FREE VIBRATION OF ISOTROPIC HALF-ELLIPTIC PLATES OF LINEARLY VARYING THICKNESS WITH CLAMPED CURVED BOUNDARY

A. P. GUPTA[1], N. BHARDWAJ[1] AND K. K.CHOONG[2]

[1]*Department of Applied Sciences and Humanities, Huda- Sector 23-A,*

Institute of Technology and Management, Gurgaon- Haryana-122017, INDIA

[2]*School of Civil Engineering, Universiti Sains Malaysia, 14300 Nibong Tebal,*

Penang, MALAYSIA

[2]*E-mail: cekkc@eng.usm.my*

Abstract: Two-dimensional boundary characteristic orthonormal polynomials are used in Rayleigh-Ritz method to study the title problem. In general, it is found that this method gives better results than the other traditional method such as boundary integral equation methods, Spline methods, Chebyshev collocation method, Frobenius method etc. The thickness is taken to be linearly varying in two orthogonal directions. Comparisons in particular cases have been made with the existing results in the literature. Convergence of frequencies of at least up to five significant figures is obtained. Results showing the variation in frequencies with taper parameters and aspect ratios are presented in tabular form. Mode shapes are shown using three-dimensional graphs of plates in displaced configurations.

Keywords: *Free vibration, isotropic half-elliptic plates, boundary characteristic, orthonormal, polynomials, Rayleigh-Ritz method*

1. INTRODUCTION

The subject of vibration of plates is an old one in which a lot of works has been carried out in the past hundred years or so. In the earlier periods, results were available for some simple cases only where the analytical solution could be found. The lack of good computational facilities made it almost impossible to get reasonably accurate results even in these cases. This explains why in spite of a lot of a theoretical development, numerical results were available only for a few cases. With the advent of fast computers, particularly after 1960s and 1970s there was a tremendous rise in the amount of research work using numerical and approximate methods. Today, very versatile methods like finite element methods, finite difference methods, boundary integral equation methods, Galerkin's method, Rayleigh-Ritz method, Spline methods and method of weighted residuals are commonly used to handle practically any geometric shape and type of boundary conditions of the plate.

Bayer *et al.* [1] have carried out a parametric study on vibrating clamped elliptical plates with variable thickness by using Moment method and Rayleigh-Ritz method. Narita [2] has studied vibration of orthotropic elliptic plates resting on point supports using Ritz-Lagrange multiplier method. Chakraverty and Petyt [3,4] have studied natural frequencies of rectangular orthotropic elliptic homogeneous and non-homogeneous plates by using boundary characteristic orthogonal polynomials in Rayleigh- Ritz method. Recently, Gupta and Bhardwaj [5] studied vibration of rectangular orthotropic elliptic plates of quadratically varying thickness resting on elastic foundation. In another paper, the same authors [6] studied vibration of orthotropic elliptic plates of varying thickness resting on elastic foundation. A lot of information is available in the literature on vibration of isotropic triangular, rectangular and circular plates of constant and varying thickness. Comparatively less work is available on isotropic elliptic plates of varying thickness and those on half elliptic plate is even meager. Leissa [7-14] gives an excellent account about clamped and free elliptic plates in Chapter III of his monograph. From 1986 onwards, quite a number of research works on vibration of elliptic plates have been carried out. Singh and Chakraverty [15-22] have analyzed transverse vibration of circular and elliptic plates of constant and variable thickness using successive approximation in Galerkin's method and boundary characteristic orthogonal polynomials in Rayleigh- Ritz method. Lam *et al.* [23] have studied vibration of circular and elliptic plates using orthogonal polynomials. They have reported the first six frequencies and shown plot results of two dimensional mode shapes. Rajalingham and Bhat [24] have analyzed axisymmetric vibration of circular plates and its analog in elliptic plates using boundary characteristic orthotrogonal polynomials in Rayleigh- Ritz method. Rajalingham *et al.* [25] have investigated vibration of clamped elliptic plates using the exact modes of circular plates as the shape function in Rayleigh- Ritz method.

In all the above-mentioned works, the researchers have analyzed full elliptic plates by taking into account various other effects. For the case of half elliptic plate with constant thickness, Liew and Lam [26] have computed the first six frequencies; Bucco, Mazumdar and Sved [27] have computed fundamental frequencies for varying aspect ratio whereas Bhattacharya and Bhowmic [28] have computed only fundamental frequency for semi-circular plate. In all these three papers, they have all computed results for ***C-F*** boundary condition only out of all nine possible combinations of boundary conditions (***C-C***, ***C-S***, ***C-F***, ***S-C***, ***S-S***, ***S-F*** and ***F-C***, ***F-S*** ***F-F***), where ***C***, ***S*** and ***F*** stand for clamped, simply supported and fixed, respectively.

In the present paper, free vibration of isotropic half elliptic plate of two-dimensional thickness variation with clamped curved boundary is considered. The first nine frequencies are computed for various values of taper parameters and aspect ratio for ***C-C***, ***C-S*** and ***C-F*** boundary conditions. Three-dimensional plots of mode shapes for these combinations are also shown. Comparisons in particular cases are made with the results of Liew and Lam [26], Bucco, Mazumdar and Sved [27] and Bhattacharya and Bhowmic [28].

