# RANKING COUNTRIES MORE RELIABLY IN THE SUMMER OLYMPICS 

Thomas L. Saaty<br>Katz Graduate School of Business<br>322 Mervis Hall, Pittsburgh, PA, USA<br>E-mail: saaty@katz.edu<br>Mujgan Sagir*<br>Eskisehir Osmangazi University<br>Eskisehir, TURKEY<br>E-mail: mujgan.sagir@gmail.com


#### Abstract

In this paper we consider the many intangible criteria that influence the outcome of the Summer Olympics by using the Analytic Network Process, and apply the ideas to evaluate the medals won and the country scores in the 2012 London Olympics. Both the categories of games and the events in each game are considered in this weighting process. Different events of the same category game could have different properties. This work shows that the current way of counting the total number of medals is not a bad way of ranking countries. With minor modifications, this systematic approach for ranking countries can be used for any Summer Olympics.


Keywords: OR in sports; country ranking; Olympic Games; Analytic Network Process; rating

## 1. Introduction

The modern Olympic Games are a major international event featuring summer and winter sports in which thousands of athletes participate in a variety of competitions. They are considered to be the world's foremost sports competitors and represent nearly 200 nations who may participate. The Games are currently held biennially, with Summer and Winter Olympic Games alternating, thus each occurring every four years. Their creation was inspired by the ancient Olympic Games which were held in Olympia, Greece for more than 1000 years from the 8th century BC to the 4th century AD.

There have been a number of studies conductedwhich have focused on the Olympic Games and other Olympic movements. For example, Andrew (2000)'s study researched why countries show different performance in Olympics. This study had three objectives with the key objective being, to examine the influences of factors affecting the Olympic performance.Two specific objectives were (i) to produce a mathematical model facilitating the prediction of the Olympic tally, and (ii) to identify the degree of factors that have influence on the Olympic performance. Bernard (2000) studied different variables in Olympic Games success investigation. On the other hand, Wade (2006)
studied the prediction of medal winners. Our motivation in this paper comes from the need for a scientific methodology to interpret the number of medals the countries have and a way to rank them.

Qualification rules for each of the Olympic sports are set by the International Sports Federations (IFs) that governs that sport's international competition. For individual sports, athletes typically qualify by attaining a certain rank in a major international event and thus gain recognition on the IF's ranking list. National Olympic committees (NOC) may enter a limited number of qualified athletes in each event.

In the ancient Olympics, no medals were awarded. First-place winners were given an olive branch to wear on their head, and second and third place winners did not receive anything. In the first modern Games held in 1896 silver and bronze medals were awarded to first and second place winners. In 1900, most winners received cups or trophies instead of medals. In the 1904 Games in St. Louis, gold replaced silver as the medal awarded for first place, followed by silver and bronze medals awarded to second and third place winners.

Nowadays, the media decides which country has won the Olympics by adding all the medals won by athletes from that country. However, this kind of practice seems selfdefeating because it assumes that all gold, silver and bronze medals should be counted as equal in merit and all games and events are assumed equally important. Nevertheless, it is only an approximate way and as it turns out, not an entirely faulty way of deciding which country is the overall winner of medals.

No methodically scientific way to deal with multicriteria ranking involving intangibles has been used to assign appropriate priority weights to each type of sport and medal won in that sport for the Olympic Games. Here we propose using the Analytic Network Process (ANP) for the measurement of intangibles, along with their dependence and feedback, to weight the criteria which we think play an important role in assigning priorities to games and events.

Our methodology consists of the following steps:

1) An ANP model is developed to assign weights to different criteria used to prioritize different games and events in each game.
2) Expert knowledge is used to define the criteria and evaluate the games and events.
3) We prioritize the significance of the three types of medals (gold, silver and bronze), weight them by the overall priorities of the events to obtain the overall priority of a medal won and add these priorities to obtain the priority rank of a country.
4) Our results do not violate intuition about the number and value of the medals and in fact take greater consideration by including the merits of the events in which they are won. Thus, we also show that the ranking of countries produces results that are reasonably close to the current results of adding all medals won but with some important exceptions.

## 2. Criteria to weight the summer Olympic Games

The priorities of different kinds of games depend on several factors. Table 1 lists the relevant basic criteria groupings or clusters and the elements in each cluster. Figure 1 represents the top level network of the Analytic Network Process model together with the criteria.

Table 1
Criteria and subcriteria

| Cluster | Elements in Cluster |
| :--- | :--- |
| Game <br> requirements | Required physical characteristics, Required training time |
| Game other <br> factors | Risk level involved, Energy spent, Duration of the act |
| People <br> involved | Number of competitors, Strength of competitors, <br> Audience reaction, popularity |
| Living <br> environment | Effects of daily life on the game, Effects on daily life, <br> Financial resources needed |
| Natural <br> environment | Season or climate suitability, Topographic pattern, <br> Absence of pollution |
| Other factors | Sport commercialization, Technology, Political factors |

According to Table 1 and Figure 1 the main cluster of criteria is "Game requirements" which consists of "Required physical characteristics" and "Required training time". For some sports, one may need physical characteristics that deal with strength. For example, in gymnastics one needs to practice for years starting at a young age.

The second cluster of criteria is "Other factors related to the game" which includes "Risk level involved", "Energy spent" and "Duration of the act". The "Risk level" involved is a criterion in our analysis because it affects people's attitudes; some people find it more challenging and encouraging to take risks. We have also included an "Energy spent" criterion because in certain games more energy is needed. For example, wrestling requires a high amount of energy spent, and certain sports also involve higher risk as in some gymnastics events. The duration of an event is another concern because some games last for a relatively long time, and in certain cases as in the marathon, a medal for this game deserves a greater value.

A third cluster of criteria is "People involved" which consists of the "Number of competitors", the "Strength of competitors" and also "Audience reaction, popularity". Here, we think that there are some games that have greater popularity and this makes these sports more attractive to attend (e.g. football, tennis). On the other hand, to be successful in a game that has many competitors is more difficult.

The fourth cluster is "Living environment", with criteria "Effects of daily life on the game", "Effects on daily life" and "Financial resources needed". Some sports can be influenced by the daily life of the competitors; for example football players are usually careful about being involved in too large amounts of social activities and entertainment
that may affect their physical strength during the game due to lack of sleep. This is equally true of organizing daily life in a way to support success in the sport, such as being careful about ones diet. Financial resources are also important in some sports both to facilitate a player's needs and to make it possible to be choosy in meeting basic needs as desired. Tennis, skiing or ice-skating need specific professional environments and specific equipment for training which can be costly.

