THE MAGIC OF THE ANALYTIC HIERARCHY PROCESS (AHP)

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ABSTRACT

Dr. Thomas Saaty developed the Analytic Hierarchy Process (AHP) with the underlying goal of making it simple and accessible to the lay user. In Saaty's own words, the AHP is based on how "ordinary people process information" and "express the strength of their judgments" (Saaty, 1994, p. 37). Because he was successful in developing the AHP in accordance with these goals, when decision makers use the AHP their experience can feel magical as they find pairwise comparisons natural and can relate to the final priorities. Careful investigation of the axioms, theorems, and proofs shows that the AHP is more than just magic and provides scientific justification of the highest order. Five important components of the AHP and some background into the history of its development are summarized and highlighted from Saaty's article, "*How to Make a Decision: the Analytic Hierarchy Process*" (Saaty, 1994).

Keywords: Analytic Hierarchy Process (AHP); Thomas Saaty

1. Introduction

There is something about magic that fascinates us. When something that is considered nearly impossible is performed, we marvel and ask, "How was it done?" I remember my first time using the Analytic Hierarchy Process (AHP) for a class assignment to model a decision about where to locate a semiconductor production facility. The process felt intuitive and natural, and the final answer made sense. As we performed the sensitivity analysis and I was able to test patterns, I was hooked. It almost seemed magical how step-by-step the process of making and aggregating the pairwise comparisons felt so natural, comfortable, and intuitive, yet somehow when all the parts were aggregated they represented an answer for a complex decision. After class, I went home and built and tested a personal decision model that worked and made sense. I knew then that it had to be more than just magic; the AHP is a complex interdisciplinary multi-criteria decision-making method composed of many axioms, theorems, and proofs. In Saaty's own words, "It must be simple and accessible to the lay user, and must have scientific justification of the highest order" (Saaty, 1994, p. 40). In his article, "*How to Make a Decision: the Analytic Hierarchy Process",* Dr. Thomas Saaty expounds on the scientific justification for the Analytic Hierarchy Process.

Rozann and Thomas Saaty constantly welcomed many, many people from all over the world into their home as they shared the AHP with them. Dr. Saaty would only ask of his

students or guests to do two things in return. First, to go and use what they had learned, and second, to share it with others, because that is the best way to get the AHP into the hands of other people. I had the opportunity on multiple occasions to hear Saaty share firsthand the journey he went through to develop this decision-making approach.

One of the most meaningful sections of his article is where he explains some of the history and background that drove him to develop the Analytic Hierarchy Process (AHP) and the Analytic Network Process (ANP). Saaty explains that his motivation to develop a "decision theory that would have lasting value" was driven by his own personal experience (Saaty, 1994, p. 37). The AHP did not come about as an interesting mathematical model developed in the abstract, but in response to Saaty's efforts to deal with real problems he faced in his job. After graduating with a PhD in mathematics from Yale University and before entering academia, Saaty spent many years working in government agencies including the U.S. State Department in the Arms Control and Disarmament Agency. In this position, he worked with some of the world's greatest minds, but the group was unsuccessful in developing ways to deal with negotiations and trade-offs in diplomatic and political decisions.

Saaty was determined to develop a method based on how humans make decisions naturally. He stepped back and asked an important question for which the answer is one of the reasons for the popularity of the AHP. He asked, "How do ordinary people process information in their minds in attempting to make a decision, and how do they express the strength of their judgments?" (Saaty, 1994, p. 37) He goes on to explain how these questions led him to consider "hierarchies and networks, paired comparisons, ratio scales, homogeneity and consistency, priorities, ranking" which are all important components of the AHP (Saaty, 1994, p. 27). Some additional details are provided by Ferretti and Saaty (2014) from an interview with Thomas Saaty. In the interview, he shared how he would wake up at night and not be able to sleep again until he would write down the thoughts that would come into his mind; it was a process that took time and many struggles. It was a real treat to hear the details behind the development of the AHP and hear the passion in his voice decades later. Because Saaty was able to answer that fundamental question, the AHP has become a popular decision-making method over the decades. In 2008, Wallenius et al. (2008) showed that the AHP/ANP was the most widely published multi-criteria decision-making method. With that important context in front of us, we can now proceed to review the other meaningful contributions in the foundational *Interfaces* article (Saaty, 1994).