Convergence of frequencies of at least up to five significant figures is observed. A maximum of twenty-two number of terms are required in the solution to achieve this convergence.

2. BASIC FORMULATION

An isotropic half elliptic plate of semi axes a, b and varying thickness $h(x,y)$ is considered as shown in Fig.1. The plate is defined with respect to a rectangular Cartesian co-ordinate system (x,y,z) by taking x,y axes to be lying along the semi-axes direction of the plate and z-axis to be along the thickness direction of the plate. Under such definition of coordinate system, the boundaries of the plate, side 2 and side 1 as shown in Fig.1 could be represented using the following expressions, respectively:

$$y = 0 \text{ (side 2)} \quad \text{and} \tag{1}$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \, ; y > 0 \text{ (side 1)} \tag{2}$$

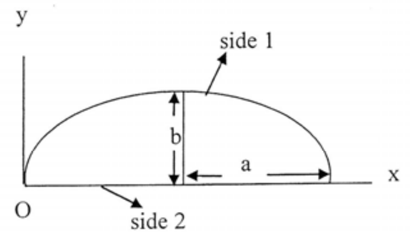

Fig. 1: An isotropic half elliptic plate

2.1 Thickness variation

The thickness of the plate is assumed to be represented by the following equation:

$$H = H_0 \, F(x,y) \tag{3}$$

where, $H=h/a$, H_0 is the thickness at the center of the plate and $F(x, y)$ is a function of the coordinates x and y. Thickness of the plate varies linearly along both axes simultaneously in the following manner:

$$F(x,y) = (1-\alpha|x|)\left(1-\frac{\beta y}{m}\right), \qquad \alpha, \beta < 1 \tag{4}$$

where m is the aspect ratio.

2.2 Generation of boundary characteristic orthogonal polynomials

The N-term approximation of the deflection function is taken as

$$W(x,y)=\sum_{j=1}^{N} c_j\,\Phi_j(x,y) \tag{5}$$

where the two dimensional boundary characteristic orthonormal polynomials $\Phi_j(x,y)$ generated by means of Grahm-Schmidt process are defined as follows:

$$\Phi_j=\phi_j\Big/\sqrt{<\phi_j,\phi_j>},\quad \phi_1=\left(1-V^2\right)^p y^q,\ j=1,2,\ldots,N \tag{6}$$

$$\phi_i=\phi_1 f_i(x,y)-\sum_{j=1}^{i-1}\frac{<\phi_1 f_i,\phi_j>}{<\phi_j,\phi_j>}\phi_j,\quad i=2,3,\ldots,N \tag{7}$$

$$<f,g>=\iint_A F(x,y)f(x,y)g(x,y)\,dy\,dx \tag{8}$$

The values of p is taken as 2 since the boundary of side 1 ($V^2=1$, $V^2=\frac{x^2}{a^2}+\frac{y^2}{b^2}$) is subjected to clamped edge conditions. The values of q will take the value of 0, 1 or 2 depending on whether side 2 ($y=0$) is subjected to free, simply supported or clamped edge conditions, respectively. The functions $f_i(x,y)$ of the form $x^{m_i}\,y^{n_i}$ are taken by the same scheme as given in reference [7].

3. RAYLEIGH –RITZ METHOD

The functional $J(W)$obtained by subtracting the maximum kinetic energy from the maximum strain energy is

$$J(W)=\frac{EH_0^3}{12(1-\nu^2)}\iint_A\left[\ F^3(x,y)\left\{\ W_{,XX}^2+W_{,YY}^2+2\,\nu W_{,XX}\,W_{,YY}+2(1-\nu)\,W_{,XY}^2\ \right\}-\Omega^2 FW^2\ \right]dYdX$$

(9)

where , E= Young's modulus, ν= Poisson ratio, ρ = mass density, ω = frequency,

$\Omega^2=12a^4\omega^2\rho(1-\nu^2)/Eh^2$ and A is the region bounded by

$$Y=0 \text{ and } V^2=1,\qquad Y>0. \tag{10}$$

Minimization of $J(W)$ as a function of the coefficients c_j based on standard procedures leads to the following standard eigenvalue problem:

$$\sum_{j=1}^{N}\left(a_{ij}-\Omega^2\,\delta_{ij}\right)c_j=0\ ,\quad i=1\,(1)\,N \tag{11}$$

where

$$a_{ij} = \iint_A \Big[F^3(x,y) \big\{ \Phi_{i,XX}\,\Phi_{j,XX} + \Phi_{i,YY}\,\Phi_{j,YY} + \nu\left(\Phi_{i,XX}\,\Phi_{j,YY} + \Phi_{i,YY}\,\Phi_{j,XX}\right)$$

$$+\ 2\,(1-\nu)\,\Phi_{i,XY}\,\Phi_{j,XY} \big\} \Big] dy\,dx \qquad (12)$$

The integrals involved in Equations (8) and (12) are evaluated by the formula

$$\iint_A \left(1-V^2\right)^c V^d x^q y^s \, dx\,dy = \frac{m^{s+1}\,\Gamma\!\left(\frac{q+s+d+2}{2}\right)\Gamma\!\left(\frac{q+1}{2}\right)\Gamma\!\left(\frac{s+1}{2}\right)\Gamma(c+1)}{2\,\Gamma\!\left(\frac{q+s}{2}\right)\Gamma\!\left(\frac{q+s+d+2c+4}{2}\right)}$$

, when q and s are even

$$= 0 \qquad \text{, otherwise} \qquad (13)$$

where $c, q, s > -1$.