The fifth cluster is "Natural environment" whose criteria are "Season or climate suitability", "Topographic pattern" and "Absence of pollution". Climate and topographic patterns have important effects on pollution which is an undesirable factor particularly for events like canoeing and marathon.

The sixth cluster is "Other factors" related to political, economic and social issues. Political factors can determine whether the games would be attended by some countries. Technology affects performance and in the long run new technology can change performance in a sport very significantly.

Figure 1 gives a screen view from the SuperDecisions software with the clusters and their criteria from Table 1 along with their interconnections.


Figure 1. ANP top level model

As an illustration, Table 2 gives a view of paired comparisons related to the effect of "Risk level involved" and "Duration of the act" on the criterion "Energy spent". "Duration of the act" has two times greater influence on the "Energy spent" criterion than "Risk level" does. Table 3 presents the criteria weights. These judgements have been obtained by interviewing different experts who have been a judge or competitor in different Olympic games, and geometric mean is used.

Table 2
Comparisons between "Risk level involved" and "Duration of the act" on "Energy spent"

| Energy spent | Risk level involved | Duration of the act | Priorities |
| :---: | :---: | :---: | :---: |
| Risk level involved | 1 | $1 / 2$ | 0.3333 |
| Duration of the act | 2 | 1 | 0.6667 |

Table 3
Criteria priorities

| Criterion | Priority | Criterion | Priority |
| :--- | :--- | :--- | :--- |
| Required physical characteristics | 0,0606 | Effects of the daily <br> life on the game | 0,0935 |
| Required training time | 0,0735 | Effects on the daily <br> life | 0,0170 |
| Risk level involved | 0,0107 | Financial resources <br> needed | 0,0136 |
| Energy spent | 0,0692 | Season or climate <br> suitability | 0,0805 |
| Duration of the act | 0,0740 | Topographic pattern | 0,0921 |
| Number of competitors | 0,0539 | Absence of pollution | 0,0673 |
| Strength of competitors | 0,0735 | Sports <br> commercialization | 0,0463 |
| Audience reaction, popularity | 0,0760 | Political factors | 0,0296 |
| Technology | 0,0679 |  |  |

## 3. How to evaluate different games and different events involved in each game

Based on the previous discussion, it appears that declaring a winning country by adding all medals may not reflect the quality of the games that are won by the athletes from that country. The difference in the quality of the sports themselves is an important factor. By using the Fundamental Scale of absolute numbers of the AHP given in Table 4, one can compare the importance of different games and the importance of the events involved in each game (Saaty, 2004).

Table 4
Fundamental scale of absolute numbers

| $\begin{array}{c}\text { Intensity of } \\ \text { Importance }\end{array}$ | Definition | Explanation |
| :---: | :--- | :--- |
| 1 | Equal importance | $\begin{array}{l}\text { Two activities contribute equally to the } \\ \text { objective }\end{array}$ |
| 2 | Weak or slight | Experience and judgment slightly favor the |
| dominance of one activity over another |  |  |$]$| 3 | Moderate importance | Experience and judgment strongly favor the <br> dominance of one activity over another |
| :---: | :--- | :--- |
| 4 | Moderate plus | An activity is favored very strongly in <br> dominating over another activity; its <br> dominance may even be demonstrated in <br> practice |
| 5 | Strong importance plus <br> demonstrated <br> importance | An activity extremely domiantes another <br> activity |
| 8 | Very, very strong | Extreme |

The experts who provided the judgments were a group of four people who have taken part in sports games as a referee, a coach and/or an athlete for over 12 years. We interviewed them for several days with regard to the criteria to be considered in the evaluation of the games and their events. We prepared a questionnaire to obtain the weights for each game and event, and then used the geometric mean to aggregate their judgments into a representative judgment for the group. When an inconcistency was discovered, we discussed the possibility of changing a judgment with the relevant person in order to reduce the inconsistency to an acceptable level and be closer to consensus on that set of judgments. We also did a literature review to obtain detailed data and information related to the criteria and the games themselves. Finally, we applied our approach to rank the winning countries for the 2012 London Summer Olympics. Table 5 lists all the games for this particular Olympics.

Table 5
2012 London Summer Olympic Games

| Archery | Cycling | Gymnastics | Shooting | Triathlon |
| :--- | :--- | :--- | :--- | :--- |
| Athletics | Diving | Handball | Swimming | Volleyball |
| Badminton | Equestrian | Judo | Synchronized <br> swimming | Water polo |
| Basketball | Fencing | Modern <br> pentathlon | Table tennis | Weightlifting |
| Boxing | Field <br> hockey | Rowing | Taekwondo | Wrestling |
| Canoe | Football | Sailing | Tennis |  |

We categorized Olympic games in terms of the "Risk levels involved", "Energy spent" and "Duration of the act". Tables 6, 7 and 8 present these categories, noticing that games could fall into different categories for different criteria.

Table 6
Summer Olympic Games according to the "Risk level involved"

| HIGH | MEDIUM |  | LOW |  |
| :--- | :--- | :--- | :--- | :--- |
| Modern <br> pentathlon | Football | Boxing | Fencing | Equestrian |
| Diving | Basketball | Canoeing | Taekwondo | Volleyball |
| Weightlifting | Water <br> polo | Cycling | Badminton | Field <br> Hockey |
| Gymnastics | Sailing | Tennis | Table tennis | Judo |
| Athletics |  |  |  |  |
| Triatlon |  | Rowing | Archery | Wrestling |
|  |  |  | Shooting | Badminton |
| Swimming |  | Synchronized <br> Swimming | Handball |  |

Table 6 implies that sports like diving and weightlifting have higher risks. They are usually considered dangerous sports that may cause harmful injuries and even death. On the other hand, according to the classification of "Energy spent" in Table 7, we obtain a different grouping of sports, i.e. fencing is a game that needs less energy when compared with other games like football.

Table 7
Summer Olympic Games according to the "Energy spent"

| HIGH |  | MEDIUM |  | LOW |
| :--- | :--- | :--- | :--- | :--- |
| Weightlifting | Modern <br> pentathlon | Sailing | Badminton | Archery |
| Athletics | Basketball | Volleyball | Table tennis | Fencing |
| Swimming | Football | Equestrian | Diving | Shooting |
| Wrestling | Water <br> polo | Canoeing | Synchronized <br> Swimming | Field <br> Hockey |
| Tennis | Rowing | Handball | Cycling | Badminton |
| Gymnastics | Triatlon | Boxing | Judo |  |
|  | Taekwondo |  |  |  |

Similarly, Table 8 groups the games according to "Duration of the act". Football and volleyball have long durations while others, like Taekwondo and wrestling, generally take shorter times.

