SEVEN

In memoriam: Thomas L. Saaty

Thomas L. Saaty will be in our hearts, not only as a great scientist, but also a peacemaker who worked hard for world peace and conflict resolution.

Mujgan Sagir Özdemir[1]
Osmangazi University
Engineering and Architecture Faculty
Industrial Engineering Department
Eskişehir, Turkey
mujgan.sagir@gmail.com

ABSTRACT

The limitations on human performance, especially on processing information, have continuously received attention since Miller's (1956) seminal article. Thomas L. Saaty added to the body of knowledge with a paper titled, '*Why the Magic Number Seven Plus or Minus Two'*, and this memoir essay summarizes that paper and its main contribution to the literature and science (Saaty & Özdemir, 2003). The paper concludes by supporting the idea that to serve both consistency and redundancy, it is best to keep the number of elements to seven or less, and therefore Miller's seven plus or minus two is indeed a limit on our ability to process information. Thomas L. Saaty was born on the 7th month of the year and passed away on August 7*2 = 14, 2017.

Keywords: Analytic Hierarchy Process; mental limits; consistency

1. Introduction

When Tom Saaty and I decided to write this paper which was my first joint paper with him, I had already been expecting how serious, impatient, quick and productive Tom would be (Saaty & Özdemir, 2003). He was a perfect organizer, a hard worker and completed tasks immediately. There was never any time, location, or business related excuses to generate an idea and not work on it. This work ethic and characteristics explain how this Great Man was so productive and able to write hundreds of papers, many books and have a great influence on thousands of people.

[1] Acknowledgment: I have carried everything I learned from Tom with me to my country and to my students. Hundreds of class projects have been completed in my creativity class, and dozens of papers using AHP and ANP have been published by my graduate students. I am grateful to him for these and for giving me a lens through which to see the world clearly and truly.

Our motivation in the paper was to discuss validity and redundancy issues by considering Miller's (1956) famous article about the "Magic Number Seven" where he mentioned variance and covariance or correlation to explain his findings. It was such an interesting topic, however; somehow, it was difficult to make the connections. The following section will briefly explain some of the main findings of the paper by making connections with Miller's channel capacity.

In 1956, George Miller conjectured that there is an upper limit on our capacity to process information on simultaneously interacting elements with reliable accuracy and with validity. He stated that the "amount of information" is exactly the same concept that we have talked about for years under the name of "variance." The equations are different, but if we hold tight to the idea that anything that increases the variance also increases the amount of information we cannot go far astray. The similarity of variance and amount of information might be explained this way. When we have a large variance, we are very ignorant about what is going to happen. If we are very ignorant, then when we make the observation it gives us a lot of information. On the other hand, if the variance is very small, we know in advance how our observation will come out, so we get little information from making the observation (Miller, 1956).

Connecting and extending this idea to decision making was an interesting topic, especially when consistency is the key issue in AHP and ANP. Our motivation in the main paper was to show that in making preference judgments on pairs of elements in a group as we do in the Analytic Hierarchy Process (AHP), the number of elements in the group should be no more than seven. This is because of the consistency of information derived from relations among the elements. The next section briefly explains the main paper (Saaty & Özdemir, 2003).

2. Main findings

The AHP relies on relative judgment about the degree or intensity of dominance of one stimulus of a pair over the other with respect to a given property present to the observer. Such a comparison is made by first identifying the smaller or lesser stimulus as the unit and then estimating how many times the greater stimulus is a multiple of that unit. When all the comparisons are made, a scale of priorities is derived from them that represents the relative dominance of the stimuli. We learn from this approach that "not only must the sensations be homogeneous or close in order for the comparisons to be meaningful, but also there must be a limit to the number we can process at one time." (Saaty, 2003, p.235) On the other hand, maintaining consistency in our judgments is also required.

We describe a method of deriving, from the observer's quantified judgments, a set of weights, w_i to be associated with individual stimuli. What this approach achieves is to put the information into usable form without deleting information residing in the qualitative judgments. As defined in detail in the main paper, let $A_1,\ldots,A_n$, be the set of stimuli. The quantified judgments on pairs of stimuli are represented by an *n-by-n matrix A.* The problem is to assign to these n stimuli a set of numerical weights, $w_1,\ldots,w_n$, that would reflect the recorded judgments. In the simplest terms, a priority vector w can be used to weight the columns of its matrix and sum the elements in each row to obtain a new priority vector and repeat the process, thus obtaining an infinite set of priority vectors. The question is which is the real priority vector? Such ambiguity is eliminated if we

ISSN 1936-6744
https://doi.org/10.13033/ijahp.v9i3.552

require that a priority vector satisfy the condition, $Aw = cw,\ c > 0$. In other words, ratios of priorities in the new vector coincide with the same ratios in the old vector. (Saaty & Özdemir, 2003).

