INTERVAL UNCERTAINTY OF ESTIMATES AND JUDGMENTS OF SUBJECT IN DECISION MAKING IN MULTI-CRITERIA PROBLEMS

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ABSTRACT

In this article, we propose a method of decision making in multi-criteria problems given an interval uncertainty of the estimates given by the subject in reference to the importance of one criterion over another and various alternatives for each criterion. The method is the development of the deterministic process of the Analytic Hierarchy Process, which uses deterministic point estimates of the importance of criteria and alternatives for each criterion for decision making in multi-criteria problems. While in the standard Analytic Hierarchy Process the values of global priorities corresponding to different alternatives are deterministic and unambiguous, in the interval process developed in this article the global priorities and alternatives are interval and uncertain. If in the standard deterministic Analytic Hierarchy Process the best alternative is selected by the maximum value of the global priority, then, to select the best interval alternative, here we introduce a criterion corresponding to the maximum values of the lower and upper boundaries of the intervals of global priorities of the alternatives. The application of the proposed method is demonstrated by a specific example.

Keywords: interval; uncertainty; estimates; decision making; analytic hierarchy process

1. Introduction

A human being cannot give accurate estimates of the absolute values of any variables or values of relative superiority (the importance, significance, intensity) of one value over another. Therefore, personal subjective estimates are usually of an uncertain interval nature, i.e., are within certain intervals. In other words, instead of an unambiguous and deterministic estimate, the subject can only indicate the boundaries of the intervals within which he believes there will be the true values of the estimated values, and the width of the estimated interval can be quite significant. By indicating the interval within which the expected value of the estimate may be, the subject implicitly assumes that it is equally likely that the true value of the estimate may be anywhere within the interval. This means that the value estimated by the subject essentially is a random variable uniformly distributed within the interval. We will refer to such an uncertainty interval as stochastic and the estimate itself interval stochastic.

Because the subject's judgments are usually based on his subjective interval stochastic estimates, the subject's conclusions and decisions in connection with one-criterion and

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multi-criteria problems will also be uncertain interval stochastic. Indeed, in making decisions in multi-criteria problems, the subject makes interval stochastic estimates in reference to various criteria and the values of relative importance (significance) of intercomparable criteria and alternatives. Therefore, the final or global criteria specific for various alternatives will not be unambiguous and deterministic either, but uncertain interval stochastic. The specified circumstance makes it significantly difficult to choose the best alternative, as the global criteria intervals may overlap and include one another, forming common areas within which various alternatives become equivalent. Therefore, to provide the subject with the opportunity to make the final selection of a compromise alternative in a multi-criteria problem given the uncertainty of subjective estimates, the relevant criteria as discussed below need to be introduced.

To make multi-criteria decisions given the uncertainty, three approaches are provisionally applied in the existing literature. In the first approach the uncertainty is simply ignored and it is assumed that all assessments of the criteria, alternatives and the inter-comparisons thereof, which are subsequently used to make decisions, are unambiguous and precisely definable, i.e., deterministic. One of the deterministic methods of decision making is the Analytic Hierarchy Process (AHP) (Saaty, 2001). Under the second approach, the interval uncertainty is specifically admissible; however, it is being reduced to complete certainty, i.e., again back to the deterministic case. For this purpose, various mathematical techniques aimed at the reduction of the uncertainty intervals to certain unambiguous point estimates are applied so that to subsequently re-apply the decision making methods applied under the first deterministic approach (Podinovski, 2007; Salo, A.A., and Hämäläinen R.P., 1995; Wang Y.-M. et al., 2005). In the third approach, the interval uncertainty is considered by fuzzy theory or stochastic formulation for the AHP (Haines, L.M., 1998; Deng, H., 1999; Lipovetsky, S., and Tishler, A., 1999; Mikhailov, L., 2004; Eskandari, H., and Rabelo, L., 2007).