In this article, Saaty begins by providing other evidence and motivation for using the AHP to solve problems. Two examples are given as follows: 1) "The Hospice Problem", where options for taking care of terminally ill patients were considered; and 2) "A Firm Evaluates its Employees for Raises" where a ratings model was used to rank employees for raises. These examples show not only how one could use the AHP, but also provide the context to present many of the key principles and theorems of the AHP in more practical terms that are accessible to a non-technical audience. Citing a book that he published in 1982, Saaty explains that a decision-making approach should have the following characteristics (Saaty, 1982, p.20):

- Be simple in construct
- Be adaptable to both groups and individuals

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- Be natural to our intuition and general thinking
- Encourage compromise and consensus building
- Not require inordinate specialization to master and communicate

The AHP possesses each of those characteristics. In line with Saaty's rule of "seven plus or minus two", there are five important contributions contained in the *Interfaces* article which will be discussed in greater detail below. These five principles are it really works, the process, from relative comparisons to deriving relative priorities, the mathematical elegance of the AHP, and the benefits from using the AHP (Saaty, 1994).

2. Five important contributions

2.1 It really works!

In addition to the two complex models presented in the article, there are two other simple examples that should be discussed. Because of their simplicity, one can quickly experience the AHP and see that it really works! Figure 1, which was printed as Figure 4 in the *Interfaces* article, contains five shapes that can be compared in terms of their areas using pairwise comparisons. The result will be a very good approximation to the relative areas of the shapes that one could obtain by using a ruler and the formulas for calculating the areas of the shapes. The figures were used in an experiment with MBA students to validate the use of the 1-9 scale (Whitaker, 2007). Saaty would also have his students do this exercise without knowing the relative weights beforehand as a part of his first lecture each semester. As the MBA students do the pairwise comparisons and see that it really works, they begin to engage and get excited about the AHP.

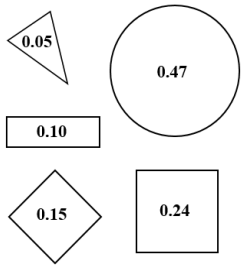

Figure 1. Five shapes to calculate relative areas

The second example is when students are asked to approximate the relative size of a cherry tomato to a watermelon. In the article, Saaty compares a blueberry to a watermelon to explain the importance of homogeneity and clustering (Saaty, 1994, p. 35). When a participant sees this example for the first time, she initially feels overwhelmed and only able to make a reasonable guess without either buying enough cherry tomatoes to try it out or potentially developing a set of measuring tools (rulers) and formulas (volume). When she is presented with the idea of comparing the cherry tomato (or blueberry) to other fruits including a plum, the pairwise comparisons suddenly feel manageable. But, it is not until the next step of comparing the plum to other objects including a grapefruit that the lightbulb comes on. After the subsequent comparisons culminating with the cluster that includes the watermelon, one feels empowered that anything can be compared through relative comparisons if the problem is structured properly. In experiencing these two examples participants informally apply many of the

principles of the AHP that were formalized and proven by Saaty and see for themselves that it really works.

