REFLECTIONS: MATHEMATICAL PRINCIPLES OF DECISION MAKING

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ABSTRACT

This article discusses my reflections on *Mathematical Principles of Decision Making* by Thomas Saaty (Saaty T. L., 2010). In this book, Saaty very clearly explains his Analytic Hierarchy Process (AHP) theory for measuring both tangible and intangible factors. Experts judgments are elicited about the dominance of a factor over another one via pairwise comparisons using an absolute scale and priorities of the factors are derived. The important concepts of the AHP such as compatibility index, validation, sensitivity analysis for testing the robustness of the priorities derived, and its generalization to structures with dependence and feedback, and the Analytic Network Process (ANP) are given. Extensions of the theory to complex decisions involving benefits, opportunities, costs and risks and applications to resource allocation and conflict resolution are included, as well as the generalization to continuous and time dependent judgments is also covered[1].

Keywords: AHP; ANP, BOCR; dynamic priorities; continuous judgments; resource allocation; conflict resolution

1. Introduction

Saaty's '*Mathematical Principles of Decision Making*' is one of my favorites of his work (Saaty T. L., 2010). Initially, I thought this book would be too difficult and mathematical to understand, yet when I started reading it I was pleasantly surprised that what I had already read in other books and articles was far better and more clearly explained here. When *IJAHP* editors decided to allocate the third issue of the journal solely to Thomas Saaty's work, I thought that I should reflect upon this book, and share my views with the audience and hopefully encourage more people to read it.

The book consists of eighteen chapters which I will try to reflect upon briefly, but I cannot omit the explanation of the Analytic Hierarchy Process (AHP) in the preface. Saaty (2010) writes,

> AHP is the thinking man's rational way to combine logic to identify connection
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> among attributes and judgments to derive priorities from causal explanation. Its
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> questions revolve around what dominates what on the average or on the whole
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> and how strongly it is expressed verbally and translated numerically with the use
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> of the absolute fundamental scale. (p. xiii)

I like to emphasize expressing judgments "verbally." Whenever I use AHP or ANP in my research, I always guide the experts to express their judgments in a verbal mode. Though this is anecdotal evidence, in almost all cases where my MBA students (quite a few of them are managers in organizations) give their judgments in a verbal mode they come up with consistent judgments if they have a choice to make about a real life multi-criteria case they have expertise about.

I will write my reflections on the book to try to emphasize the new concepts he introduced and parts I found interesting. I would especially like to communicate to our future generations. As a person who is more interested in solid applications, implementations of the theory and methodology, I want to encourage people who apply AHP/ANP to read the book so that they understand the measurement scales, how to construct logical and complete models, how to elicit pairwise comparisons without causing expert fatigue, how to test robustness of the results and how to validate it. Structuring is the most creative and challenging part of model building. One should not neglect a sensitivity analysis to test the robustness of the priorities derived for real life decisions.

2. Reflections on the book

"Judgments, as well as the priorities derived from them use absolute numbers," Saaty (2010, p. 24) writes in Chapter 1. "One can see from the literature of scales that scales derived as in the AHP are a new paradigm in measurement," he continues, and asserts that "many people do not understand well" (Saaty, 2010, p. 24). I concur with this statement considering the variety of articles I have read and reviewed for journals that were written by people, who use the AHP/ANP method but do not work directly in AHP/ANP field. I think this book would be very useful for them if they can be convinced to not be afraid of reading it like I was years ago.

I like the comments about intuition in this chapter that follow; "…..intuition is very reliable when a knowledgeable person provides judgments…." (Saaty, 2010, p. 25). I completely agree; I have observed so many experts comparing criteria and/or alternatives pairwise and giving their judgments who were very pleased with the priorities developed

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as a result of these comparisons. The market share models of McDonald's, Arby's and Burger King are another good example. As many readers of this book will know, market share derived using expert's pairwise comparisons of criteria and alternatives give results very close to the actual market share.

I had not thought about the relationship between how the brain works, "how it is characterized as a continuous response mechanism that operates through the electrical firings of neurons," and AHP/ANP until I read chapter two (Saaty, 2010, p. 29). He writes, "…there is a law of firing and synthesis of signals in the brain that we can derive from generalizing our representation of the process of creating priorities from pairwise judgments in decision making." (Saaty, 2010, p. 35) There is similarity and difference as well. In my understanding, if the total stimulus is not strong enough (is not over a threshold) the firing of the neuron may not occur and this signal may be lost. In our AHP/ANP synthesizing, however, all signals, pairwise judgments contribute to creating priorities. It is fascinating to read this chapter; I hope some researchers will explore this relationship further and discover new information in this area.