The eigenvalues (Ω) and the eigenvectors (c_j) are computed using Jacobi method. The mode shapes are computed using Equation (5).

4. RESULTS AND DISCUSSION

Table 1 shows the convergence of Ω. It can be seen that for the first nine modes, Ω show convergence of up to five significant figures when the number of terms in Equation 5 is taken as 19, 19 and 22 for ***C-C***, ***C-S*** and ***C-F*** plates, respectively.

Comparison of Ω with the results of Liew and Lam [26] for isotropic half elliptic ***C-F*** plate of uniform thickness when $\alpha = \beta = 0$, $m = 2/3$ and $\nu_x = 0.3$ are shown in Table 2. Table 3 shows comparison of Ω for isotropic half elliptic ***C-F*** plate of uniform thickness and varying aspect ratio with (i) the results of both Bucco, Mazumdar and Sved [27] when $\alpha = \beta = 0$ and $\nu_x = 0.3$ and (ii) the results of Bhattacharya and Bhowmic [28] for isotropic semi-circular ***C-F*** plate when $\alpha = \beta = 0$ and $\nu_x = 0.3$. A very close agreement is found in almost all the cases and the present results are found to be better, especially in higher modes, even for those cases where lesser number of terms is taken into consideration in the solution.

Table 1: Convergence of Ω for isotropic half elliptic plate when $\alpha = \beta = 0.5$ and $\nu_x = 0.3$.

Edge Conditions	N	Ω_1	Ω_2	Ω_3	Ω_4	Ω_5	Ω_6	Ω_7	Ω_8	Ω_9
C-C	15	51.607	56.503	65.201	75.360	89.297	141.17	147.47	163.49	164.22
	16	51.604	56.503	65.200	75.360	89.296	105.01	141.17	147.64	163.40
	17	51.604	56.501	65.199	74.971	89.292	105.01	140.58	147.64	160.74
	18	51.604	56.501	65.199	74.971	89.292	104.61	140.58	147.58	160.74
	19	51.604	56.501	65.199	74.971	89.292	104.01	140.58	147.58	160.61
C-S	15	31.016	38.942	48.629	60.643	75.577	110.52	119.34	135.51	168.34
	16	31.012	38.940	48.627	60.634	74.006	93.595	110.52	119.37	135.11
	17	31.012	38.933	48.625	59.925	73.923	93.032	110.42	119.37	134.41
	18	31.012	38.897	48.615	59.923	73.746	91.031	110.42	119.37	134.33
	19	31.012	38.897	48.615	59.923	73.746	91.031	110.42	119.37	134.33
C-F	18	5.3149	12.734	22.050	32.353	42.449	48.546	55.786	63.165	73.292
	19	5.2655	12.731	21.852	32.340	42.378	48.181	55.786	63.165	70.113
	20	5.2655	12.644	21.849	32.324	42.311	48.170	55.785	63.071	70.092
	21	5.2599	12.644	21.827	32.324	42.299	47.973	55.785	63.071	70.063
	22	5.2599	12.644	21.827	32.324	42.299	47.973	55.785	63.071	70.063

Table 2: Comparison of Ω for isotropic half elliptic plate of constant thickness for *C-F* edge condition when $\alpha = \beta = 0$, $m = 2/3$ and $\nu_x = 0.3$.

Ref.	Number of Terms	Ω_1	Ω_2	Ω_3	Ω_4	Ω_5	Ω_6
[26]	40	9.8771	16.7973	27.0056	41.8201	56.5984	60.0828
Present	28	9.8771	16.7953	27.004	41.820	56.590	60.081

Table 3: Comparison of Ω for clamped isotropic half elliptic plate of constant thickness for *C-F* edge condition when $\alpha = \beta = 0$, and $\nu_x = 0.3$.

1/m	Ref. [27]	Ref. [28]	Present
1.0	28.652	28.293	28.113
1.1	33.807	-	32.980
1.2	39.515	-	38.331
1.3	45.716	-	44.159
1.4	52.358	-	50.455
1.5	59.400	-	57.217
2.0	101.256	-	97.897
3.0	231.802	-	213.340
4.0	411.089	-	374.310
5.0	630.549		580.99

Table 4 shows the variation of Ω with taper parameters α for ***C-C***, ***C-S*** and ***C-F*** plates when $\beta = 0$, $m = 0.5$ and $\nu_x = 0.3$. Similar results of comparison with taper parameter β when $\alpha = 0$, $m = 0.5$ and $\nu_x = 0.3$ is summarized in Table 5. It is observed that the frequencies decrease when the thickness of plate varies from thick to thin. This is due to the fact that the stiffness of the plate decreases continuously with increasing α or β.