2.2 The process itself

Saaty explains, "I believe that making decisions in real life situations depends on the depth and sophistication of the structures decision makers use to represent a decision or prediction problem" (Saaty, 1994, p. 39). To help guide decision makers through the process of structuring a problem Saaty provides ten suggestions (Saaty, 1994, p. 22):

(1) Identify the overall goal. What are you trying to accomplish? What is the main question?
(2) Identify the sub-goals of the overall goal. If relevant, identify time horizons that affect the decision.
(3) Identify criteria that must be satisfied to fulfill the sub-goals of the overall goal
(4) Identify sub-criteria under each criterion. Note that criteria or sub-criteria may be specified in terms of ranges of values of parameters or in terms of verbal intensities such as high, medium, low.
(5) Identify the actors involved.
(6) Identify the actor's goals.
(7) Identify the actor's policies.
(8) Identify options or outcomes.
(9) For yes-no decisions, take the most preferred outcome and compare the benefits and costs of making the decision with those of not making it.
(10) Do a benefit/ cost analysis using marginal values.

These suggestions follow a logical process that decision makers can use to approach and think about decisions in general. It is also no coincidence that this process naturally flows into a hierarchy. This "ten step cheat sheet" can be used as a guide to structure personal decisions or when guiding a group through the decision-making process. These steps should not be taken for granted or glossed over; as Saaty said, "Perhaps the most creative part of decision making that has a significant effect on the outcome is modeling the problem" (Saaty, 1994, p. 22). This is another advantage of using the AHP to make decisions, the process that you go through helps you thoughtfully consider/discover what matters and what impacts the decision as you design the hierarchy. Through the pairwise comparison process there is a way to quantify the relative importance and impact of each of the elements that were identified. This process of following the structured ten steps and creating a hierarchy and making pairwise comparisons also leads to meaningful discussions. Further discussions and insight occur when exploring the robustness of the decision using sensitivity analysis. In one sense, the process itself is as important as the method.

2.3 From relative comparisons to priorities

Making relative comparisons is an innate ability that humans have and is at the core of the AHP. Saaty often used the example of comparing apples with respect to their size without using rulers and/or scales, but by making relative comparisons to demonstrate this point. Here is where Saaty brought psychology and math together to take advantage of an innate human ability to obtain priority vectors. Saaty defined priorities as numerical ranks measured on a relative absolute scale with a ratio property. "A ratio scale is a set of positive numbers whose ratios remain the same if all the numbers are multiplied by an

arbitrary positive number", he said (Saaty, 1994, p. 28). Furthermore, he states, "Ratio scales are the fundamental tool of the mind that people use to understand magnitudes" (Saaty, 1994, p. 40). The process of obtaining priorities is broken down into smaller steps. Saaty explains, "The AHP is based on the innate human ability to make sound judgments about small problems" (Saaty, 1994, p. 21); he defines a judgment or comparison as "the numerical representation of a relationship between two elements that share a common parent" (Saaty, 1994, p. 25). In other words, the decision can be decomposed into smaller subsections with as many levels as the decision maker needs until the elements are "small" enough that sound judgements can be made as the elements are compared. The numerical relationships are elicited through relative comparisons with respect to importance, preference, or likelihood, and this is where the psychology comes in. "Cognitive psychologists have recognized for some time that people are able to make two kinds of comparisons, absolute and relative. In absolute comparisons, people compare alternatives with a standard in their memory that they have developed through experience. In relative comparisons, they compare alternatives in pairs according to a common attribute" (Saaty, 1994, p. 33).

Relative comparisons also allow us to measure "intangibles" and even combine tangible and intangible criteria within the same model. In the apple example, the weight or size are "intangible" if one does not have rulers or scales, but that does not limit the ability of one who is familiar with size and weight to derive meaning and make relative comparisons. The priority vector from the relative comparisons will be very close to that obtained from first weighing the apples on a scale and then expressing the weights relative to the other apples. Intangible properties have a unit of measurement or else even relative comparisons could not be made. Another way of looking at intangibles in the AHP is to think of them as properties where using a formalized measurement tool is prohibitive because of reasons like cost, current technological limitations, or that it is socially inhibitive to use a formalized measuring tool. By breaking the intangibles down into smaller problems and making relative comparisons between two elements meaningful priorities can be obtained. This leads us to the beauty of the math of the AHP and how to get final priorities.