The article proposes the method for solving multi-criteria problems given the uncertainty of the estimates given by the subject, which are interval stochastic. The Analytic Hierarchy Process, which is initially deterministic and uses unambiguous estimates of relative importance of both the criteria and alternatives for the criteria, is the basis of multi-criteria optimization(Saaty, 2001). To support decision making in multi-criteria problems given the interval uncertainty of the estimates, the article introduces a criterion which corresponds to the maximum values of the upper and lower boundaries of the intervals of global priorities. The application of the interval stochastic Analytic Hierarchy Process developed in this article is considered using a specific example.

2. Interval stochastic Analytic Hierarchy Process

The normal Analytical Hierarchy Process AHP is applied for decision-making in multi-criteria problems, and it is deterministic (Saaty, 2001). In other words, it is assumed that all estimates of relative importance of the criteria and alternatives for the criteria are deterministic and unambiguously defined. The Analytic Hierarchy Process is based on the construction of deterministic pairwise comparison matrices for the criteria and the alternatives with respect to each criterion, followed by determination of the eigenvectors corresponding to the maximum eigenvalue of these matrices. The elements of the eigenvectors pairwise comparison matrices are relative importance coefficients of the criteria and alternatives assessed in terms of each criterion. The Analytic Hierarchy Process (AHP) is finalized by the calculation of global priorities for each alternative and selection of the best of them corresponding to the maximum value of a global priority. The AHP assumes that the subject is capable of carrying out pairwise comparisons with sufficient accuracy of any two factors (both quantitative and qualitative), and furthermore also of indicating the precise value of superiority of one factor over another using the fundamental scale (Saaty, 2001).

Meanwhile, the subject in reality is not capable of providing accurate estimates. He can only conclude that in his opinion the degree of superiority of one factor over another is, for example, *somewhere between* the weak degree of superiority (2 points based on the fundamental scale) and slightly above the average degree (4 points) of superiority. Therefore, subjective estimates of the degree of superiority of one factor over another, as well as the values of any variables/factors at all are fundamentally blurred and uncertain. Instead of the exact value of some variable, the subject can only specify a subjective interval of its possible variation, implicitly implying that within this interval the estimated value can equally likely take any value. It should be noted that the subject basically has no reason to believe that his subjective interval stochastic estimate has some other probability density (e.g., triangular, normal, log-normal, etc.) different from the uniform probability density, and the judgments of the subject trying to assess a priori the type of probability density of estimates tend to be wrong (Kahneman, D. et al., 2001).

Thus, subjective estimates are interval stochastic. Therefore, the pairwise comparison matrices, their eigenvalues and eigenvectors, the coefficients of relative importance of criteria and alternatives, as well as the global priorities of alternatives in the AHP are also interval stochastic. Because of this, the final decision will be determined by not strictly defined deterministic estimates of global priorities, as is the case in the deterministic AHP, but by their interval stochastic estimates.

The mathematical model $\{X, F, \succ_X, a(\omega)\}$ of multi-criteria decision making given the interval stochastic uncertainty of estimates is considered, where(Madera, A.G., 2010):

$X = (x_1, x_2, \ldots, x_n)$ – multitude of possible decisions,
$F = (f_1, f_2, \ldots, f_m)$ – vector criteria,
$\succ_X$ – the ratio of the preference given on the set of possible solutions X,
$a(\omega)$ – subjective interval stochastic estimate of the value, which is firstly interval $a(\omega) \in [\underline{a}, \overline{a}]$, where $\underline{a}$ and $\overline{a}$ – the lowest and highest boundaries of the interval, and secondly a stochastic variable uniformly distributed in the interval $[\underline{a}, \overline{a}]$ with the probability density $p(a) = 1/\Delta$, $a \in [\underline{a}, \overline{a}]$ and $p(a) = 0$, $a \notin [\underline{a}, \overline{a}]$, where $\Delta = \overline{a} - \underline{a}$ – the width of the interval, $\omega \in \Omega$ – elementary events in the space of elementary events Ω (Ross, S., 1993).