Table 6 shows the variation of Ω with aspect ratio m for ***C-C***, ***C-S*** and ***C-F*** plates when $\alpha = \beta = -0.8$ and $\nu_x = 0.3$. It can be seen that Ω increases as m decreases from 1. The rate of increase is maximum in the case of ***C-C*** plates and minimum in the case of ***C-F*** plates.

Three-dimensional plots for the first nine normal mode shapes for ***C-C***, ***C-S*** and ***C-F*** plates are shown in Fig. 2,3 and 4, respectively when $\alpha = \beta = 0.5$, $\nu = 0.3$ and $m = 0.5$.

Table 4: Variation of Ω with α for isotropic half elliptic plate when $\beta = 0.0$ and $\nu_x = 0.3$.

Edge Conditions	α	Ω_1	Ω_2	Ω_3	Ω_4	Ω_5	Ω_6	Ω_7	Ω_8	Ω_9
C-C	-0.8	112.66	146.93	197.37	250.61	293.82	384.57	395.08	481.71	490.30
	-0.6	108.87	139.30	183.12	231.19	285.42	360.01	362.16	444.06	446.18
	-0.4	105.16	131.76	169.12	211.96	277.19	324.17	340.08	398.21	411.76
	-0.2	101.52	124.27	155.43	192.97	268.78	288.01	318.09	353.22	378.74
	0.0	97.897	116.14	142.14	174.32	249.02	262.90	294.04	311.51	347.27
	0.2	94.200	109.14	129.39	156.10	212.88	254.10	258.58	282.89	317.56
	0.4	90.224	101.12	117.26	138.36	177.02	217.59	245.17	262.17	288.92
	0.6	85.538	92.189	105.41	120.77	143.85	179.03	233.70	242.53	264.43
	0.8	78.948	81.214	91.800	102.12	118.57	147.94	214.42	217.45	237.48
C-S	-0.8	82.233	112.36	155.83	202.27	243.83	321.63	329.32	419.12	428.45
	-0.6	79.074	106.24	144.34	186.00	236.22	298.73	303.29	385.86	386.60
	-0.4	75.934	100.18	133.08	169.98	228.79	267.82	285.30	343.42	355.13
	-0.2	72.915	94.141	122.09	154.33	221.36	236.84	267.67	301.24	324.91
	0.0	69.857	88.061	111.45	139.15	204.55	215.31	250.18	259.78	296.12
	0.2	66.732	81.816	101.21	124.59	174.04	207.72	217.96	234.23	269.01
	0.4	63.416	75.184	91.332	110.70	144.32	178.61	199.87	217.60	244.03
	0.6	59.690	67.813	81.225	96.895	117.48	142.74	190.10	200.47	221.49
	0.8	54.980	59.052	69.175	80.579	97.631	116.03	175.01	179.29	195.68
C-F	-0.8	26.148	43.582	67.652	104.59	118.94	149.58	160.55	222.27	254.32
	-0.6	24.404	40.563	62.780	95.906	114.16	136.96	151.82	205.77	231.50
	-0.4	22.679	39.572	57.943	87.387	109.48	124.57	143.23	189.61	208.32
	-0.2	20.974	34.600	53.132	79.079	104.85	112.45	134.72	173.87	184.54
	0.0	19.289	31.632	48.323	71.035	99.980	100.94	126.19	158.68	159.99
	0.2	17.627	28.640	43.471	63.286	89.216	95.633	117.42	134.89	144.10
	0.4	15.997	25.590	38.504	55.741	78.130	90.704	107.33	110.83	129.87
	0.6	14.414	22.448	33.316	47.895	66.617	85.273	87.071	98.134	112.00
	0.8	12.908	19.162	27.674	38.759	52.921	70.597	78.634	85.666	91.407

Table 5: Variation of Ω with β for isotropic half elliptic plate when $\alpha = 0.0$ and $\nu_x = 0.3$.