Table 8
Summer Olympic Games according to the "Duration of the act"

| HIGH |  | MEDIUM |  | LOW |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Athletics | Triatlon | Canoeing | Field Hockey | Archery | Fencing |
| Basketball | Voleyball | Gymnastic | Rowing | Boxing | Shooting |
| Cycling |  | Swimming | Sailing | Badminton | Taekwondo |
| Football |  | Handball |  | Diving | Water polo |
| Modern <br> pentathlon | Table tennis |  | Equestrian | Weightlifting |  |
| Tennis |  | Synchronized <br> Swimming |  | Judo | Wrestling |

There were 29 games in the 2012 London Olympics. In order to prioritize them we used the ANP ratings module by evaluating them one at a time. The categories "low, medium, high" or "low, medium, high, very high" were chosen for the 17 criteria with an appropriate adjustment for the number of categories in each group (3 or 4). Tables 9(a) and 9(b) show a screen view of the rating module from the Super Decisions software. Five different experts from different professions were consulted.

Table 9 (a)
Screen view of the rating module to weight the games (for the criteria 1-7)


International Journal of the Analytic Hierarchy Process

Table 9(b)
Screen view of rating module to weight the games (for the criteria 8-17)

|  |  |  | cisions Ratings |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Priorities | Totals | $\begin{aligned} & 32 \text { Strenght of com } \\ & 0.073581 \end{aligned}$ | 33 Audiences' read 0.076012 | $\begin{aligned} & \text { 41_Effects of daily } \\ & 0.093591 \end{aligned}$ | 42 Effects to the d 0.017005 | $\left\{\begin{array}{l} 52 \text { Topographic pA } \\ 0.092124 \end{array}\right.$ | 51_Season or clima 0.080547 | 61_The sport comn 0.046377 | $\begin{aligned} & 53 \text { Absence of poll } \\ & 0.067333 \end{aligned}$ | 62_Technology 0.067916 | $\left\lvert\, \begin{aligned} & 63 \text { Political factors } \\ & 0.029642 \end{aligned}\right.$ |
| A1_Archery | 0.019187 | 0.236120 | Lo | Lo | Lo | Lo | Lo | Lo | Lo | Lo | Lo | Lo |
| A2_Athletics | 0.053145 | 0.653998 | Hi | Med | Med | Hi | Med | Hi | Hi | Lo | Lo | Med |
| A3_Badminton | 0.022586 | 0.277947 | Lo | Lo | Lo | Lo | Lo | Lo | Lo | Lo | Lo | Lo |
| A4_Basketball | 0.045458 | 0.559407 | Hi | Hi | Lo | Med | Lo | Lo | Hi | Lo | Lo | Lo |
| A5_Boxing | 0.023097 | 0.284226 | Med | Med | Lo | Med | Lo | Lo | Lo | Lo | Lo | Lo |
| A6_Canoeing | 0.043378 | 0.533808 | Med | Lo | Lo | Lo | Hi | Hi | Med | Lo | Hi | Lo |
| A7_Cycling | 0.049236 | 0.605895 | Med | Hi | Lo | Lo | Med | Hi | Hi | Med | Med | Lo |
| A8_Diving | 0.021667 | 0.266627 | Lo | Med | Lo | Med | Lo | Lo | Lo | Lo | Lo | Lo |
| A9_Equestrian | 0.024044 | 0.295879 | Med | Med | Lo | Lo | Lo | Med | Lo | Med | Lo | Lo |
| A10_Fencing | 0.024625 | 0.303028 | Lo | Lo | Lo | Lo | Lo | Lo | Hi | Lo | Lo | Lo |
| A11_Field Hockey | 0.026627 | 0.327670 | Lo | Lo | Lo | Lo | Med | Med | Lo | Med | Med | Lo |
| A12_Football | 0.053561 | 0.659122 | Hi | Hi | Med | Med | Lo | Med | Hi | Lo | Lo | Med |
| A13_Gymnastics | 0.047840 | 0.588714 | Hi | Med | Lo | Hi | Lo | Lo | Med | Lo | Hi | Lo |
| A14_Handball | 0.021194 | 0.260813 | Lo | Lo | Lo | Lo | Lo | Lo | Lo | Lo | Lo | Lo |
| A15_Judo | 0.024049 | 0.295940 | Lo | Lo | Med | Med | Lo | Lo | Lo | Lo | Lo | Lo |
| A16_Modern Penth | 0.037247 | 0.458354 | Med | Lo | Med | Med | Lo | Lo | Lo | Lo | Lo | Lo |
| A17_Rowing | 0.042079 | 0.517816 | Lo | Lo | Lo | Med | Hi | Hi | Lo | Hi | Hi | Lo |
| A18_Sailing | 0.038750 | 0.476859 | Lo | Lo | Lo | Lo | Med | Hi | Hi | Med | Hi | Lo |
| A19_Shooting | 0.023009 | 0.283143 | Lo | Lo | Lo | Lo | Med | Med | Lo | Lo | Med | Lo |
| A20_Swimming | 0.046161 | 0.568055 | Hi | Hi | Med | Hi | Lo | Lo | Med | Lo | Lo | Hi |
| A21_Synchronized | 0.030647 | 0.377137 | Med | Med | Lo | Med | Lo | Lo | Lo | Lo | Lo | Med |
| A22_Table Tennis | 0.021839 | 0.268744 | Med | Lo | Lo | Lo | Lo | Lo | Lo | Lo | Lo | Lo |
| A23_Taekwondo | 0.024043 | 0.295868 | Med | Med | Lo | Med | Lo | Lo | Lo | Lo | Lo | Lo |
| A24_Tennis | 0.061200 | 0.753129 | Hi | Hi | Hi | Hi | Lo | Med | Hi | Hi | Lo | Med |
| A25 Triathlon | 0.045919 | 0.565074 | Med | Lo | Med | Hi | Med | Hi | Med | Lo | Lo | Lo |
| A.26_Volleyball | 0.037779 | 0.464908 | Very high | Hi | Lo | Med | Lo | Lo | Lo | Lo | Lo | Med |
|  | 0.028980 |  | Med | Lo | Lo | Med | Lo | Lo | Med | Lo | Lo | Lo |
| A27_Water Polo |  |  | Med | Med | Med | Med | Lo | Lo | Med | Lo | Lo | Med |
| A.28_Weightifiting | 0.032941 | 0.405365 | Lo | Lo | Med | Med | Lo | Lo | Lo | Lo | Lo | Lo |

Table 10 summarizes all the game priorities.

