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Solution of Some Application of System of Ordinary Initial Value Problems Using Osculatory Interpolation Technique

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Abstract

The aim of this paper is to find a new method for solving a system of linear initial value problems of ordinary differential equation using approximation technique by two-point osculatory interpolation with the fit equal numbers of derivatives at the end points of an interval [0, 1] and compared the results with conventional methods and is shown to be that seems to converge faster and more accurately than the conventional methods.

Key words : Initial value problems , Approximation , Osculatory interpolation

1- Introduction

Systems of ordinary differential equations (ODEs) arise in mathematical models throughout science and engineering. When an explicit condition (or conditions) that a solution must satisfy is specified at one value of the independent variable, usually its lower bound, this is referred to as an initial value problem (IVP) and a system of ordinary differential equations is a system of equations relating several unknown functions $y_i(x)$ of an independent variable x, some of the derivatives of the $y_i(x)$, and possibly x itself. [1] .Initial-value problems for systems of differential equations permeate many areas of mathematics: such problems arise naturally in modelling the evolution of dynamical processes in economics, engineering, and the physical , biological sciences [2]. In this paper we introduce reactor problem which introduced in [3] Kehoe and Butt have studied the kinetics of benzene hydrogenation on asupported Ni/kieselguhr catalyst. In the presence of a large excess of hydrogen, the reaction is pseudo-first-order at temperatures below 200°C with the rate given by

$$-r = P_{H2} k_0 K_0 T \exp[(-Q-E_a)/R_g T)] C_B \quad \text{mol /(g catalyst s)}$$

where

R_g = gas constant, 1.987 cal/(mole'K)
$- Q - E_a$ = 2700 cal/mole

P_{H2} = hydrogen partial pressure (torr)

k_o = 4.22 mole/(gcat·s·torr)
K_o = 2.63 X 10-6 cm3/(mole'K)
T = absolute temperature (K)
C_B = concentration of benzene (mole/cm3).

Price and Butt [4] studied this reaction in a tubular reactor. If the reactor is assumed to be isothermal, we can calculate the dimensionless concentration profile of benzene in their reactor given plug flow operation in the absence of inter- and intraphase gradients. Using a typical run,

P_{H2} = 685 torr
P_B = density of the reactor bed, 1.2 gcat/cm3
e = contact time, 0.226 s
T = 150°C

And if we now consider the reactor to be adiabatic instead of isothermal, then an energy balance must accompany the material balance. Formulate the system of governing differential equations.

The data of this problem

C_p = 12.17 X 104 J/(kmole'°C)
- Δ.H_r = 2.09 x 108 J/kmole

$T^* = \dfrac{T}{T^0}, T^0 = 423K(150^0 C)$. For the "short" reactor, .

We have system of initial value problem

$$\frac{dy}{dx} = -0.1744\exp[\frac{3.21}{T^*}]y \qquad \text{(material balance)}$$

$$\frac{dT^*}{dx} = 0.06984\exp[\frac{3.21}{T^*}]y \qquad \text{(energy balance)}$$

with I.C y(0) =1 , $T^*(0)$ =1

2- Problem definition

In this section we can explain the way through the application of this system, of initial value problem:

$$\left.\begin{aligned} \frac{dy}{dx} &= -0.1744\exp[\frac{3.21}{T^*}]y \\ \frac{dT^*}{dx} &= 0.06984\exp[\frac{3.21}{T^*}]y \end{aligned}\right\rangle \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

with I.C y(0) =1 , $T^*(0)$ =1

In this paper we are particularly concerned with fitting function values and derivatives at the two end points of a finite interval, say [0,1],wherein a useful and succinct way of writing a osculatory interpolant $P_{2n+1}(x)$ of degree 2n + 1 was given for example by Phillips [5] as:

$$P_{2n+1}(x)=\sum_{j=0}^{n}\{y^{(j)}(0)\, q_j(x)+(-1)^j\, y^{(j)}(1)\, q_j(1-x)\}\text{............}(2)$$

$$q_j(x)=(x^j/j!)(1-x)^{n+1}\sum_{s=0}^{n-j}\binom{n+s}{s}x^s=Q_j(x)/j!\quad\text{.............}(3)$$

so that (2) with (3) satisfies

$$y^{(r)}(0)=P_{2n+1}^{(r)}(0)\ ,\quad y^{(r)}(1)=P_{2n+1}^{(r)}(1)\ ,\qquad r=0,1,2,\ldots,n.$$

