THE LEGACY OF THE ANALYTIC HIERARCHY/NETWORK PROCESS

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"In memory of my mentor and friend Tom Saaty"

ABSTRACT

There is no doubt that the Analytic Hierarchy Process is in itself a remarkable contribution to the field of decision making. In this paper, I have tried to point out what I consider to be Tom Saaty's three most important contributions within his Analytic Hierarchy Process: group decision making, conflict resolution and the fundamental equation of pairwise comparisons. The third contribution occupied him most of his life in an attempt to connect pairwise comparisons with brain activity related to cognitive processes.

Keywords: Analytic Hierarchy Process; group decision making; conflict resolution; fundamental equation of pairwise comparisons

1. Introduction

One of the most difficult tasks modelers face is the incorporation of human behavior into decision-making. It is known that human behavior is not always rational in the way it is assumed by the rational choice school with the axiom of transitivity. In recent years, a new way of thinking has evolved using psychology and economics that is trying to show that transitivity need not always be satisfied to be a rational decision maker. Kahneman and Tversky (1979) showed the many problems that expected utility theory has as a descriptive theory of behavior leading to preference reversals; and Tversky and Thaler (1990) gave some plausible explanations as to how preference reversals may occur when people make decisions. Richard Thaler, the 2017 Nobel Prize in Economics, has demonstrated that mankind is afflicted by emotion and irrationality, which influences their decision making about everything from retirement savings, to health-care policy, to professional sports. This is in complete agreement with what Thomas L. Saaty has been saying for years.

Saaty's theory (1977, 1980, 1986), the Analytic Hierarchy Process (AHP), is based on the idea that making decisions need not assume transitivity. One could go one step further and imply that lack of transitivity in preferences may lead to rank reversals. It is one of the reasons why Saaty's theory has been criticized. However, a theory of decision-making should allow for intransitivity if we expect to capture what Thaler (2017) calls "predictably irrational" behavior. Thaler does not believe that human beings are randomly irrational. He is not the only one who believes this to be the case. Ariely (2008) also challenges the assumptions about making decisions based on rational thought.

We believe we make decisions by comparing alternatives in pairs according to different criteria, but for every pairwise comparison, we have one and only one criterion in mind. We perform all the comparisons according to all the criteria, and somehow, we synthesize all the comparisons in our brain to arrive at the final decision. Saaty created his theory to help model this process and to incorporate the experience, talent and knowledge of the actors involved in the resolution process. However, as a theory of decision-making, the AHP and its extension to networks in the Analytic Network Process (ANP) approximate how we really make decisions.

The methodology developed by Thomas L. Saaty provides the flexibility of accepting or rejecting transitivity in the modelling process. We know that a necessary and sufficient condition for rank preservation in a single matrix of pairwise comparisons is that row dominance be satisfied (Saaty & Vargas, 2012). Nonetheless, people are not always transitive, and hence, they violate the fundamental premise of the rational choice school and Thaler's "predictable irrational" behavior follows. When Tom Saaty conceived the AHP, he envisioned three fundamental problems that needed to be addressed:

(1) Group decision-making,
(2) Conflict resolution, and
(3) The role of pairwise comparisons, in particular in neural and brain activity.

2. Three important areas

2.1 Group decision-making

Now more than ever, group decision-making is critical at all societal levels. Problems are becoming more complex requiring multiple experts to understand all the dimensions of the problem, and the implications of decisions are multidimensional. We need to be able to make decisions in groups without the fear of having a decision being imposed on us. This would be the case, if all we do is rank alternatives, because then we could fall under the umbrella of Arrow's Impossibility theorem (Arrow, 1951). The starting point of Arrow's theory is a set of at least three alternatives and at least two individuals in a group. The individuals rank order the alternatives according to their preferences. The objective of group decision making is to find a procedure, known as a social welfare function (SWF), which combines the individual preferences into a group preference. Arrow (1951) imposed the following conditions to SWF: unrestricted domain (i.e., all orderings are possible), unanimity (i.e., if all the individual members prefer an alternative over another, the group should also maintain that preference), independence from irrelevant alternatives (i.e., the addition or deletion of an alternative should not alter the group ordering of the remaining alternatives) and non-dictatorship (i.e., there is no individual in the group whose preferences coincide with the group preference). Under these conditions Arrow proved that no SWF exists.