In chapter 3, Saaty explains structures in decision making. In the field of decision making a structure is the first step in organizing, representing and solving a problem. This is the creative part of decision making. As he explains it, structure must be logical and it must relate the items accurately and be complete, including everything that has important influence. I think everybody will concur with this statement, yet it is difficult to test if the structure has all these traits. Besides, if the problem is structured as a network with dependence and feedback we may not omit any judgment even if it has very little influence since locally unimportant judgments may become quite important globally because of feedback and dependence.

I like the title of chapter 3, "On the Subjective Geometry of Hierarchies and Networks." Here, Saaty relates the geometry we know with subjective geometry, vertices and edges that connect our subjective thoughts with paths and cycles. I had never related the two until I read this chapter though I understand that Saaty and H.S. Shih (Saaty & Shih, 2009) published an article on this subject in the *European Journal of Operations Research*.

I would like to write about the absolute scale, one of the scales of measurement discussed in Chapter 4, since I think this is generally one that people are the least familiar with it. I will share comments Saaty made on the Likert scale, since it is widely used in statistical analysis, with regard to rating versus making comparisons, since some people believe that rating-based ranking is a more objective approach. I will also introduce the concept of compatibility since it is used much less in the applied articles published in journals.

Saaty writes, "An absolute scale is a special instance of a ratio scale with the constant multiplier equal to one…it is dimensionless." (Saaty, 2010, p.101) Authors generally refer to ratio scale any time they introduce AHP and/or ANP in a publication, yet I have

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not read any article other than that belonging to Thomas Saaty that mentions the absolute scale.

The Likert scale is a psychometric scale commonly used in questionnaires. "In a rather haphazard way people have the habit of using ordinal numbers to create scales," Saaty writes (Saaty, 2010, p. 109). "In addition they perform meaningless arithmetic operation on such numbers that makes no sense," he continues. On the technical side, it is simply wrong to treat evaluations on a Likert scale, such as a 1 or a 5 as real numbers that can be multiplied and summed. If '5' stands for 'Excellent' and '4' stands for 'Very Good,' we know that '5' is better than '4, but we don't know by how much. We can't even assume that the distance between '5' and '4' is equal to the distance between '4' and '3.' Many users of Likert scale are simply unaware of this technical problem.

We can use a rating approach for ranking alternatives. "Rating is a quick but not accurate way of ranking," Saaty asserts (Saaty, 2010, p. 111). "If we have many alternatives, it is convenient and efficient although often misleading," he continues (Saaty, 2010, p.111). I agree with him. I would only use the rating approach for ranking many alternatives, eliminate the worst ones, then continue with pairwise comparisons for the remaining few manageable number, for example, the best ten alternatives. If I use the rating approach I would, at least, determine intensity of scales with pairwise comparisons.

Checking the consistency of judgments is a well-known concept for AHP/ANP applications. Authors always illustrate the consistency ratio (C.R) of pairwise comparisons. What we should do if the C.R. is beyond the desired 0.10 for practical applications, is well explained in chapter 4 of the book. Though almost all authors denote the consistency of the judgments, very few of them refer to compatibility. The compatibility index illustrates how close two vectors are. If we have two experts from whom we are eliciting judgments we can compare the closeness of their priorities for the alternatives via compatibility analysis.

One practical concern while implementing ANP is the number of judgments required and their validity. The ANP requires quite a few judgments, and experts may not have the patience to provide all of them. Chapter 5 addresses the expert fatigue problem and how to expedite making the decision without loss of validity. The best approach in my judgment is to construct as small of a model as possible, yet it should be a valid representation of the reality. Once you have the most compact model, with the smallest number of components, and the smallest number of elements in each component, you can reduce number of judgments elicited from the experts by comparing, for example, all the elements in the top row as that set of n-1 judgments forms a spanning tree. Of course, the most scientific approach would be to apply Harker's algorithm explained in this chapter. Harker suggests that decision maker should provide at least one judgment in each column.

Chapter 6 covers the axioms of AHP/ANP, whereas chapter 7 deals with how to construct a group judgment from the judgments of individuals comprising this group. A group

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judgment also has to satisfy the reciprocal property. It is very difficult to summarize and comment on them in this reflection. These two chapters are quite mathematical; they are for those who want to learn mathematical principals of decision making.

Validation is relevant in social as well as physical sciences, as Saaty writes in chapter 8 (Saaty, 2010). He gives examples for validation for a single matrix, for hierarchies, as well as for networks. He and a colleague find the relative wealth of seven nations as a result of pairwise comparisons with respect to relative power and standing with no direct consideration of the economic data. The judgements were based on their perception of "soft-power". They illustrate how close the values are to values obtained by normalizing the GNP of the nations. He does not do any compatibility analysis of these two vectors. I did not either; I believe he did this exercise before he came up with the concept of compatibility; yet it would be interesting to calculate one. My gut feeling, looking at the numbers, says that it is much less than 1.1. As I was writing about this, I realized that relative power needs to be defined. I think GNP is one factor of relative power, but it has other components too. I would not think that priorities developed comparing different nations with respect to relative power and standing illustrates priorities of the GNP. I would have identified factors influencing GNP and compared different nations with respect to these factors. I wished I could have discussed it with him because I hope I am not missing something.