Decision making based on the interval stochastic AHP method developed in the article is carried out according to the algorithm:

(1) the subject specifies the intervals of estimates for the relative importance of the criteria relative to each other and the alternatives for each criterion. It should be noted

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that the interval estimates of the values of relative importance of the criteria and each alternative by the criteria are statistically independent (Madera, A.G., 2014);

(2) interval stochastic pairwise comparison matrices for the criteria and for each alternative of all criteria are constructed;

(3) statistical measures of interval stochastic eigenvectors: mathematical expectations (ME), the variance (VAR) and standard deviations (SD) are determined for each constructed interval stochastic pairwise comparison matrix. In the AHP method, the elements of eigenvectors of pairwise comparison matrices for criteria and alternatives for the criteria are also the coefficients of relative importance of the criteria and alternatives for each criterion;

(4) statistical measures of global priorities for each alternative are calculated based on the estimated values of statistical measures of eigenvectors of pairwise comparison matrices,

(5) the boundaries of the intervals of global priorities are determined, as ME ± ε·SD, where ε determines the width of the interval global priority for this confidence probability P_ε;

(6) the final decision is made based on the location of the boundaries of the intervals of global priorities of alternatives.

Let's consider the determination of the statistical measures of interval stochastic variables used for the implementation of steps (1) – (6) of the algorithm.

2.1 Determining statistical measures for interval stochastic eigenvector of interval stochastic pairwise comparison matrix

The main characteristics defined in the deterministic AHP are eigenvectors of pairwise comparison matrices compiled for both the criteria and the alternatives with respect to each criterion. The elements of eigenvectors are priorities, or coefficients of importance, criteria and alternatives in terms of each criterion.

In the interval stochastic AHP discussed in this article, the eigenvectors are interval stochastic, and methods to determine their statistical measures are required to define their characteristics. The interval stochastic pairwise comparison matrix for the considered mathematical model is as follows:

$$A(\omega) = \begin{pmatrix} 1 & a_{12}(\omega) & \cdots & a_{1n}(\omega) \\ a_{21}(\omega) & 1 & \cdots & a_{2n}(\omega) \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1}(\omega) & a_{n2}(\omega) & \cdots & 1 \end{pmatrix}, \tag{1}$$

where $a_{ij}(\omega) \in [\underline{a}_{ij}, \overline{a}_{ij}]$ – interval estimate of the relative importance of i factor over j factor with their pairwise comparison carried out by the fundamental scale (Saaty, 2001); $a_{ji}(\omega) = 1/a_{ij}(\omega)$ (i, j = 1, 2,…, n) for each implementation of $\omega \in \Omega$, and the variation interval boundaries $a_{ji}(\omega)$ are equal to $[1/\overline{a}_{ij}, 1/\underline{a}_{ij}]$.

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In the deterministic AHP, the normalized elements w_i of the eigenvector $W = (w_1, w_2, \ldots, w_n)$ the pairwise comparison matrices A can be calculated with sufficient accuracy by the formula (Saaty, 2001):

$$w_i = \left(\prod_{j=1}^{n} a_{ij}\right)^{\frac{1}{n}} \Big/ \sum_{i=1}^{n} \left(\prod_{j=1}^{n} a_{ij}\right)^{\frac{1}{n}}. \qquad (2)$$

For the considered interval stochastic AHP, the formulas (2) remain valid for each implementation of $\omega \in \Omega$, so the random elements $w_i(\omega)$ of the random eigenvector $W(\omega) = (w_1(\omega), w_2(\omega), \ldots, w_n(\omega))$ in the pairwise comparison matrix $A(\omega)$ (1) are calculated based on the following formula:

$$w_i(\omega) = \left(\prod_{j=1}^{n} a_{ij}^{1/n}(\omega)\right) \Big/ \sum_{i=1}^{n} \left(\prod_{j=1}^{n} E\{a_{ij}^{1/n}(\omega)\}\right), \qquad (3)$$

where $E\{a_{ij}^{1/n}(\omega)\}$ – mathematical expectation of random variable $a_{ij}^{1/n}(\omega)$.