Edge Conditions	β	Ω_1	Ω_2	Ω_3	Ω_4	Ω_5	Ω_6	Ω_7	Ω_8	Ω_9
C-C	-0.8	133.72	158.18	190.88	237.85	325.63	359.58	392.77	422.48	489.98
	-0.6	125.14	148.23	179.03	222.33	306.64	336.38	367.68	395.08	454.75
	-0.4	116.35	138.05	166.99	206.60	287.64	312.60	343.01	367.48	419.07
	-0.2	107.29	127.59	154.72	190.60	268.54	288.14	318.59	339.60	383.10
	0.0	97.897	116.14	142.14	174.32	249.02	262.90	294.04	311.51	347.27
	0.2	88.066	105.47	129.12	157.73	228.50	236.89	268.16	284.03	312.09
	0.4	77.620	93.517	115.43	140.86	204.92	211.19	239.05	258.52	278.19
	0.6	66.220	80.582	100.71	123.71	175.68	186.99	205.65	235.24	245.49
	0.8	53.073	66.074	84.468	106.39	140.66	162.66	167.77	211.76	214.13
C-S	-0.8	101.40	123.79	152.31	188.79	266.26	300.12	333.43	352.88	398.68
	-0.6	93.764	115.09	142.36	176.46	251.34	279.51	314.09	328.34	372.23
	-0.4	85.987	106.25	132.27	164.07	236.16	258.60	294.50	303.84	346.30
	-0.2	78.034	97.212	121.98	151.63	220.61	237.26	273.75	280.40	320.95
	0.0	69.857	88.061	111.45	139.15	204.55	215.31	250.18	259.78	296.12
	0.2	61.391	78.585	100.59	126.59	187.67	192.75	224.68	241.09	271.37
	0.4	52.522	68.729	89.341	113.88	166.98	172.26	197.75	223.64	245.63
	0.6	43.043	58.319	77.583	100.90	141.13	154.79	169.04	207.64	217.14
	0.8	32.523	47.190	65.203	87.807	111.77	138.18	139.69	185.41	193.65
C-F	-0.8	33.006	46.926	65.862	91.315	125.87	149.30	179.92	193.53	219.28
	-0.6	29.443	42.941	61.336	86.167	119.35	137.22	166.67	185.38	204.02
	-0.4	25.959	39.052	56.901	81.090	112.98	125.05	153.30	177.18	188.84
	-0.2	22.567	35.276	52.561	76.061	106.74	112.75	139.81	168.79	173.75
	0.0	19.289	31.632	48.323	71.035	99.980	100.94	126.19	158.68	159.99
	0.2	16.153	28.140	44.190	65.938	87.454	94.772	112.44	143.50	150.56
	0.4	13.204	24.826	40.154	60.664	74.343	88.928	98.582	128.04	140.46
	0.6	10.515	21.706	36.161	55.087	60.661	83.092	84.544	112.25	130.17
	0.8	8.1994	18.778	31.924	46.208	49.100	70.304	76.913	96.997	120.75

Table 6: Variation of Ω with m for isotropic half elliptic plate when $\alpha = \beta = -0.8$ and $\nu_x = 0.3$.

Edge Conditions	m	Ω_1	Ω_2	Ω_3	Ω_4	Ω_5	Ω_6	Ω_7	Ω_8	Ω_9
C-C	1.0	47.379	74.168	106.88	118.40	145.21	181.36	195.14	219.57	251.41
	0.8	67.554	97.971	138.75	169.94	187.91	238.82	267.01	314.90	335.75
	0.6	110.98	149.52	201.80	265.77	287.45	378.85	397.32	489.98	494.96
	0.4	233.74	295.70	383.99	485.97	621.04	738.98	778.57	909.43	984.15
	0.2	892.22	1081.4	1380.1	1694.2	2402.4	2606.9	2857.4	3260.5	3670.5
C-S	1.0	38.292	62.991	99.178	102.65	129.48	156.98	174.27	201.25	245.42
	0.8	53.456	80.877	117.87	144.15	161.95	205.76	235.36	288.66	308.22
	0.6	86.176	119.61	165.25	218.16	241.68	326.08	341.17	413.61	438.11
	0.4	178.31	228.42	301.29	377.02	518.55	597.75	670.46	748.06	805.82
	0.2	669.28	805.33	1043.9	1251.8	1962.2	2042.8	2339.2	2628.4	2985.7
C-F	1.0	18.142	35.303	52.617	62.364	82.167	99.761	113.42	129.26	155.37
	0.8	22.604	40.332	64.834	78.800	100.58	115.64	141.43	168.50	189.62
	0.6	32.581	51.752	78.608	118.37	126.58	167.62	170.93	235.47	272.09
	0.4	61.138	85.550	117.54	164.99	224.16	263.67	330.38	388.97	432.57
	0.2	209.67	263.89	331.82	434.52	555.60	940.53	987.42	1183.5	1198.7

5. CONCLUSIONS

As already pointed out, there are a number of papers on vibrations of isotropic elliptic plates where most of them are dealing with constant thickness and only a few are on variable thickness. In the particular case of half elliptic plates, only two papers are found and those too are dealing with constant thickness. Furthermore, in those papers, results are computed only for one boundary condition out of the nine possible combinations.

The current paper deals with free vibration of isotropic half elliptic plates of linearly varying thickness with clamped-curved boundary using Rayleigh-Ritz method. This method describes a procedure that can be applied to obtain approximations for the frequencies up to practically any order. Convergence of frequencies of at least up to five significant figures is observed. A maximum of twenty-two number of terms are required in the solution to achieve this convergence. The main advantage of the proposed method is that comparison of consecutive approximation makes it possible to monitor the rate of convergence. Also, it gives an idea to obtain results for a particular mode with the desired accuracy.

