,3)-arc.

41. If $\alpha = 8$, $\beta = 7$, then the cubic curve:
C_{41} = {1,2,11,21,3,12,20,37,54,64, 80,89}, which is incomplete cubic, and by adding to it the point 86, we obtain a complete (13,3)-arc.

42. If $\alpha = 8$, $\beta = 7$, then the cubic curve:
C_{42} = {1,2,11,21,3,12,20,43,45,55,70,72}, which is incomplete cubic, and by adding to it the point 87, we obtain a complete (13,3)-arc.

when we checked each set of the cubic curves to find, if the points of each curve lead to maximum completeness, we found that the only (16,3)-arcs are complete, and we found that there exist complete (13,3)-arcs and complete (15,3)-arcs.

References

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(GF(9),∗)

∗	1	2	3	4	5	6	7	8
1	1	2	3	4	5	6	7	8
2	2	1	6	8	7	3	5	4
3	3	6	4	7	1	8	2	5
4	4	8	7	2	3	5	6	1
5	5	7	1	3	8	2	4	6
6	6	3	8	5	2	4	1	7
7	7	5	2	6	4	1	8	3
8	8	4	5	1	6	7	3	2

(GF(9),+)

+	0	1	2	3	4	5	6	7	8
0	0	1	2	3	4	5	6	7	8
1	1	2	0	4	5	3	7	8	6
2	2	0	1	5	3	4	8	6	7
3	3	4	5	6	7	8	0	1	2
4	4	5	3	7	8	6	1	2	0
5	5	3	4	8	6	7	2	0	1
6	6	7	8	0	1	2	3	4	5
7	7	8	6	1	2	0	4	5	3
8	8	6	7	2	0	1	5	3	4

Points and Lines of PG(2,9)

i	P_i	ℓ_i									
1	(1,0,0)	2	11	20	29	38	47	56	65	74	83
2	(0,1,0)	1	11	12	13	14	15	16	17	18	19
3	(1,1,0)	4	11	22	30	44	55	63	68	9	87
4	(2,1,0)	3	11	21	31	41	51	61	71	81	91
5	(3,1,0)	9	11	27	34	40	53	60	66	82	86
6	(4,1,0)	6	11	24	37	45	49	59	70	80	84
7	(5,1,0)	8	11	26	32	46	52	58	69	75	90
8	(6,1,0)	7	11	25	36	39	50	64	67	78	89
9	(7,1,0)	5	11	23	35	42	54	57	73	76	88
10	(8,1,0)	10	11	28	33	43	48	62	72	77	85
11	(0,0,1)	1	2	3	4	5	6	7	8	9	10
12	(1,0,1)	2	13	22	31	40	49	58	67	76	85
13	(2,0,1)	2	12	21	30	39	48	57	66	75	84
14	(3,0,1)	2	18	27	36	45	54	63	72	81	90
15	(4,0,1)	2	15	24	33	42	51	60	69	78	87
16	(5,0,1)	2	17	26	35	44	53	62	71	80	89
17	(6,0,1)	2	16	25	34	43	52	61	70	79	88
18	(7,0,1)	2	14	23	32	41	50	59	68	77	86
19	(8,0,1)	2	19	28	37	46	55	64	73	82	91
20	(0,1,1)	1	29	30	31	32	33	34	35	36	37
21	(1,1,1)	4	13	21	29	46	54	62	70	78	86
22	(2,1,1)	3	12	22	29	42	52	59	72	82	89
23	(3,1,1)	9	18	25	29	44	51	58	73	77	84
24	(4,1,1)	6	15	28	29	40	50	63	71	75	88
25	(5,1,1)	8	17	23	29	43	49	64	66	81	87
26	(6,1,1)	7	16	27	29	41	55	57	69	80	85
27	(7,1,1)	5	14	26	29	45	48	60	67	79	91
28	(8,1,1)	10	19	24	29	39	53	61	68	76	90
29	(0,2,1)	1	20	21	22	23	24	25	26	27	28
30	(1,2,1)	3	13	20	30	43	50	60	73	80	90
31	(2,2,1)	4	12	20	31	45	53	64	69	77	88
32	(3,2,1)	7	18	20	34	46	48	59	71	76	87
33	(4,2,1)	10	15	20	37	44	52	57	67	81	86
34	(5,2,1)	5	17	20	32	39	51	63	70	82	85