Let's find formulas for determining the statistical measures (ME, VAR and SD) of the elements of the interval stochastic eigenvector $W(\omega)$. Considering that in each i row of the matrix $A(\omega)$ (1) random values $a_{i1}(\omega)$, $a_{i2}(\omega)$, …, $a_{in}(\omega)$ are statistically independent we obtain according to (3) that values $\mathrm{ME}_{wi} = E\{w_i(\omega)\}$, $\mathrm{VAR}_{wi} = E\{(w_i(\omega) - E\{w_i(\omega)\})^2\}$ and $\mathrm{SD}_{wi} = (\mathrm{VAR}_{wi})^{1/2}$ of the stochastic elements $w_i(\omega)$, $i = 1, 2, \ldots, n$, the eigenvector $W(\omega)$ may be calculated based on the following formulas:

$$\mathrm{ME}_{wi} = \left(\prod_{j=1}^{n} E\{a_{ij}^{1/n}(\omega)\}\right) \Big/ \sum_{i=1}^{n} \left(\prod_{j=1}^{n} E\{a_{ij}^{1/n}(\omega)\}\right), \qquad (4)$$

$$\mathrm{VAR}_{wi} = E\{w_i^2(\omega)\} - \mathrm{ME}_{wi}^2, \qquad (5)$$

where

$$E\{w_i^2(\omega)\} = \left(\prod_{j=1}^{n} E\{a_{ij}^{2/n}(\omega)\}\right) \Big/ \left(\sum_{i=1}^{n} \left(\prod_{j=1}^{n} E\{a_{ij}^{1/n}(\omega)\}\right)\right)^2. \qquad (6)$$

In the formulas (4) – (6), there may be the mathematical expectations of various degrees of variables $a_{ij}(\omega)$, namely, $a_{ij}^{1/n}(\omega)$, $a_{ij}^{-1/n}(\omega)$, $a_{ij}^{2/n}(\omega)$, $a_{ij}^{-2/n}(\omega)$. Since the probability densities of interval stochastic variables $a_{ij}(\omega)$ are uniform, the corresponding values of the mathematical expectations will be determined only by the lower and upper boundaries of the intervals $[\underline{a}_{ij}, \overline{a}_{ij}]$, namely:

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$$E\{\, a_{ij}^{\pm 1/n}(\omega) \,\} = \left((\bar{a}_{ij})^{1 \pm 1/n} - (\underline{a}_{ij})^{1 \pm 1/n} \right) \Big/ \Delta_{ij} \left(1 \pm 1/n \right), \tag{7}$$

$$E\{\, a_{ij}^{\pm 2/n}(\omega) \,\} = \left((\bar{a}_{ij})^{1 \pm 2/n} - (\underline{a}_{ij})^{1 \pm 2/n} \right) \Big/ \Delta_{ij} \left(1 \pm 2/n \right). \tag{8}$$

Having calculated the values (7) and (8) and plugging them into (4) – (6), we will obtain the formulas for determining the values of ME_{wi}, VAR_{wi} and SD_{wi} of the interval stochastic elements $w_i(\omega)$, $i = 1, 2,\ldots, n$ of the eigenvector $W(\omega)$ of the interval stochastic matrix $A(\omega)$.

2.2 Determining statistical measures for vectors of criteria priorities and vectors of priorities of alternatives for each criterion

The formulas for calculating the values of ME, VAR and SD of the interval stochastic eigenvector $W(\omega)$ of the interval stochastic pairwise comparison matrix $A(\omega)$ were obtained above (Section2.1).

In the deterministic AHP, a matrix of pairwise comparisons (A_F) for the criteria $F = (f_1, f_2, \ldots, f_m)$ and m of the pairwise comparisons matrices (A_X^F) for the alternatives $X = (x_1, x_2, \ldots, x_n)$ relative to each criterion from the vector F is compiled. Then, for each pairwise comparison matrix, their eigenvectors are determined, which are the vectors of priorities of the criteria $w_F = (w_{f\,1}, w_{f\,2}, \ldots, w_{f\,m})$ and the vectors of priorities of the alternatives $x_1, x_2, \ldots, x_n$relative to each criterion f_k ($k = 1, 2, \ldots, m$), i.e., w_X^{fk} = ($w_{x1}^{fk}, w_{x2}^{fk}, \ldots, w_{xn}^{fk}$).