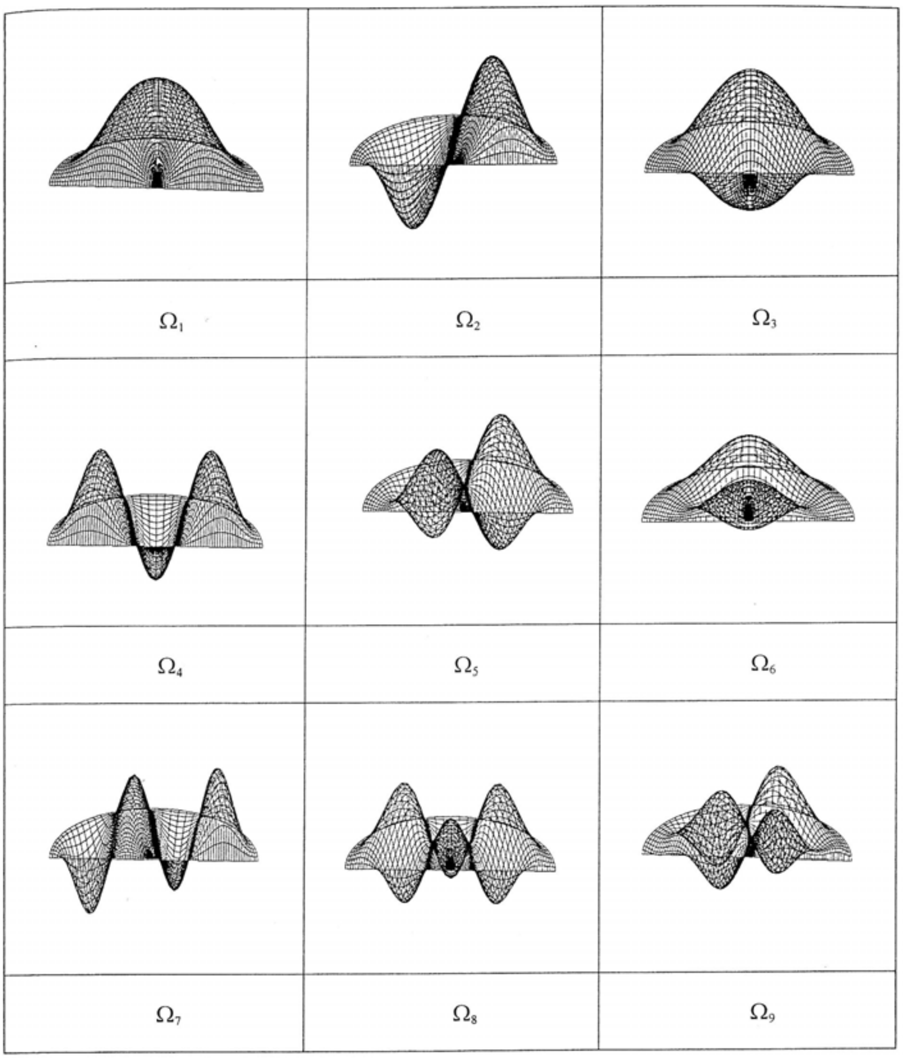

Fig. 2: Three dimensional plots for the first nine normal modes of vibration for *C-C* plate when $\alpha = \beta = 0.5$, $\nu = 0.3$ and $m = 0.5$.