Table 10
2012 London Summer Olympic Games weights

| Olympic <br> game | Weight | Olympic <br> game | Weight | Olympic <br> game | Weight | Olympic <br> game | Weight |
| :--- | :--- | :--- | :---: | :--- | :--- | :--- | :---: |
| Archery | 0.019 | Equestrian | 0.024 | Rowing | 0.042 | Triathlon | 0.045 |
| Athletics | 0.053 | Fencing | 0.024 | Sailing | 0.038 | Volleyball | 0.038 |
| Badminton | 0.022 | Field <br> Hockey | 0.027 | Shooting | 0.023 | Water polo | 0.029 |
| Basketball | 0.045 | Football | 0.054 | Swimming | 0.046 | Weightlifting | 0.032 |
| Boxing | 0.023 | Gymnastics | 0.048 | Synchronized <br> swimming | 0.030 | Wrestling | 0.029 |
| Canoe | 0.043 | Handball | 0.021 | Table tennis | 0.021 |  |  |
| Cycling | 0.049 | Judo | 0.024 | Taekwondo | 0.024 |  |  |
| Diving | 0.021 | Modern <br> pentathlon | 0.037 | Tennis | 0.061 |  |  |

International Journal of the
Analytic Hierarchy Process

Tables 11 and 12 below present examples of category comparisons from the rating scale for the criteria "Energy spent" and "Physical characteristics", respectively.

Table 11
Category comparisons on the criteria "Energy spent"

| Energy spent | High | Medium | Low | Priorities |
| :---: | :---: | :---: | :---: | :---: |
| High | 1 | 2 | 3 | 0.5396 |
| Medium | $1 / 2$ | 1 | 2 | 0.2970 |
| Low | $1 / 3$ | $1 / 2$ | 1 | 0.1634 |

Table 12
Category comparisons on the criteria "Physical characteristics"

| Energy spent | Very high | High | Medium | Low | Priorities |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Very high | 1 | 2 | 4 | 6 | 0.4990 |
| High | $1 / 2$ | 1 | 3 | 5 | 0.3129 |
| Medium | $1 / 4$ | $1 / 3$ | 1 | 2 | 0.1202 |
| Low | $1 / 6$ | $1 / 5$ | $1 / 2$ | 1 | 0.0679 |

Table 13 presents the priorities of the events.
Table 13
2012 London Summer Olympic Events Priorities

| Games | Events | Event priorities | Games | Events | Event priorities |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Archery | Archery |  | Modern <br> Pentathlon | Modern <br> Pentathlon | 0,011862 |
| Athletics | Decathlon | 0.014525 | Rowing | Double Sculls | 0,012572 |
|  | Heptathlon | 0.013967 |  | Eight | 0.012273 |
|  | Jump | 0.012536 |  | Four | 0.01159 |
|  | Marathon | 0.016925 |  | Lightweight Four | 011405 |
|  | Pole Vault | 0.012636 |  | Pair | $\begin{aligned} & 0,012273 \\ & 0.012273 \\ & \hline \end{aligned}$ |
|  | Relay | 0.009546 |  | Quadruple Sculls | 0,012273 |
|  | Shot Put | 0.012382 |  | Single Sculls | 0,013401 |
|  | Throw | 0.0091 |  |  |  |
|  | Track | 0.011834 |  |  |  |
|  | Walk | 0.010107 |  |  | 0,012572 |
| Badminton |  | 0.007193 | Sailing | 470 | 0.008177 |

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| Field Hockey | Field Hockey | 0,00848 | Volleyball | Beach | 0.012032 |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | Indoor | 0.010881 |
| Football | Football | 0,017058 | Water Polo | Water Polo | 0,009229 |
| Gymnastic | Artistic | 0.015236 | Weightliftin g | 48 | 0.00652 |
|  | Rhythmic | 0.013007 |  | 53 | 0.00652 |
|  | Trampoline | 0.01403 |  | 56 | 0.00652 |
|  |  |  |  | 58 | 0.00652 |
|  |  |  |  | 62 | 0.00652 |
|  |  |  |  | 63 | 0.00652 |
|  |  |  |  | 69 | 0.00652 |
|  |  |  |  | 75 | 0.00652 |
|  |  |  |  | +75 | 0.006704 |
|  |  |  |  | 77 | 0.006704 |
|  |  |  |  | 85 | 0.006704 |
|  |  |  |  | 94 | 0.009347 |
|  |  |  |  | 105 | 0.010491 |
|  |  |  |  | 105+ | 0,010491 |
| Handball | Handball | 0,00675 | Wrestling | Freestyle | 0.009463 |
|  |  |  |  | Greco-Roman | 0.005714 |
|  |  |  |  | $\begin{gathered} \hline \text { Greco-Roman } \\ 120 \end{gathered}$ | 0.005714 |
|  |  |  |  | $\begin{gathered} \text { Greco-Roman } \\ 60 \\ \hline \end{gathered}$ | 0.005994 |
|  |  |  |  | Greco- <br> Roman84 | 0.008799 |
| Judo | J1 48 | 0.005035 |  |  |  |
|  | J2 52 | 0.005035 |  |  |  |
|  | J3 57 | 0.005035 |  |  |  |
|  | J4 60 | 0.005546 |  |  |  |
|  | J5 63 | 0.005546 |  |  |  |
|  | J6 66 | 0.006 |  |  |  |
|  | J7 70 | 0.006287 |  |  |  |
|  | J8 73 | 0.006287 |  |  |  |
|  | J9 78 | 0.006287 |  |  |  |
|  | J10 81 | 0.005777 |  |  |  |
|  | J11 90 | 0.007659 |  |  |  |
|  | J12 100 | 0.007659 |  |  |  |
|  | J13 100+ | 0.007659 |  |  |  |

## 4. How to obtain medal weights and their priorities for each event under different games

The relative values of gold, silver and bronze medals were studied in an earlier work as follows (Saaty, 2008). Thirteen sets of comparisons and their actual outcomes are shown in Tables 14-19. From the pairwise comparison judgments between different types of medals, one derives the priorities of different types of medals under 13 possible situations and then averages them to obtain the priorities of gold, silver and bronze medals.

In Table 14, a gold medal is very slightly favored over a silver medal and is not dependent on whether it is moderately or extremely favored over a bronze medal.