In the interval stochastic AHP, the pairwise comparison matrices are interval stochastic, and their eigenvectors $w_F = w_F(\omega)$ and $w_X^{fk} = w_X^{fk}(\omega)$ – interval stochastic. Therefore, for the complete characterization of random vectors, one needs to have their statistical measures (ME, VAR and SD = $(\mathrm{VAR})^{1/2}$) for:

– the criteria

$$\mathrm{ME}_{w\,f\,k} = E\{w_{f\,k}(\omega)\},\ \mathrm{VAR}_{w\,f\,k} = E\{(w_{f\,k}(\omega) - E\{w_{f\,k}(\omega)\})^2\};$$

– alternatives for each criterion f_k ($k = 1, 2, \ldots, m$)

$$\mathrm{ME}_{wxi}^{fk} = E\{\, w_{xi}^{fk}(\omega)\},\ \mathrm{VAR}_{wxi}^{fk} = E\{(\, w_{xi}^{fk}(\omega) - E\{\, w_{xi}^{fk}(\omega)\})^2\},$$

which are calculated according to the formulas of the type (4) – (8).

2.3 Determining statistical measures for the vector of global priorities of alternatives

In the deterministic AHP, the values of global priorities (GPs) of alternatives are used for the final selection of the best decision in a multi-criteria problem. The GPs are calculated as the sum of the multiplications of the priorities of this alternative with respect to each criterion by the priorities of relevant criteria.

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In the interval stochastic AHP, the priorities of both the criteria and alternatives for the criteria are interval stochastic values. Therefore, the GPs of alternatives will also be interval stochastic G_{xi} (ω), which for each alternative x_i (i = 1, 2,…,n) will be determined by the following formula:

$$G_{xi}(\omega) = w_{xi}^{f1}(\omega)\cdot w_{f1}(\omega) + w_{xi}^{f2}(\omega)\cdot w_{f2}(\omega) + \cdots + w_{xi}^{fm}(\omega)\cdot w_{fm}(\omega), \qquad (9)$$

where $w_{xi}^{f1}(\omega)$, $w_{xi}^{f2}(\omega)$, … , $w_{xi}^{fm}(\omega)$ – interval stochastic values of the priorities of the alternative xi relative to the criteria f_1, f_2, … , f_m; $w_{f1}(\omega)$; $w_{f2}(\omega)$, … , $w_{fm}(\omega)$ – interval stochastic values of the criteria priorities. All priorities are statistically independent, pairwise and in combination.

Since the stochastic values of the priorities of the alternatives and criteria in (9) are mutually independent, for the random variable G_{xi} (ω) the values of the statistical measures ME_{Gi}, VAR_{Gi} and $\mathrm{SD}_{Gi} = (\mathrm{VAR}_{Gi})^{1/2}$ will be determined by the following formulas:

$$\mathrm{ME}_{Gi} = \sum_{k=1}^{m} E\{w_{xi}^{fk}(\omega)\}\cdot E\{w_{fk}(\omega)\}, \qquad (10)$$

$$\mathrm{VAR}_{Gi} = \sum_{k=1}^{m}\{\mathrm{VAR}_{wxi}^{fk}\cdot \mathrm{VAR}_{wfk} + \mathrm{VAR}_{wxi}^{fk}\cdot \mathrm{ME}_{wfk}^{2} + \mathrm{VAR}_{wfk}\cdot(\mathrm{ME}_{wxi}^{fk})^2\}. \qquad (11)$$