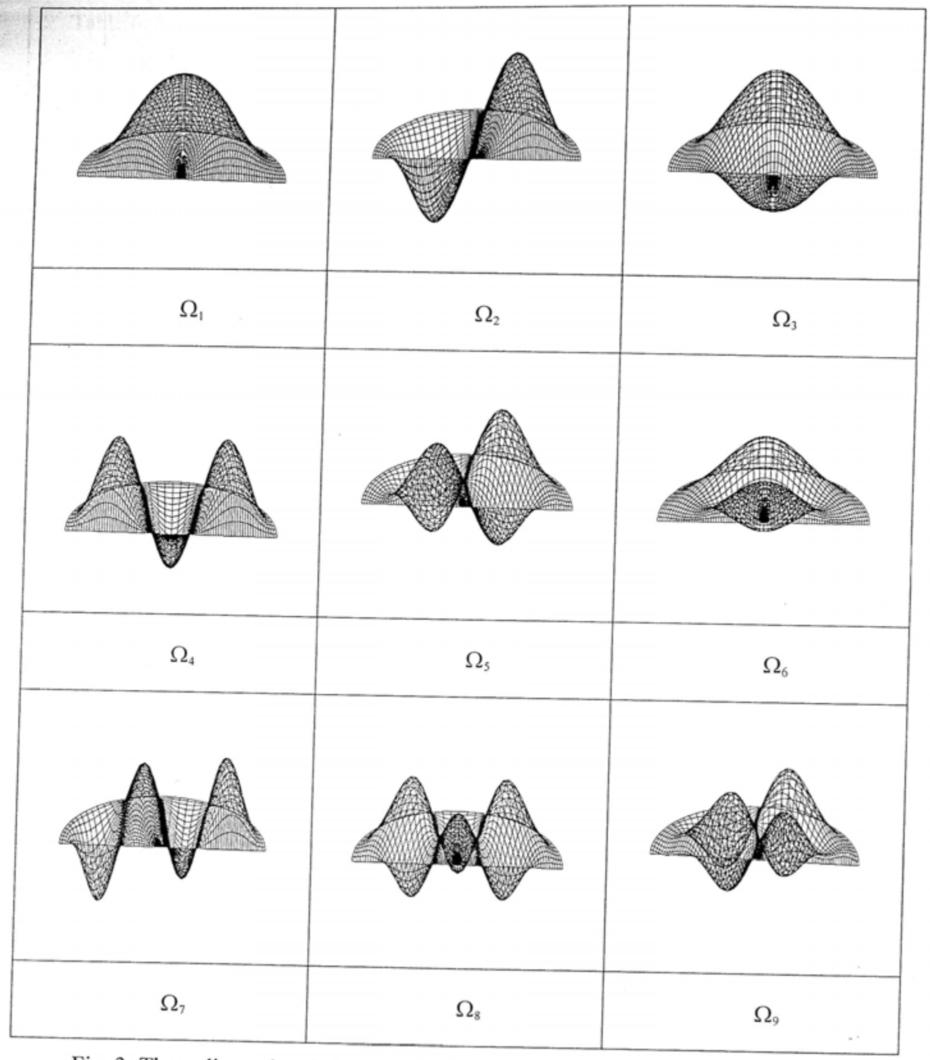

Fig. 3: Three dimensional plots for the first nine normal modes of vibration for *C-S* plate when $a = \beta = 0.5$, $\nu = 0.3$ and $m = 0.5$.

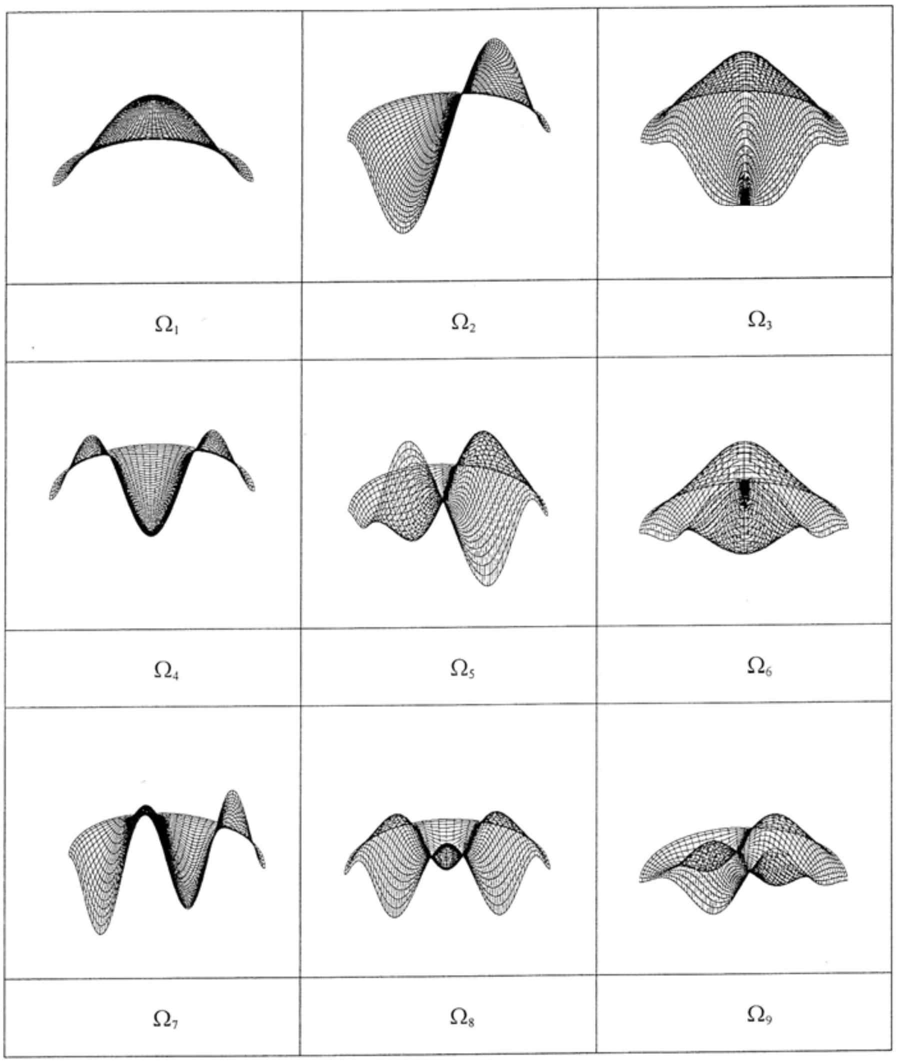

Fig. 4: Three dimensional plots for the first nine normal modes of vibration for *C-F* plate when $a = \beta = 0.5$, $\nu = 0.3$ and $m = 0.5$.

REFERENCES

[1] I. Bayer, U Güven and G. Atlay, "A parametric study on vibrating clamped elliptical plates with variable thickness". Journal of Sound and Vibration, 254(1), 179-188,2002.

[2] Y. Narita, "Free vibration analysis of orthotropic elliptic plates resting on arbitrary distributed point supports". *Journal of Sound and Vibration*, 108(1), 1-10, 1986.