We can be write the equation (2) directly in terms of the Taylor coefficients a_i and b_i about x = 0 and x = 1 respectively, as

$$P_{2n+1}(x)=\sum_{j=0}^{n}\{a_j\, Q_j(x)+(-1)^j b_j\, Q_j(1-x)\}.\ \text{....}(4)$$

The simple idea of this paper is to replace y(x) in problem (1) by a P_{2n+1} in equation (3) .The first step therefore is to construct the P_{2n+1} . To do this we need the Taylor coefficients of y (x) and $T^*(x)$ respectively about x=0

$$y(x)=a_0+a_1x+\sum_{i=2}^{\infty}a_ix^i\quad\text{............}\quad(5a)$$

Where $y(0)=a_0$, $y'(0)=a_1$. …… $y^{(j)}(0)/i!=a_i$ i=2.3,…..

And

$$T^*(x)=b_0+b_1x+\sum_{i=2}^{\infty}b_ix^i\quad\text{............}\quad(5b)$$

Where $T^*(0)=b_0$, $T^{*'}(0)=b_1$. …… $T^{*(j)}(0)/i!=b_i$ i=2.3,…..

Also we need the Taylor coefficients of y (x) and $T^*(x)$ respectively about x=1

$$y(x)=c_0+c_1(x-1)+\sum_{i=2}^{\infty}c_i(x-1)^i\quad\text{...........}(6a)$$

Where $y_1(1)=c_0$, $y'(1)=c_1$. …… $y^{(j)}(1)/i!=c_i$ i=2.3,….. ………… (6b)

$$T^*(x)=d_0+d_1(x-1)+\sum_{i=2}^{\infty}d_i(x-1)^i$$

Where $T^*(1)=d_0$, $T^{*'}(1)=d_i$. …… $T^{*(j)}(1)/i!=d_i$ i=2.3,…..

Then we simply insert the series forms in (5a) in to equation (1) and equate coefficients of x to obtain a_1 , then derive equation (1) and insert the series in to (5a) and equate coefficients of x to obtain a_2 and soon to obtain a_3 , a_4 …..
Then equation (5b) in the same manner to obtain b_2, b_3,…..
and simply insert the series forms in (6a) in to equation (1) and equate coefficients of(x- 1) to obtain c_1 , then derive equation (1) and insert the series in to (6a) and equate coefficients of x to obtain c_2 and soon to obtain c_3 , c_4 ….. .Then equation (6b) in the same manner to obtain d_2, d_3,….

The resulting system of equations can be solved to obtain (a_0, a_1, a_i) for all $i \geq 2$.

The notation implies that the coefficients depend only on the indicated unknowns a_0, b_0, c_0, d_0 .

We note here there are only two variables c_0, d_0 because all the unknowns in terms of a_0, b_0 so requires then presence of only two equations.
Now integrate equation (1) to obtain :

$$c_0-a_0+\int_0^1 f_1(x,y,T^*)\,dx=0\quad\text{......................}(7a)$$

$$d_0-b_0+\int_0^1 f_2(x,y,T^*)\,dx=0\quad\text{......................}(7b)$$

and replacement P_{2n+1} , $\widetilde{p}_{2n+1}$ of y ,T^* in (7a) and (7b) respectively and insert c_0 and d_0 and a_i' s , b_i's , c_i's , d_i's in to P_{2n+1} , $\widetilde{p}_{2n+1}$

Then solve system of algebraic equation using matlab to obtain c_0 and d_0 and insert into (4) which represent the solution of (1) .