Based on the calculated values of the statistical measures (10), (11), ε-intervals are constructed, within which the GPs will be with the confidence level of P_ε, namely $G_{xi}(\omega) \in [\underline{G}_{xi}, \overline{G}_{xi}]$, for the GPs of alternatives. The lower $\underline{G}_{xi}$ and upper $\overline{G}_{xi}$ boundaries of the GP-intervals of alternatives are equal to:

$$\underline{G}_{xi} = \mathrm{ME}_{Gi} - \varepsilon\cdot\mathrm{SD}_{Gi},\ \overline{G}_{xi} = \mathrm{ME}_{Gi} + \varepsilon\cdot\mathrm{SD}_{Gi}. \qquad (12)$$

The values G_{xi} (ω) for the alternative xi will be within the interval $\mathrm{ME}_{Gi} - \varepsilon\cdot\mathrm{SD}_{Gi} \le G_{xi}(\omega) \le \mathrm{ME}_{Gi} + \varepsilon\cdot\mathrm{SD}_{Gi}$. The number ε should be selected as equal to 3 when the probability of finding the GP value outside the interval $[\mathrm{ME}_{Gi} \pm \varepsilon\cdot\mathrm{SD}_{Gi}]$ will not exceed 1/9.

2.4 Making the best decisions in the interval stochastic AHP

The intervals estimating the GPs of various alternatives may have varied mutual arrangements (Fig. 1). In general, this leads to a multitude of selected decisions which may contain interval decisions incomparable with each other, which considerably complicates the final selection of the best alternative.

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In the considered interval stochastic AHP, the obtained intervals of any possible GP values are the basis for determining the best compromise alternative, which is selected based on the relative arrangement of the lower and upper boundaries of the GP intervals for various alternatives. The article assumes that it is reasonable to select such an interval decision, for which the lower and upper boundaries of the GP interval have maximum values of all other boundaries of the GP intervals of alternatives. On Fig. 1*a*, such is the GP interval corresponding to the alternative *x*', because its lower and upper boundaries have maximum values (shifted to the right, Fig. 1*a*). Accordingly, an alternative (*x*'', Fig. 1*a*) for which the GP interval boundaries are minimal (shifted to the left, Fig. 1*a*) will be the worst one. This is also true for those occasions when either the upper (Fig. 1*b*) or the lower (Fig. 1*c*) boundaries match in two inter-compared GP-intervals. Thus, if the upper boundaries of two GP intervals match, the decision with the lower boundary value which is higher (*x*', Fig. 1*b*) is selected, and, if the lower GP boundaries match, the best decision will the one with the highest value of the upper boundary (*x*'', Fig. 1*c*). We should especially note the case when the lengths of the GP-intervals are equal and their lower and upper boundaries match in two compared decisions. In this case, both decisions are equivalent by all the criteria, and the adoption of one of them is determined solely by the opinion and preferences of the subject making the decision.

Cases of "nested" (Fig. 1*d*) GP-intervals $[\underline{G}_{x'}, \overline{G}_{x'}] \subset [\underline{G}_{x''}, \overline{G}_{x''}]$ when the length of the GP-interval $[\underline{G}_{x'}, \overline{G}_{x'}]$ of one decision (*x*') is less than the length of the GP-interval $[\underline{G}_{x''}, \overline{G}_{x''}]$ of another decision (*x*'') are possible as well, and there are disparities between the lower and upper boundaries: $\underline{G}_{x''} < \underline{G}_{x'} < \overline{G}_{x'} < \overline{G}_{x''}$. With the nested GP-

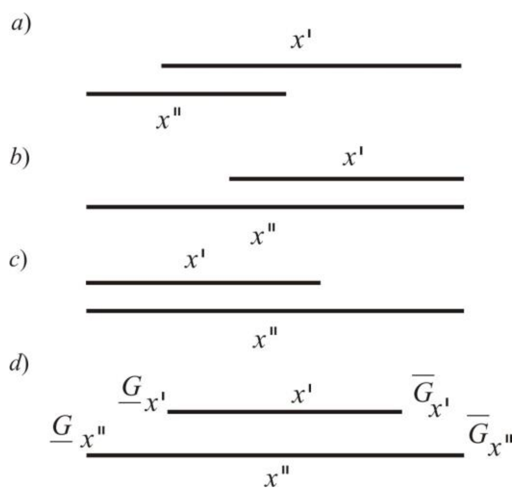

Figure 1. Variants of relative arrangement of GP-intervals corresponding to two compared alternatives *x*' and *x*''