[3] S. Chakraverty and M. Petyt, "Natural frequencies for free vibration of non-homogeneous elliptic and circular plates using two dimensional orthogonal polynomials". Applied Mathematical Modelling, 21, 53-67, 1997.

[4] S. Chakraverty and M. Petyt, "Free vibration analysis of elliptic and circular plates having rectangular orthotropy". Structural Engineering and Mechanics, 7, 53-67, 1999.

[5] A.P. Gupta and N. Bhardwaj, "Vibration of rectangular orthotropic elliptic plates of quadratically varying thickness resting on elastic foundation". Journal of Vibration and Acoustic, 126, 132-140, 2004.

[6] A. P. Gupta and N. Bhardwaj, "Vibration of orthotropic elliptic plates of varying thickness resting on elastic foundation". ICIAM-2003, ICL 09-J-051, 2003.

[7] A. W. Leissa, "Vibration of plates". Washington: Office of Technology Utilization, NASA; NASA SP-160, 1969.

[8] A. W. Leissa, "Recent research in plate vibrations: Classical theory". The Shock and Vibration Digest, 9(10), 3-24, 1977.

[9] A. W. Leissa, "Recent research in plate vibrations, 1973-1976: Complicating effects". The Shock and Vibration Digest, 9(11), 1-35, 1977.

[10] A. W. Leissa, "Plate vibration research, 1976-1980: Classical theory". The Shock and Vibration Digest, 13(9), 11-22, 1981.

[11] A. W. Leissa, "Plate vibration research, 1976-1980: Classical theory". *The Shock and Vibration Digest*, 13(10), 19-36, 1981.

[12] A. W. Leissa, "Recent studies in plate vibrations: 1981-1985 part I, Classical theory". *The Shock and Vibration Digest*, 19(2), 11-18, 1987.

[13] A. W. Leissa, "Recent studies in plate vibrations: 1981-1985 part II, Complicating effects". *The Shock and Vibration Digest*, 19(3), 10-24, 1987.

[14] A. W. Leissa, "Vibration of Plates. *Acoustic Society of America*". Sewickley, 1993.

[15] B. Singh and S. Chakraverty, "Transverse vibration of circular and elliptic plates with variable thickness". *Indian. J. of pure. Appl. Math.*, 22(9) 787-803, 1991.

[16] B. Singh and S. Chakraverty, "Transverse vibration of completely free elliptic and circular pates using orthogonal polynomials in Rayleigh- Ritz method". *Int. J. Mech. Sci.*, 33 741-751, 1991.

[17] B. Singh and S. Chakravery, "Transverse vibration of circular and elliptic plates with quadratically varying thickness". *Applied Mathematical Model,* 16, 269-274, 1992.

[18] B. Singh and S. Chakraverty, "Transverse vibration of completely free elliptic and circular plates using orthogonal polynomials in the Rayleigh-Ritz method". *International Journal of mechanical Sciences*, 33, 741-751, 1991.

[19] B. Singh and S. Chakraverty, "On the use of orthogonal polynomials in the Rayleigh-Ritz method for the study of transverse vibration of elliptic plates". *Computer and Structure*, 43(3), 439-443, 1992.

[20] B. Singh and S. Chakraverty, "Transverse vibration of simply supported elliptical and circular plates using boundary characteristic orthogonal polynomials in the two variables". *Journal of Sound and Vibration*, 152(1), 149-156, 1992.

[21] B. Singh and S. Chakraverty, "Transverse vibration analysis of annular circular and elliptic plates using the characteristic orthogonal polynomials in two dimensions". *Journal of Sound and Vibration*, 162(3), 537-546, 1993.

[22] B. Singh and S. Chakraverty, "Use of characteristic orthogonal polynomials in two dimensions for transverse vibration of elliptic and circular plates with variable thickness". *Journal of Sound and Vibration*, 173(3), 289-299, 1994.

[23] K. Y. Lam, K. M. Liew and S. T. Chow, "Use of two-dimensional orthogonal polynomials for vibration analysis of circular and elliptical plates". Journal of Sound and Vibration, 154(2), 261-269, 1992.

[24] C. Rajalingham and R. B. Bhat, "Axisymmetric vibration of circular plates and its analog in elliptical plates using characteristic orthogonal polynomials". *Journal of Sound and Vibration*, 161(1), 109-118, 1993.

[25] C. Rajalingham, R. B. Bhat and G. D. Xistris, "Vibration of clamped elliptic plates using exact circular plates mode as shape function in Rayleigh-Ritz method". *Int. J. of Mech. Sci.*, 36, 231-246, 1994.

[26] K. M. Liew and K. Y. Lam, "On the use of 2-D orthogonal polynomials in the Rayleigh-Ritz method for flexural vibration of annular sector plates of arbitrary shape". *International Journal of Mechanical Science*, 35(2), 129-139, 1993.

[27] D. Bucco, J. Majumdar and G. Sved, "Vibration analysis of plates of arbitrary shape - a new approach". Journal of Sound and Vibration, 67(2), 253-262, 1979.

[28] A.P. Bhattacharya and K.N. Bhowmic, "Free vibration of a sectorial plate". Journal of Sound and Vibration, 41, 503-505, 1975.