Table 14
Gold slightly over Silver

|  | Gold | Silver | Bronze | Relative <br> Values |
| :---: | :---: | :---: | :---: | :---: |
| Gold | 1 | 2 | 3 | 0.55 |
| Silver | $1 / 2$ | 1 | $3 / 2$ | 0.27 |
| Bronze | $1 / 3$ | $2 / 3$ | 1 | 0.18 |


|  | Gold | Silver | Bronze | Relative <br> Values |
| :---: | :---: | :---: | :---: | :---: |
| Gold | 1 | 2 | 9 | 0.61 |
| Silver | $1 / 2$ | 1 | 5 | 0.32 |
| Bronze | $1 / 9$ | $1 / 5$ | 1 | 0.07 |

Table 15 shows that the gold medal is moderately favored over the silver medal and from very strongly to extremely over the bronze medal, and is noy dependent on whether a silver medal is moderately or strongly favored over a bronze medal.

Table 15
Gold moderately over Silver

|  | Gold | Silver | Bronze | Relative <br> Values |
| :---: | :---: | :---: | :---: | :---: |
| Gold | 1 | 3 | 7 | 0.64 |
| Silver | $1 / 3$ | 1 | 3 | 0.26 |
| Bronze | $1 / 5$ | $1 / 3$ | 1 | 0.10 |


|  | Gold | Silver | Bronze | Relative <br> Values |
| :---: | :---: | :---: | :---: | :---: |
| Gold | 1 | 3 | 7 | 0.65 |
| Silver | $1 / 3$ | 1 | 5 | 0.28 |
| Bronze | $1 / 7$ | $1 / 5$ | 1 | 0.07 |


|  | Gold | Silver | Bronze | Relative <br> Values |
| :---: | :---: | :---: | :---: | :---: |
| Gold | 1 | 3 | 9 | 0.67 |
| Silver | $1 / 3$ | 1 | 3 | 0.27 |
| Bronze | $1 / 9$ | $1 / 3$ | 1 | 0.06 |

In Table 16, the strength of a gold medal over a silver medal increases even more to between moderately and strongly and a gold medal is favored nearly very strongly to extremely over a bronze medal, while a silver medal is only moderately favored over a bronze medal in both cases.

Table 16
Gold between moderately and strongly over Silver

|  | Gold | Silver | Bronze | Relative <br> Values |
| :---: | :---: | :---: | :---: | :---: |
| Gold | 1 | 4 | 6 | 0.69 |
| Silver | $1 / 4$ | 1 | 3 | 0.22 |
| Bronze | $1 / 6$ | $1 / 3$ | 1 | 0.09 |


|  | Gold | Silver | Bronze | Relative <br> Values |
| :---: | :---: | :---: | :---: | :---: |
| Gold | 1 | 4 | 9 | 0.73 |
| Silver | $1 / 4$ | 1 | 3 | 0.20 |
| Bronze | $1 / 9$ | $1 / 3$ | 1 | 0.07 |

In Table 17, a gold medal is strongly favored over a silver medal and very strongly to extremely favored over a bronze medal.

Table 17
Gold strongly over Silver

|  | Gold | Silver | Bronze | Relative <br> Values |
| :---: | :---: | :---: | :---: | :---: |
| Gold | 1 | 5 | 7 | 0.72 |
| Silver | $1 / 5$ | 1 | 4 | 0.21 |
| Bronze | $1 / 7$ | $1 / 4$ | 1 | 0.07 |


|  | Gold | Silver | Bronze | Relative <br> Values |
| :---: | :---: | :---: | :---: | :---: |
| Gold | 1 | 5 | 9 | 0.74 |
| Silver | $1 / 5$ | 1 | 4 | 0.19 |
| Bronze | $1 / 9$ | $1 / 4$ | 1 | 0.07 |

In Table 18, a gold medal is considered strongly more important than a silver medal and extremely more than a bronze medal while a silver medal is first moderately and then between moderately and strongly more important over a bronze medal.

Table 18
Gold very strongly over Silver

|  | Gold | Silver | Bronze | Relative <br> Values |
| :---: | :---: | :---: | :---: | :---: |
| Gold | 1 | 7 | 9 | 0.79 |
| Silver | $1 / 7$ | 1 | 3 | 0.15 |
| Bronze | $1 / 9$ | $1 / 3$ | 1 | 0.06 |


|  | Gold | Silver | Bronze | Relative <br> Values |
| :---: | :---: | :---: | :---: | :---: |
| Gold | 1 | 7 | 9 | 0.78 |
| Silver | $1 / 7$ | 1 | 4 | 0.16 |
| Bronze | $1 / 9$ | $1 / 4$ | 1 | 0.06 |

In Table 19, a gold medal is extremely important over a silver medal and a bronze medal while a silver medal is first strongly important and then extremely important than a bronze medal.

Table 19
Gold extremely over Silver

|  | Gold | Silver | Bronze | Relative <br> Values |
| :---: | :---: | :---: | :---: | :---: |
| Gold | 1 | 9 | 9 | 0.80 |
| Silver | $1 / 9$ | 1 | 5 | 0.15 |
| Bronze | $1 / 9$ | $1 / 5$ | 1 | 0.05 |


|  | Gold | Silver | Bronze | Relative <br> Values |
| :---: | :---: | :---: | :---: | :---: |
| Gold | 1 | 9 | 9 | 0.78 |
| Silver | $1 / 9$ | 1 | 9 | 0.18 |
| Bronze | $1 / 9$ | $1 / 9$ | 1 | 0.04 |

The tables given above give the priorities of different types of medals under 13 different situations shown in Tables 14-19, and then the priorities of different types of medals are
obtained by taking the geometric mean of the priorities derived from the judgment matrices above as seen in Table 20.