intervals and the boundaries which do not match, the selection of the best alternative is difficult, which is due to the fact that two alternatives *x*' and *x*'' with the nested GP-intervals are incomparable with each other. Indeed (Fig. 1*d*), the GP values of both alternatives *x*' and *x*'' from the nested GP-interval $[\underline{G}_{x'}, \overline{G}_{x'}]$ completely match, making the alternatives *x*' and *x*'' indistinguishable from each other in the impacts. At the same

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time, the alternative x'' with large GP-interval $[\underline{G}_{x''}, \overline{G}_{x''}]$ has the GP values which, on the one hand, *are higher* than the upper boundary $\overline{G}_{x'}$ of the nested GP-interval $[\underline{G}_{x'}, \overline{G}_{x'}]$, making the alternative x'' better than the alternative x', and, on the other hand, less than the lower boundary $\underline{G}_{x'}$ of the nested GP-interval $[\underline{G}_{x'}, \overline{G}_{x'}]$, making the alternative x'' worse than the alternative x'. For the final selection of the best alternative in the considered case, subjective interval estimates and judgments provided by this subject making the decision to clarify them and narrow uncertainty intervals should be additionally analyzed. The practical calculations show that it is usually possible to reduce alternatives characterized by uncertain GP-intervals to a comparable type in which the best alternative will have the maximal lower and upper GP-interval boundaries.

3. Example of application of interval stochastic AHP

Let's consider the multi-criteria problem of selecting the best place in the region (of the three proposed x_1, x_2, x_3) for the placement of a publicly significant object assessed in terms of the infrastructural maturity (the criterion f_1), the solvent demand of the population (the criterion f_2), the competitive environment density (the criterion f_3), the estimated cost of future construction (the criterion f_4).

The subject cannot know apriori the exact values of the factors of a multi-criteria problem at the time of the decision, and the subject can only roughly estimate the boundaries of the intervals, within which, in his opinion, certain factors can assume values. Moreover, the possible values of the factors in their variation intervals can be located at any point with equal probability. Due to the interval stochastic nature of the estimates of the subject, the pairwise comparison matrices of relative importance of the criteria and alternatives in terms of each criterion are interval stochastic matrices, each element of which is an interval stochastic variable uniformly distributed within its variation interval.

Let's assume that the interval stochastic pairwise comparison matrices in the this example are:

– for the criteria f_1, f_2, f_3, f_4

$$A_F = \begin{pmatrix} 1 & [2, 3] & [4, 6] & [5, 7] \\ [1/3, 1/2] & 1 & [3, 5] & [6, 9] \\ [1/6, 1/4] & [1/5, 1/3] & 1 & [5, 7] \\ [1/7, 1/5] & [1/9, 1/6] & [1/7, 1/5] & 1 \end{pmatrix},$$

– for the alternatives $x1$, $x2$, $x3$ relative to each criterion f_1, f_2, f_3, f_4

$$A_X^{f1} = \begin{pmatrix} 1 & [3, 5] & [5, 7] \\ [1/5, 1/3] & 1 & [4, 5] \\ [1/7, 1/5] & [1/5, 1/4] & 1 \end{pmatrix}, \ A_X^{f2} = \begin{pmatrix} 1 & [1/6, 1/3] & [1/8, 1/6] \\ [3, 6] & 1 & [1/4, 1/2] \\ [6, 8] & [2, 4] & 1 \end{pmatrix},$$

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$$A_X^{f3} = \begin{pmatrix} 1 & [1/6, 1/4] & [2, 4] \\ [4, 6] & 1 & [7, 9] \\ [1/4, 1/2] & [1/9, 1/7] & 1 \end{pmatrix}, \; A_X^{f4} = \begin{pmatrix} 1 & [1/8, 1/6] & [1/7, 1/5] \\ [6, 8] & 1 & [2, 3] \\ [5, 7] & [1/3, 1/2] & 1 \end{pmatrix}.$$

