THE SERIOUS OMISSION OF COMPARISONS IN ARISTOTLE'S LAWS OF THOUGHT

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ESSAY

Comparisons are the bedrock of all human thinking no matter where it is done on this earth in every culture. We compare things with one another and with themselves through memory to know if they are what we are looking for. We need memory and time to do that regardless of how brief and spontaneous a comparison may be. Thus thought is a dynamic process in its minutest form and needs time to be connected and unfolded to draw meaning as we compare it to other thoughts.

Saaty lecturing at ISAHP2011 in Sorrento, Italy.

The original three laws of thought due to Aristotle are fundamental axiomatic rules on which Western Philosophy rational discourse through reasoning has been primarily based for more than two millennia. They have been used in our logic, mathematics and scientific thinking, and they implicitly assume an individual can compare a thing against his memory of a thing to determine if what he sees or perceives is or is not the same as his memory of it; if not, then it is something else, or if it does not have the properties it is remembered to have, then it is not the same thing. Here we will discuss the three laws and show that a fourth law should be added, the law of comparisons, which in fact should precede the other three. Being able to compare is a talent we have that is necessary for us to recognize and make distinctions among things; and it must come first because without it we would not know how to use the other three laws.

1. The law of identity

A necessary and sufficient condition that two entities x and y be identical (or the same) is that they share the same properties, and thus can be said to be indiscernible.

According to Leibnitz the law of identity is the first primitive truth of reason which is affirmative. But absolute identity has no meaning unless one thinks of relative identity, which means relative to other things. This involves the even more basic assumption that things are implicitly comparable. We need to compare x and y to see if they are identical or not. There is no other way to know.

2. The law of non-contradiction

Either x or not x is true and there is no other possibility. This means that opposite assertions cannot be true at the same time.

Here again one must compare x to not x to determine which case one is dealing with.

3. The law of the excluded middle

An entity *x* has or does not have a property P. Conversely, a property P is either possessed by an entity x or it is not possessed by x.

Here x is compared with itself having the property P and not having the property P to determine the case.

The significant conclusion is that to know if anything is identical with itself, it is necessary to compare it with itself and with other things with respect to a certain property P and also compare P with other properties as to which is possessed by x and which is not. Thus comparisons are intertwined with all three laws of thought. All Meaning derives from Comparison in some way.

In light of this we suggest that the zeroth law of thought, one of the basic definitions in the Analytic Hierarchy Process, should come first.

0. The law of comparisons

Let $\mathfrak{A}$ be a finite set of n elements called alternatives. Let $\mathfrak{C}$ be a set of properties or attributes with respect to which elements in $\mathfrak{A}$ are compared. A property is a feature that an object or individual possesses even if we are ignorant of this fact, whereas an attribute is a feature we assign to some object: it is a concept. Here we assume that properties and attributes are interchangeable, and we generally refer to them as criteria. A criterion is a primitive concept.

When two objects or elements in $\mathfrak{A}$ are compared according to a criterion C in $\mathfrak{C}$, we say that we are performing binary comparisons. Let $>_C$ be a binary relation on $\mathfrak{A}$ representing "more preferred than" or "dominates" with respect to a criterion C in $\mathfrak{C}$. Let $\sim_C$ be the binary relation "indifferent to" with respect to a criterion C in $\mathfrak{C}$. Hence, given two elements, $A_i, A_j \in \mathfrak{A}$, either $A_i >_C A_j$ or $A_j >_C A_i$ or $A_i \sim_C A_j$ for all $C \in \mathfrak{C}$. We use $A_i \geq_C A_j$ to indicate either more preferred or indifferent. A given family of binary relations with respect to a criterion C in $\mathfrak{C}$ is a primitive concept. We shall use this relation to derive the notion of priority or importance both with respect to one criterion and also with respect to several.

In passing we note that induction has also been proposed as another law of thought.

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http://dx.doi.org/10.13033/ijahp.v3i2.124