35	(6,2,1)	9	16	20	36	42	49	62	68	75	91
36	(7,2,1)	8	14	20	35	40	55	61	72	78	84
37	(8,2,1)	6	19	20	33	41	54	58	66	79	89
38	(0,3,1)	1	74	75	76	77	78	79	80	81	82
39	(1,3,1)	8	13	28	34	45	51	57	68	74	89
40	(2,3,1)	5	12	24	36	43	55	58	71	74	86
41	(3,3,1)	4	18	26	37	42	50	61	66	74	85
42	(4,3,1)	9	15	22	35	41	48	64	70	74	90
43	(5,3,1)	10	17	25	30	40	54	59	69	74	91
44	(6,3,1)	3	16	23	33	46	53	63	67	74	84
45	(7,3,1)	6	14	27	31	39	52	62	73	74	87
46	(8,3,1)	7	19	21	32	44	49	60	72	74	88
47	(0,4,1)	1	47	48	49	50	51	52	53	54	55
48	(1,4,1)	10	13	27	32	42	47	64	71	79	84
49	(2,4,1)	6	12	25	35	46	47	60	68	81	85
50	(3,4,1)	8	18	24	30	41	47	62	67	82	88
51	(4,4,1)	4	15	23	34	39	47	58	72	80	91
52	(5,4,1)	7	17	22	33	45	47	61	73	75	86
53	(6,4,1)	5	16	28	31	44	47	59	66	78	90
54	(7,4,1)	9	14	21	37	43	47	63	69	76	89
55	(8,4,1)	3	19	26	36	40	47	57	70	77	87
56	(0,5,1)	1	65	66	67	68	69	70	71	72	73
57	(1,5,1)	9	13	26	33	39	55	59	65	81	88
58	(2,5,1)	7	12	23	37	40	51	62	65	79	90
59	(3,5,1)	6	18	22	32	43	53	57	65	78	91
60	(4,5,1)	5	15	27	30	46	49	61	65	77	89
61	(5,5,1)	4	17	28	36	41	52	60	65	76	84
62	(6,5,1)	10	16	21	35	45	50	58	65	82	87
63	(7,5,1)	3	14	24	34	44	54	64	65	75	85
64	(8,5,1)	8	19	25	31	42	48	63	65	80	86
65	(0,6,1)	1	56	57	58	59	60	61	62	63	64
66	(1,6,1)	5	13	25	37	41	53	56	72	75	87
67	(2,6,1)	8	12	27	33	44	50	56	70	76	91
68	(3,6,1)	3	18	28	35	39	49	56	69	79	86
69	(4,6,1)	7	15	26	31	43	54	56	68	82	84
70	(5,6,1)	6	17	21	34	42	55	56	67	77	90
71	(6,6,1)	4	16	24	32	40	48	56	73	81	89
72	(7,6,1)	10	14	22	36	46	51	56	66	80	88
73	(8,6,1)	9	19	23	30	45	52	56	71	78	85
74	(0,7,1)	1	38	39	40	41	42	43	44	45	46
75	(1,7,1)	7	13	24	35	38	52	63	66	77	91
76	(2,7,1)	9	12	28	32	38	54	61	67	80	87
77	(3,7,1)	10	18	23	31	38	55	60	70	75	89
78	(4,7,1)	8	15	21	36	38	53	59	73	79	85
79	(5,7,1)	3	17	27	37	38	48	58	68	78	88
80	(6,7,1)	6	16	26	30	38	51	64	72	76	86
81	(7,7,1)	4	14	25	33	38	49	57	71	82	90
82	(8,7,1)	5	19	22	34	38	50	62	69	81	84
83	(0,8,1)	1	83	84	85	86	87	88	89	90	91

84	(1,8,1)	6	13	23	36	44	48	61	69	82	83
85	(2,8,1)	10	12	26	34	41	49	63	73	78	83
86	(3,8,1)	5	18	21	33	40	52	64	68	80	83
87	(4,8,1)	3	15	25	32	45	55	62	66	76	83
88	(5,8,1)	9	17	24	31	46	50	57	72	79	83
89	(6,8,1)	8	16	22	37	39	54	60	71	77	83
90	(7,8,1)	7	14	28	30	42	53	58	70	81	83
91	(8,8,1)	4	19	27	35	43	51	59	67	75	83

بناء الاقواس (k,3) في المستوي الإسقاطي PG(2,9) باستخدام طريقة هندسية

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استلم البحث في :27 آذار 2001 ، قبل البحث في :28 آيار 2001

الخلاصة

يتضمن هذا البحث، بناء الاقواس (k,3) اسقاطية مختلفة في المستوي الاسقاطي PG(2,9) بتطبيق طريقة هندسية. ويتم بناء المنحنيات التكعيبية باستخدام المعادلة العامة التكعيبية. لقد وجدنا أن الاقواس – (13,3) كاملة، وكذلك الاقواس – (15,3) كاملة، و وجدنا ان اكبر الاقواس الكاملة هي الاقواس – (16,3).

الكلمات المفتاحية : اقواس ،قاطع ،مستوي اسقاطي