Table 20
The 13 vectors of priorities from Tables 10-15 and their average

|  |  |  |  |  |  |  |  |  |  |  |  |  |  | Total | GEO. <br> MEAN |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{G}^{*}$ | 0.55 | 0.61 | 0.64 | 0.65 | 0.67 | 0.69 | 0.73 | 0.72 | 0.74 | 0.79 | 0.78 | 0.80 | 0.78 | 9.15 | 0.6900 |
| S | 0.27 | 0.32 | 0.26 | 0.28 | 0.27 | 0.22 | 0.20 | 0.21 | 0.19 | 0.15 | 0.16 | 0.15 | 0.18 | 2.86 | 0.2000 |
| B | 0.18 | 0.07 | 0.10 | 0.07 | 0.06 | 0.09 | 0.07 | 0.07 | 0.07 | 0.06 | 0.06 | 0.05 | 0.04 | 0.99 | 0.0060 |

*G: Gold, S: Silver, B: Bronz
Now we re-rank the countries that won medals in the 2012 London Summer Olympics according to a our method which considers not only the total medals won but also the weighted priority of each game and event under each game. The rank of the countries when simply counting medals won is shown in the sixth column of Table 21. Following the traditional way of counting the total number of medals won, the USA is the top ranked country followed by China and Russia. However, the ranking of the countries that won medals in the 2012 London Summer Olympics (shown in the last column of Table 21 ) is different when based on our finer approach. For example, the Ukraine won 20 medals in boxing, canoeing, fencing, gymnastic, rowing, shooting, weighlifting and wrestling. This includes 6 gold medals 5 silver medals, and 9 bronze medals. When counting the total number of medals won, the Ukraine is ranked $10^{\text {th }}$ place (if we use the total number of gold medals as a second criterion for the countries when the total number of medals are the same, then Ukraine ranked as 12th place in the current methodology, South Korea is $9^{\text {th }}$, Italy is $10^{\text {th }}$, Netherland is $11^{\text {th }}$ and Ukraine is $12^{\text {th }}$ ), but when considering the priorities of the events and the games by proposed methodology the Ukraine's rank moves to $15^{\text {th }}$ place. This is because gymnastics is one of the important games considered in this research, and the Ukraine won a bronze medal in gymnastics. On the other hand, they won 5 medals in boxing and 2 medals in fencing events which have relatively lower priorities. Another example is Latvia which won only two medals, one in cycling (gold) and the other in beach volleyball (bronze). When considering the number of medals won, Latvia is one of the lowest ranked countries by current ranking system. As we explained above in the Ukraine example, the countries that have an equal number of medals are ranked as the same. Latvia, Bulgaria, Indonesia, Dominican Republic and so on are ranked $25^{\text {th }}$ by the current ranking. However, although Latvia is ranked $25^{\text {th }}$, it is actually $57^{\text {th }}$ if we use the number of gold medals as the second criterion (when we just count the countries above Latvia in the current order, Latvia is $57^{\text {th }}$ not $\left.25^{\text {th }}\right)$. When we look at it like this, we can more easily interpret the difference from the proposed rank and say that Latvia deserves a better rank even though it has just two medals because the priorities of those games are high. Our methodology ranked Latvia $42^{\text {th }}$ because the priorities of the events in which Latvia won medals are relatively high. A similar example is Tunisia which won only three medals. These medals were won in the marathon, track and swimming, all events with higher priorities as compared to other games.

Table 21
2012 London Olympics medals current and proposed ranking comparisons

| Current Methodology |  |  |  |  |  | Proposed Methodology |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Country | Gold | Silver | Bronze | Total Medal <br> Count | Current <br> Ranking | Total Priority Score | Proposed Ranking |
| USA | 46 | 29 | 29 | 104 | 1 | 0.4348 | 1 |
| China | 38 | 27 | 23 | 88 | 2 | 0.2515 | 3 |
| Russian | 24 | 25 | 32 | 81 | 3 | 0.2237 | 4 |
| Great Britain | 29 | 17 | 19 | 65 | 4 | 0.2868 | 2 |
| Germany | 11 | 19 | 14 | 44 | 5 | 0.1288 | 5 |
| Japan | 7 | 14 | 17 | 38 | 6 | 0.0756 | 8 |
| Australia | 7 | 16 | 12 | 35 | 7 | 0.0899 | 7 |
| France | 11 | 11 | 12 | 34 | 8 | 0.1248 | 6 |
| South Korea | 13 | 8 | 7 | 28 | 9 | 0.0710 | 9 |
| Italy | 8 | 9 | 11 | 28 | 9 | 0.0686 | 11 |
| Netherlands | 6 | 6 | 8 | 20 | 10 | 0.0502 | 13 |
| Ukraine | 6 | 5 | 9 | 20 | 10 | 0.0404 | 15 |
| Hungary | 8 | 4 | 6 | 18 | 11 | 0.0687 | 10 |
| Canada | 1 | 5 | 12 | 18 | 11 | 0.0356 | 20 |
| Spain | 3 | 10 | 4 | 17 | 12 | 0.0402 | 16 |
| Brazil | 3 | 5 | 9 | 17 | 12 | 0.0256 | 27 |
| Cuba | 5 | 3 | 7 | 15 | 13 | 0.0258 | 26 |
| Kazakhstan | 7 | 1 | 5 | 13 | 14 | 0.0396 | 17 |
| New Zeland | 6 | 2 | 5 | 13 | 14 | 0.0536 | 12 |
| Belarus | 2 | 5 | 5 | 12 | 15 | 0.0384 | 18 |
| Iran | 4 | 5 | 3 | 12 | 15 | 0.0300 | 23 |
| Jamaica | 4 | 4 | 4 | 12 | 15 | 0.0450 | 14 |
| Kenya | 2 | 4 | 5 | 11 | 16 | 0.0345 | 21 |
| Czech Republic | 4 | 3 | 3 | 10 | 17 | 0.0278 | 24 |
| Azerbaijan | 2 | 2 | 6 | 10 | 17 | 0.0202 | 32 |
| Poland | 2 | 2 | 6 | 10 | 17 | 0.0209 | 31 |
| Romania | 2 | 5 | 2 | 9 | 18 | 0.0238 | 29 |
| Denmark | 2 | 4 | 3 | 9 | 18 | 0.0358 | 19 |