As an example, for hierarchical validation he analyzes the decision of the U.S. Congress towards China in 2000 to grant the country Permanent Normal Trade Relations (PNTR) status. He then illustrates that with a benefits, opportunities, costs and risks (BOCR) model PNTR is a dominant alternative. I wished that Congress had actually made a decision based on the result of a BOCR analysis that they had participated as experts in giving pairwise comparisons in the BOCR model. That would have been a far more convincing validation example for me.

One ANP validation example that is given predicted the market share of airlines and compared it with the actual market share. Validation is far more challenging, in my judgment, when we don't know the results, when experts compare qualitative factors pairwise. Saaty concludes this chapter by asserting that group decision making "tends to give better results because of the broader knowledge and because of the possibility of debate" (Saaty, 2010, p. 282).

We can use priorities of the alternatives as objective function coefficients of a Linear Programming (LP) problem and determine the best resource allocation. An example of resource allocation with AHP is given in chapter 9, concerning human resource recruiting at Biological Detection Systems (BDS). Since there are 31 applicants, Saaty used the rating approach to rank them. Then these priorities are used as objective function coefficients and solved by LP subject to salary, and some other constraints. Such constraints are, for example, there should be just one marketing assistant, at most one shipping clerk, at most one quality specialist and so on. He concludes the chapter saying, "many real-world problems cannot be readily solved by LP since they usually contain

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intangible variables" (Saaty, 2010, p.300). Combining LP with AHP and or ANP "makes it possible to deal with all optimization problems whether the variables are tangible or intangible." (Saaty, 2010, p. 300) I hope I created some interest that will cause you to want to read this book because of what I commented on so far. I will be far briefer for the following eight chapters since my expertise is more limited on conflict resolution, time dependent decisions, and continuous judgments.

Chapter 10 mostly reports Saaty's conflict resolution endeavors for the Middle East Israeli-Palestinian conflict. This project, mainly funded in its later years by a well-known foundation, "involved a mixed group of Palestinians, knowledgeable pro-Israeli experts, and others from the outside, like China, Saudi Arabia, Turkey and the US." (Saaty, 2010, p.317) Over a three-day period, the team structured the problem and developed several potential alternatives. The purpose was to formulate packages of tradeoffs, comprised of both tangible and intangible factors, so that both parties would feel they were fairly treated. I think it would be interesting to many AHP scholars to read the entire process from the book as I cannot summarize it in a couple of paragraphs. At the end, the BOCR synthesis suggested that "the best policy to resolve the Middle East Conflict is to establish a two state solution." (Saaty, 2010, p.340) Sensitivity analysis suggests that results were robust. Saaty reports that "the most significant results come from the efforts and road-map generated." (p.340)

Saaty asserts, in chapter 11 that our political bodies need a formal approach for prioritization. He explains how different decision making is in politics from decision making in business. He prioritized most of the bills that were before the U.S. Congress in 2005. First, he prioritized major areas of government. Each major criterion has sub-criteria with priorities. The alternatives were rated with respect to the sub-criteria using an intensity scale determined by making pairwise comparisons. Of course, he is well aware that he needs "to seek further agreement from many more people in government to fine-tune the results he obtained." (p.365) He concludes that, "voting on bills in government is a very complicated process." (p. 365) An official who has to vote needs to "consider national objectives, constituency interest and commitments to party and to other senators or representatives." (p. 368)

Chapters 12, 13, and 14 deal with uncertainty and time dependence of judgments and answer criticisms of the AHP. He introduces the "other" criterion if the expert is unsure of his/her structure, and illustrates with a sensitivity analysis how priorities of the alternatives might change with the introduction of "other" criterion. I have not seen the "other" criterion or "other" alternative so far among the research I have read on AHP/ANP applications and/or theory. I have seen relatively little research, again so far, on time-dependent decision making, or dynamic priorities in the AHP/ANP. Chapter 13 could be a good start on these topics. There is a lot to be done on generalizing from discrete to continuous judgments, as response to stimuli.

3. Conclusion

As Saaty writes in the conclusion of the book, it takes little time to learn the mechanics of AHP/ANP, yet structuring the problem is far more challenging, requiring expertise, creativity and good knowledge about the problem. One should not ignore checking how robust the priorities are to changes in expert judgments by performing a sensitivity analysis of the problem. Readers will need to explore some more recent publications to understand sensitivity analysis in ANP models. As I wrote above, there is a lot of work to be done on continuous as well as time-dependent judgments.

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