From equations (2) , (3) we have the solution when n=3,4 :

P_7=-.6200000e-18X^7-1.110814X^6+3.35442X^5-.5768X^4-.3313X^3 +.134693e-2X^2+4.14525*x+.24730108e-3

P_9=-.821544e-17X^9+.233415X^8-20059X^7+.1197551X^6+2.52655X^5+.265788e-4X^4-5.1622X^3+.326077e-X^2+3.202X+.345602e-5

And

$\widetilde{p}_7$=-.122040e-4X^7+.48829e-3X^6-.8651138e-3X^5-.123418e-1X^4 +.166667e-1X^3+.155100X^2-.103300X-.30012

$\widetilde{p}_9$=.19740e-6X^9-.833318e-X^8+.577163e-X^7+.345225e-3X^6 -.832243e-3X^5-. 125044e-1X^4+.163267e-1X^3+.1521000X^2-.10010X-.3354

It is clear that from table 2 , the suggested method is more accurate that the other results and converge faster and easy implementation.

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The results of solution given in the following table :

Table 1 :The result of the methods for n = 3, 4 of example

	P7	P9	$\widetilde{p}_7$	$\widetilde{p}_9$
c0	1.32667921198	1.32668875443	1.32667925438	1.32668875487
d0	-2.6828498769	-2.6833878269	-2.682849108	-2.6833878142
X	P_7	P_9	$\widetilde{p}_7$	$\widetilde{p}_9$
0	1.0000000000	1.0000000000	1.0000000000	1.0000000000
0.1	0.7503665788	0.7003665774	1.1199859943	1.1199859943
0.2	0.5291928846	0.5291928843	1.1889942932	1.1889942911
0.3	0.4137326433	0.4137326475	1.2349569243	1.2349569603
0.4	0.3299212549	0.3299212598	0.2685940556	0.2685940302
0.5	0.2364347965	0.2456766932	1.2934506588	1.2934506322
0.6	0.2172183536	0.2172183562	1.3134460345	1.3134460212
0.7	0.2178286933	0.2178286938	1.3290470766	1.3290470733
0.8	0.1469447534	0.1469447562	1.3416557878	1.3416557893
0.9	0.2244532838	0.1216292831	1.3517234577	1.3517234588
1	0.1009877458	0.1009877429	1.3600221044	1.36002210432

Now we give a comparison between the solution of suggested method and solution of other methods in the following table

Table 2: A Comparison between P_9 and other methods of example

X	DVERK, TOL= (-6) y	DGEAR (MF =21), TOL =(-4) y	P_9 by using Osculatory interpolation	DVERK, TOL= (-6) T^*	DGEAR (MF =21), TOL =(-4) T^*	$\widetilde{p}_9$ by using Osculatory interpolation
0	1.000000	1.000000	1.0000000000	1.00000	1.00000	1.0000000000
0.1	0.700367	0.700468	0.7003665774	1.11999	1.11994	1.1199859943
0.2	0.529199	0.529298	0.5291928843	1.18853	1.18849	1.1889942911
0.3	0.413737	0.413775	0.4137326475	1.23477	1.23475	1.2349569603
0.4	0.329919	0.329864	0.3299212598	0.26833	1.26836	0.2685940302
0.5	0.266492	0.266349	0.2456766932	1.29373	1.29379	1.2934506322
0.6	0.217208	0.217070	0.2172183562	1.31347	1.31353	1.3134460212
0.7	0.178209	0.178076	0.2178286938	1.32909	1.32914	1.3290470733
0.8	0.146943	0.146801	0.1469447562	1.34161	1.34167	1.3416557893
0.9	0.121629	0.121495	0.1216292831	1.35175	1.35180	1.3517234588
1	0.100980	0.100864	0.1009877429	1.36002	1.36006	1.36002210432

حل بعض تطبيقات منظومة من مسائل القيم الابتدائية الاعتيادية باستخدام تقنية الاندراج التماسي

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الخلاصة

الهدف من هذا البحث هو إيجاد طريقة جديدة لحل منظومة من مسائل القيم الابتدائية المعادلةالتفاضلية الاعتيادية ، إذ استعملت تقنية التقريب ذا الاندراج التماسي ذي النقطتين التي تتفق فيها الدالة وعدد متساو من المشتقات المعرفة عند نقطتي نهاية المدة [0,1] مع البيانات المعطاة وقورنت الطريقة المقترحة مع الطرائق التقليدية وقد ظهرت النتائج بأن الطريقة المقترحة ذو تقارب أسرع و أكثر دقة من الطرائق التقليدية.

كلمات مفتاحية : مسائل القيم الابتدائية ، التقريب ، الاندراج التماسي