We need to ensure that a decision by a group is not a dictatorial decision. Saaty and Vargas (2012) showed that it is possible to make decisions in groups without being dictatorial so long as the intensity of preference given by the individual judgments is represented with an absolute scale, and the social welfare function is the geometric mean

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of the individual judgments. This seminal paper in group decision making on which Saaty and I collaborated was published in the journal of *Social Choice and Welfare* in 2012.

2.2 Conflict resolution

In 1981, the book "Getting to YES" revolutionized the way conflicts were looked at (Fisher & Ury 1981). Fisher and Ury introduced the concept of principled negotiation in which the participants are problem solvers. The approach is based on four principles: (1) Separate the people from the problem, (2) Focus on interests not positions, (3) Invent options for mutual gain, and (4) Insist on using objective criteria. In this approach, the parties do not see each other as adversaries, but as collaborators in search of a fair solution.

However, the approach does not measure gains and losses of the parties for different options. Thus, the parties may not be able to perceive how fair the proposed solution is. What is needed is the development of scales that represent the preferences of the parties. It is not enough to assign numbers to preferences without any mathematical assumptions because we want to ensure that the results belong to a measurement scale. This is a difficult problem if the dimensions of the conflict involve intangibles, which by definition are considered not to have a scale of measurement. Pairwise comparisons from Saaty's absolute scale can be used to build such relative measurement scales. In Saaty et al. (2017), this approach was used to show that a fair solution (developed by those involved in the process) exists.

This is just one of many examples that show that to deal with conflicts the negotiation approach needs to be measurement based. Since intangibles are always involved, we need pairwise comparisons to build measurement scales, which we can use to compute gain/loss ratios of tradeoffs from each party's perspective. Gain/loss ratios are not symmetric, and the tradeoffs are non-zero sums. Hence, measurement allows for the selection of tradeoffs for which both parties benefit equally through a MaxMin optimization model.

2.3 Pairwise comparisons and neural activity

The nervous system uses its own kind of mathematical function patterns to deal with both external and internal realities. The conscious part of the nervous system is there to respond to what happens outside by regulating externally received information signals from the senses and the skin and muscles of the body itself. To do that, it needs to communicate with its subconscious using the familiar language of neural firing. Saaty and Vargas (2017) show that because reciprocal pairwise comparisons are performed at the neural level, the division algebra of octonions, in which commutativity and associativity are not satisfied, provides the structure needed to represent mental processes (Baez, 2001).

Tom showed, by extending the discrete pairwise comparisons to continuous spaces, that the response of a neuron in spontaneous activity, *w(s),* is an eigenfunction solution of a Fredholm's integral equation of the second kind if and only if it satisfies the functional equation $\mathbf{w}(as) = b\mathbf{w}(s)$, where s represents stimuli (Saaty, 2015; 2017a,b). He called this equation *the fundamental equation of pairwise comparisons*. Its solution in the space of octonions is given by $\tilde{w}(u) = a^{\left(\frac{\ln b}{\ln a}\right)u} P(u) \oplus P(u) a^{\left(\frac{\ln b}{\ln a}\right)u}$, where *P(u)* is a periodic function of period 1. It satisfies the condition $\tilde{w}(uv) = \tilde{w}(u)\tilde{w}(v)$ if and only *P(u)* satisfies the semigroup condition *P(u+v) = P(u)P(v),* and it can generate the group of automorphisms, G_2.

In G_2, these functions are given by $\tilde{w}(u) = b^u e^{2n\pi u}$, and they are dense in the space of continuous functions defined on the octonions. Thus, all continuous functions could be expressed as linear combinations of the solution of the equation, and they could generate the group of automorphisms. In sum, any representation of brain activity with octonions could be expressed with the solution of the equation $\mathbf{w}(as) = b\mathbf{w}(s)$. According to this result, the firing of neurons through the continuous paired comparison process generates a smooth G_2-manifold in which cognition and the representations of our thoughts could take place (Saaty and Vargas, 2017).

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