2.4 The mathematical elegance

Saaty touched on the axioms, theorems, and proofs of the AHP at a very high and practitioner-oriented level throughout the article. The real meat of the mathematics can be found in various other publications, but key principles like redundancy and the consistency ratio were presented. Redundancy among the pairwise comparisons and the consistency ratio are both important in that they help increase the decision maker's confidence in the decision. Another important mathematical property of the AHP that was discussed was the three ways that can be used to get a priority vector from a perfectly consistent pairwise comparison matrix. In the hospice example, Saaty explains that one can add the values in each row of the pairwise comparison matrix and divide that summation by the sum of all the entries in the pairwise comparison matrix. There you have it! This is the easiest way to get the priority vector; however, it is no coincidence that there are other ways to get the same result. Two other methods include solving the equation $Ax = \lambda_{max}x$; or raising the matrix to large powers and normalizing the sums of the rows when it converges. These ways, which work for a consistent matrix, also work if there is some inconsistency and they take into account all the intransitivities between the elements (Saaty, 2013). Each solution could be considered the most elegant

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for different reasons, but what is most important is the idea that there is more than one way to get the same result. In some ways, this is like the Pythagorean theorem that has been proven in many different ways. There is value in knowing the solution is not a special case.

2.5 Five benefits

Even though human behavior does not always follow theoretical and normative considerations, Saaty argues, based on his experience, that an analytical approach can still be very useful. Saaty concludes by sharing five benefits of using the "descriptive analytical approach" to decision-making (Saaty, 1994, p. 40):

- First, the morphological way of thoroughly modeling the decision, induces people to make explicit their tacit knowledge.
- Second, particularly in the framework of hierarchies and feedback systems, the process permits decision makers to use judgments and observations to surmise relations and strengths of relations in the flow of interacting forces moving from the general to the particular, and to make predictions of most likely outcomes.
- Third, people are able to incorporate and trade off values and influences with greater accuracy of understanding than they can by using language alone.
- Fourth, people are able to include judgments that result from intuition and emotion as well as those that result from logic.
- Finally, a formal approach allows people to make gradual and more thorough revisions and to combine the conclusions of different people studying the same problem in different places (Saaty & Alexander 1989).

The benefits from the morphological way of modeling and the framework were touched upon in the prior sections of this paper where the value of the structure and process in the AHP were discussed. The third and fourth benefits are related, and deal with the ability to make pairwise comparisons. The integration of language and mathematics provides a richer model than if either were used alone. Recognizing and incorporating the decision maker's intuition and emotion is critical. First, because both intuition and emotion are required in making a decision, a quantitatively optimal decision may no longer be optimal if it makes one feel miserable. Additionally, by being able to incorporate intuition and emotion into the decision, the feeling of trust through validation and buy-in from the decision maker can increase. Additional buy-in is due in part to acknowledging and incorporating important mental components of human decision making into the decision. The ability to synthesize input from different people or get them to come to consensus is self-evident in how the AHP has been so widely applied in group decision-making.

3. Conclusion

These fundamentals of the AHP reviewed herein, from the ability to make pairwise comparisons to the psychological and structural benefits of using the AHP, should not be taken for granted. Nor can we forget our initial experience with the AHP, the moment where we saw that it really works, and let that excitement diminish. In spite of the ease provided by modern software, we should not overlook the elegant math behind the AHP. These concepts are at the very core of the AHP and can almost be taken for granted the more familiar one becomes with the process, but if they are taken for granted, glossed

over, and not given the proper attention, then the AHP would become just another multi-criteria decision-making method. As Saaty said, "I believe that making decisions in real life situations depends on the depth and sophistication of the structures decision makers use to represent a decision or prediction problem" (Saaty, 1994, p. 39). These fundamental principles of the AHP facilitate meaningful decision-making. Saaty changed the way we measure things; the greatest way to recognize his contributions is to use them to tackle the complex decisions in our day and spread awareness of the "magic" of the method.

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