International Journal of the Analytic Hierarchy Process

| Colombia | 1 | 3 | 4 | 8 | 19 | 0.0190 | 34 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Sweden | 1 | 4 | 3 | 8 | 19 | 0.0190 | 34 |
| Ethiopia | 3 | 1 | 3 | 7 | 20 | 0.0343 | 22 |
| Mexico | 1 | 3 | 3 | 7 | 20 | 0.0177 | 37 |
| Georgia | 1 | 3 | 3 | 7 | 20 | 0.0074 | 50 |
| North Korea | 4 | 0 | 2 | 6 | 21 | 0.0195 | 33 |
| Croatia | 3 | 1 | 2 | 6 | 21 | 0.0215 | 30 |
| South Africa | 3 | 2 | 1 | 6 | 21 | 0.0275 | 25 |
| India | 0 | 2 | 4 | 6 | 21 | 0.0146 | 40 |
| Mongolia | 0 | 2 | 3 | 5 | 22 | 0.0043 | 56 |
| Turkey | 2 | 2 | 1 | 5 | 22 | 0.0163 | 38 |
| Lithuania | 2 | 1 | 2 | 5 | 22 | 0.0094 | 45 |
| Ireland | 1 | 1 | 3 | 5 | 22 | 0.0040 | 57 |
| Trinidad And Tobago | 1 | 0 | 3 | 4 | 23 | 0.0095 | 44 |
| Switzerland | 2 | 2 | 0 | 4 | 23 | 0.0189 | 35 |
| Norway | 2 | 1 | 1 | 4 | 23 | 0.0186 | 36 |
| Slovenia | 1 | 1 | 2 | 4 | 23 | 0.0048 | 59 |
| Argentina | 1 | 1 | 2 | 4 | 23 | 0.0098 | 43 |
| Serbia | 1 | 1 | 2 | 4 | 23 | 0.0063 | 53 |
| Malaysia | 0 | 1 | 3 | 4 | 23 | 0.0021 | 64 |
| Finland | 0 | 1 | 2 | 3 | 24 | 0.0037 | 58 |
| Tunisia | 1 | 1 | 1 | 3 | 24 | 0.0254 | 28 |
| Uzbekistan | 1 | 0 | 2 | 3 | 24 | 0.0085 | 47 |
| Armenia | 0 | 1 | 2 | 3 | 24 | 0.0160 | 39 |
| Belgium | 0 | 1 | 2 | 3 | 24 | 0.0087 | 47 |
| Thailand | 0 | 2 | 1 | 3 | 24 | 0.0024 | 63 |
| Indonesia | 0 | 1 | 1 | 2 | 25 | 0.0020 | 65 |
| Dominic Republic | 1 | 1 | 0 | 2 | 25 | 0.0083 | 48 |
| Latvia | 1 | 0 | 1 | 2 | 25 | 0.0109 | 42 |
| Chinese Tapei | 0 | 1 | 1 | 2 | 25 | 0.0004 | 73 |
| Estonia | 0 | 1 | 1 | 2 | 25 | 0.0027 | 61 |
| Puerto Rico | 0 | 1 | 1 | 2 | 25 | 0.0031 | 60 |
| Bulgaria | 0 | 1 | 1 | 2 | 25 | 0.0068 | 51 |

International Journal of the Analytic Hierarchy Process

| Egypt | 0 | 2 | 0 | 2 | 25 | 0.0025 | 62 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Moldova | 0 | 0 | 2 | 2 | 25 | 0.0016 | 67 |
| Qatar | 0 | 0 | 2 | 2 | 25 | 0.0020 | 64 |
| Greece | 0 | 0 | 2 | 2 | 25 | 0.0019 | 66 |
| Singapore | 0 | 0 | 2 | 2 | 25 | 0.0014 | 69 |
| Algeria | 1 | 0 | 0 | 1 | 26 | 0.0083 | 47 |
| Bahamas | 1 | 0 | 0 | 1 | 26 | 0.00668 | 52 |
| Botswana | 0 | 1 | 0 | 1 | 26 | 0.0024 | 63 |
| Guatemala | 0 | 1 | 0 | 1 | 26 | 0.0020 | 65 |
| Grenada | 1 | 0 | 0 | 1 | 26 | 0.0083 | 49 |
| Uganda | 1 | 0 | 0 | 1 | 26 | 0.0118 | 41 |
| Slovakia | 0 | 1 | 0 | 1 | 26 | 0.0050 | 55 |
| Montenegro | 0 | 1 | 0 | 1 | 26 | 0.0014 | 69 |
| Venezuela | 1 | 0 | 0 | 1 | 26 | 0.0055 | 54 |
| Bahrain | 0 | 0 | 1 | 1 | 26 | 0.0090 | 46 |
| Gabon | 0 | 1 | 0 | 1 | 26 | 0.0015 | 68 |
| Morocco | 0 | 0 | 1 | 1 | 26 | 0.0012 | 70 |
| Portugal | 0 | 1 | 0 | 1 | 26 | 0.0022 | 64 |
| Tajikistan | 0 | 0 | 1 | 1 | 26 | 0.0003 | 74 |
| Cyprus | 0 | 1 | 0 | 1 | 26 | 0.0025 | 62 |
| Afghanistan | 0 | 0 | 1 | 1 | 26 | 0.0007 | 71 |
| Hong Kong | 0 | 0 | 1 | 1 | 26 | 0.0016 | 67 |
| Saudi Arabia | 0 | 0 | 1 | 1 | 26 | 0.0006 | 72 |
| Kuwait | 0 | 0 | 1 | 1 | 26 | 0.0007 | 71 |

* The rows in bold show that the rank of a country obtained by the proposed method is the same as its current rank


## 5. Conclusion

Training for competition in the Olympics requires time and resources with different types of events having different characteristics. Individual games require more concentration while team games require more cooperation. To become successful in gymnastics, one usually has to start training at a very young age (five or six). The duration of a volleyball game is usually about one to two hours, and the marathon takes about two hours and requires more energy when compared to other events. On the other hand, archery takes only a few seconds. Thus, a medal should be given a different value depending on which game it is won for. We propose that ranking countries in the Olympics should not only be decided by counting the total medals won, but also by the type of game in which the medal was won. In this study we prioritized different games and the events under each
game. Our elaborate approach to the Olympics shows that counting the total numbers of medals won is not a bad way of ranking countries. Finally, while it is known that multicriteria decision making is very important in optimal allocation of limited resources, it may not always produce radically better results than much simpler existing ways of ranking. For the last winter Olympics (2014) in Sochi, Russia, there were more noticeable differences in the two rankings methods so that Norway which ranked third according to the number of medals won, actually ranked first according to prioritization of the different kinds of games. This is a significant finding because ranking first would have been a very distinguished and celebrated outcome for Norway. We also performed Compatibility Index calculations to compare actual ranking and the estimated one as shown in Appendix. Since the ranks are slightly different, the index is obtained was 1,22 which could be acceptable and reasonable.