The calculations of the statistical measures of interval stochastic eigenvectors for interval stochastic pairwise comparison matrices were carried out according to the formulas obtained above (Sections2.1 – 2.3). The results of the calculations of the statistical measures ME and SD for the coefficients of relative importance of the criteria and the alternatives relative to the criteria are given in Table 1, and for the GPs of alternatives in

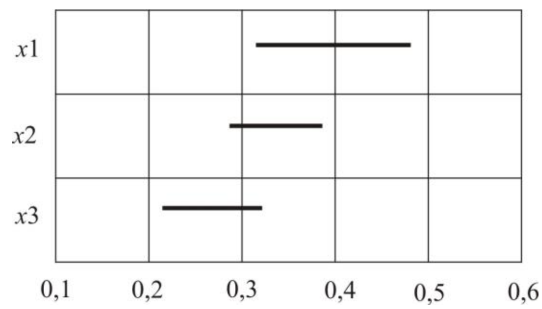

Figure 2. Boundaries of the GP-intervals of alternatives obtained by the interval stochastic AHP method developed in this article

Table 2. Table 2 also presents the calculated variation GP-intervals for various alternatives.

Table 1
Statistical measures ME and SD of the coefficients of relative importance of criteria and alternatives for each criterion

Criteria	f_1		f_2		f_3		f_4	
	ME	SD	ME	SD	ME	SD	ME	SD
	0,507	0,024	0,321	0,018	0,129	0,007	0,043	0,002
Alternatives	ME	SD	ME	SD	ME	SD	ME	SD
x_1	0,676	0,039	0,0759	0,006	0,183	0,014	0,068	0,003
x_2	0,246	0,013	0,273	0,025	0,741	0,034	0,614	0,029
x_3	0,079	0,003	0,651	0,046	0,076	0,005	0,318	0,016

Table 2
Statistical measures ME and SD and variation GP-intervals of alternatives

Global priorities of alternatives	Statistical measures of global priorities		Variation GP-intervals of alternatives
	ME	SD	
G_{x1}	0,394	0,026	[0.315, 0.472]
G_{x2}	0,334	0,015	[0.290, 0.378]
G_{x3}	0,273	0,019	[0.216, 0.330]

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Figure 2 shows the calculated boundaries of the GP-intervals of alternatives x_1, x_2, x_3. The resulting intervals of possible GP values (Table 2) serve as the basis for selecting the best compromise alternative (Section2.4). In this example, the best alternative is $x1$ (Fig. 2) since the lower and upper boundaries of the interval of its GPs have maximal values of all the other GP-intervals of alternatives. The worst alternative will be the one for which the GP-interval boundaries are minimal (the alternative x_3, Fig. 2).

4. Conclusion

The interval stochastic AHP method developed in the article is an extension of the deterministic AHP, also applying to the interval stochastic uncertainty, which is closer in nature to subjective evaluation and decision-making. With the obtained formulas, the statistical measures (mathematical expectations ME, variances VAR and standard deviations SD) for both interval stochastic eigenvectors of pairwise comparison matrices and intervals of global priorities GPs of various alternatives can be determined. Based on the determined values of statistical measures, the intervals of possible GPs are measured, namely: $\mathrm{ME} - \varepsilon \cdot \mathrm{SD} \leq G(\omega) \leq \mathrm{ME} + \varepsilon \cdot \mathrm{SD}$ corresponding to various alternatives. The best compromise decision corresponds to the alternative with such a GP-interval whose lower and upper boundaries are the highest among the boundaries of all other GP-intervals of alternatives. This can always be achieved through interactions with the subject making decisions, by specifying his subjective estimates. The interval stochastic AHP method proposed in this article allows one to make the best compromise decision in multi-criteria problems given the interval stochastic uncertainty of subjective estimates, which reflects the real psychology of the subject carrying out the estimation, selection and decision-making.

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