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## APPENDIX

COMPATIBILITY INDEX ANALYSIS

| Pair | wis | Comparis | on Matrix from | Actu | ual Data |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | A1 |  | A2 |  | A3 |  | A4 |  |  | A5 |  |  | A6 |  | A7 | A8 |
| A1 | 1 |  | 1,14433 |  | 1,32934 |  | 1,59712 |  |  | 2,00000 |  |  | 2,67470 |  | 3,96429 | 7,92857 |
| A2 | 0,8738739 |  | 1,00000 |  | 1,16168 |  | 1,39568 |  |  | 1,74775 |  |  | 2,33735 |  | 3,46429 | 6,92857 |
| A3 | 0,7522523 |  | 0,86082 |  | 1,00000 |  | 1,20144 |  |  | 1,504504505 |  |  | 2,01205 |  | 2,98214 | 5,96429 |
| A4 | 0,6261261 |  | 0,71649 |  | 0,83234 |  | 1 |  |  | 1,25225 |  |  | 1,67470 |  | 2,48214 | 4,96429 |
| A5 | 0,5 |  | 0,57216 |  | 0,66467 |  | 0,79856 |  |  | 1 |  |  | 1,33735 |  | 1,98214 | 3,96429 |
| A6 | 0,3738739 |  | 0,42784 |  | 0,49701 |  | 0,59712 |  |  | 0,747747748 |  |  | 1 |  | 1,48214 | 2,96429 |
| A7 | 0,2522523 |  | 0,28866 |  | 0,33533 |  | 0,40288 |  |  | 0,504504505 |  |  | 0,674699 |  | 1 | 2,00000 |
| A8 | 0,1261261 |  | 0,14433 |  | 0,16766 |  | 0,20144 |  |  | 0,252252252 |  |  | 0,337349 |  | 0,5 | 1 |
| Pairwise Comparison Matrix from Estimated Data |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | A1 |  | A2 | A3 |  | A4 |  | A5 |  |  | A6 |  | A7 |  | A8 |  |
| A1 | 1 |  | 1,743141 | 1,959768 |  | 1,528591 |  | 1 3,403727 |  |  |  | 7 7 5,798942 |  | 4,876529 | 3,512821 |  |
| A2 | 0,573677 |  | 1,00000 | 1,12427 |  | 0,87692 |  | 1,95264 |  |  | 3,32672 |  |  | 2,79755 | 2,01522 |  |
| A3 | 0,510265 |  | 0,88946 | 1,00000 |  | 0,77999 |  | 1,736801 |  |  | 1 2,95899 |  |  | 2,48832 | 1,79247 |  |
| A4 | 0,654197 |  | 1,14036 | 1,28207 |  | 1 |  | 2,22671 |  |  |  | 3,79365 |  | 3,19021 | 2,29808 |  |
| A5 | 0,293796 |  | 0,51213 | 0,57577 |  | 0,44909 |  | 1 |  |  |  | 1,70370 |  | 1,43270 | 1,03205 |  |
| A6 | 0,172445 |  | 0,30060 | 0,33795 |  | 0,26360 |  | 0,265569 |  |  | 9 |  |  | 0,84093 | 0,60577 |  |
| A7 | 0,205064 |  | 0,35746 | 0,40188 |  | 0,31346 |  | 0,697981 |  |  | 1 1,189153 |  | 1 |  | 0,72035 |  |
| A8 | 0,284672 |  | 0,49622 | 0,55789 |  | 0,43515 |  | 0,968944 |  |  | 4 1,650794 |  | 1,388209 |  | 1 |  |
| Transpose of Comparison Matrix from Estimated Data |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | A1 |  | A2 | A3 |  |  |  | A4 |  | A5 |  |  | A6 |  | A7 | A8 |
| A1 | 1 |  | 0,573677 | 0,510264599 |  |  | 0,654197 |  |  | 0,29379562 |  |  |  | 0,172445 | 0,20506387 | 0,284672 |
| A2 | 1,7431412 |  | 1,00000 | 0,88946 |  |  | 1,14036 |  |  | 0,51213 |  |  |  | 0,30060 | 0,35746 | 0,49622 |
| A3 | 1,9597675 |  | 1,12427 | 1,00000 |  |  |  | 1,2820 |  |  | 0,57577 |  |  | 0,33795 | 0,40188 | 0,55789 |
| A4 | 1,5285914 |  | 0,87692 | 0,77999 |  |  | 1 |  |  |  | 0,44909 |  |  | 0,26360 | 0,31346 | 0,43515 |
| A5 | 3,4037267 |  | 1,95264 | 1,736801242 |  | 2,22671 |  |  |  | 1 |  |  |  | 0,265569 | 0,69798137 | 0,968944 |
| A6 | 5,7989418 |  | 3,32672 | 2,95899 |  | 3,79365 |  |  |  | 1,70370 |  |  | 1 |  | 1,18915344 | 1,650794 |
| A7 | 4,8765295 |  | 2,79755 | 2,48832 |  | 3,19021 |  |  |  | 1,43270 |  |  |  | 0,84093 | 1 | 1,388209 |
| A8 | 3,5128205 |  | 2,01522 | 1,79247 |  | 2,29808 |  |  |  | 1,03205 |  |  |  | ,60577 | 0,72035 | 1 |
| Res |  | Hadamar | d (Cell-wise) M | Multipl | lication | of Pre | viou | us Tw | o Ma | ice |  |  |  |  |  |  |
|  |  | A1 | A2 |  | A3 |  |  | A4 |  |  | A5 |  |  | A6 | A7 | A8 |
|  | A1 | 1 | 0,6564758 |  | 0,678315 | 5814 |  | 1,044 | 833 |  | 0,587 | 591241 |  | 0,461239 | 0,81293176 | 2,257039 |
|  | A2 | 1,52329 | 1 |  | 1,03327 |  |  | 1,591 |  |  | 0,895 |  |  | 0,70260 | 1,23833 | 3,43811 |
|  | A3 | 1,47424 | 0,96780 |  | 1 |  |  | 1,540 | 334 |  | 0,866 |  |  | 0,67998 | 1,19846 | 3,32742 |
|  | A4 | 0,95709 | 0,62831 |  | 0,649209 | 9948 |  | 1 |  |  | 0,562 |  |  | 0,44145 | 0,77805 | 2,16019 |
|  | A5 | 1,70186 | 1,11723 |  | 1,154400 | 082 |  | 1,778 | 163 | 1 | 1 |  |  | 0,35516 | 1,38350 | 3,84117 |
|  | A6 | 2,16807 | 1,42329 |  | 1,47064 |  |  | 2,265 |  |  | 1,273 |  |  | 1 | 1,76250 | 4,89342 |
|  | A7 | 1,23012 | 0,80754 |  | 0,83441 |  |  | 1,285 |  |  | 0,722 |  |  | 0,56738 | 1 | 2,77642 |
|  | A8 | 0,44306 | 0,29086 |  | 0,30053 |  |  | 0,462 |  |  | 0,260 |  |  | 0,20436 | 0,36018 | 1 |

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609

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