In light of this, for the validity of the vector of priorities to describe the response, greater redundancy is needed and therefore a large number of comparisons. On the other hand, for consistency a small number of comparisons is needed. To find the optimum number, the psychological idea of the consistency of judgments and its measurement is discussed in the paper together with the central concept in matrix theory, and the size of our channel capacity to process information (Saaty & Özdemir, 2003). It is the principal eigenvalue of a matrix of paired comparisons. It is shown that every 2-by-2 positive reciprocal matrix is consistent, though not every 3-by-3 positive reciprocal matrix is consistent, but in this case we are fortunate to have explicit formulas for the principal eigenvalue and eigenvector (Saaty, 1996).

As we know, the consistency ratio (C.R.) of a pairwise comparison matrix is the ratio of its consistency index to the corresponding random index value in Table 1. Figure 1 below is a plot of the first two rows of Table 1.

Table 1
Random Index

Order	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
R.I.	0	0	0.52	0.89	1.11	1.25	1.35	1.40	1.45	1.49	1.52	1.54	1.56	1.58	1.59
First Order Differences		0	0.52	0.37	0.22	0.14	0.10	0.05	0.05	0.04	0.03	0.02	0.02	0.02	0.01

The notion of order of magnitude is essential in any mathematical consideration of changes in measurement. When one has a numerical value, say between 1 and 10 for some measurement, and wishes to determine whether change in this value is significant or not, one reasons as follows: a change of a whole integer value is critical because it changes the magnitude and identity of the original number significantly. If the change or perturbation in value is of the order of a percent or less, it would be so small (by two orders of magnitude) and would be considered negligible. However, if this perturbation is a decimal (one order of magnitude smaller) we are likely to pay attention to modify the original value by this decimal without losing the significance and identity of the original number as we first understood it to be.

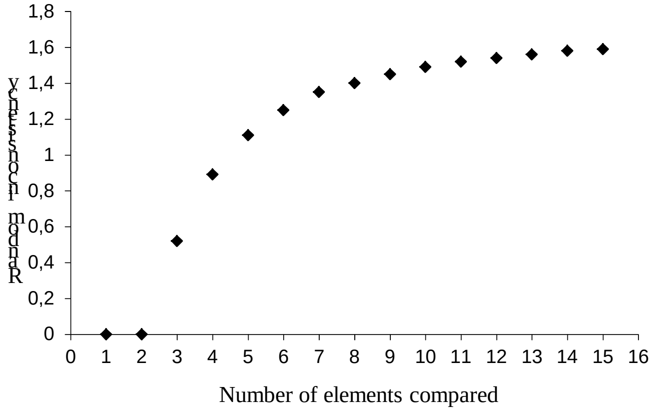

Figure 1. Plot of random inconsistency

Thus, in synthesizing near consistent judgment values, changes that are too large can cause dramatic change in our understanding, and values that are too small cause no change in our understanding. We are left with only values of one order of magnitude smaller that we can deal with incrementally to change our understanding. It follows that our allowable consistency ratio should be not more than about .10.

In the light of the above explanations, the quality of response to stimuli is determined by three factors. These factors are accuracy or validity, consistency, and efficiency or amount of information generated. Our judgment is much more sensitive and responsive to large perturbations. When we speak of perturbation, we have in mind numerical change from consistent ratios obtained from priorities. The larger the inconsistency and hence the larger the perturbations in priorities, the greater our sensitivity to make changes in the numerical values assigned. Conversely, the smaller the inconsistency, the more difficult it is for us to know where changes should be made to produce not only better consistency but also better validity of the outcome.

Similarly, the third row of Table 1 gives the differences between successive numbers in the second row. Figure 2 is a plot of these differences and shows the importance of the number seven as a cutoff point beyond which the differences are less than 0.10 where we are not sufficiently sensitive to make accurate changes in judgment on several elements simultaneously.

Stability of the principal eigenvector also imposes a limit on channel capacity and highlights the importance of homogeneity. To a first order approximation, perturbation Δw_1 in the principal eigenvector w_1 due to a perturbation ΔA in the matrix A where A is consistent as given by Wilkinson (1965):

$$\Delta w_1 = \sum_{j=2}^{n} (v_j^T \Delta A\, w_1 / (\lambda_1 - \lambda_j) v_j^T w_j) w_j$$

Here, T indicates transposition. The eigenvector w_1 is insensitive to perturbation in A; if the number of terms n is small, if the principal eigenvalue λ_1 is separated from the other eigenvalues λ_j, and, if none of the products $v_j^T w_j$ of left and right eigenvectors is small but if one of them is small, they are all small. However, $v_1^T w_1$, the product of the normalized left and right principal eigenvectors of a consistent matrix is equal to n that as an integer is never very small. If n is relatively small and the elements being compared are homogeneous, none of the components of w_1 is arbitrarily small and correspondingly, none of the components of v_1^T is arbitrarily small. Their product cannot be arbitrarily small, and thus w is insensitive to small perturbations of the consistent matrix A. The conclusion is that n must be small, and one must compare homogeneous elements.

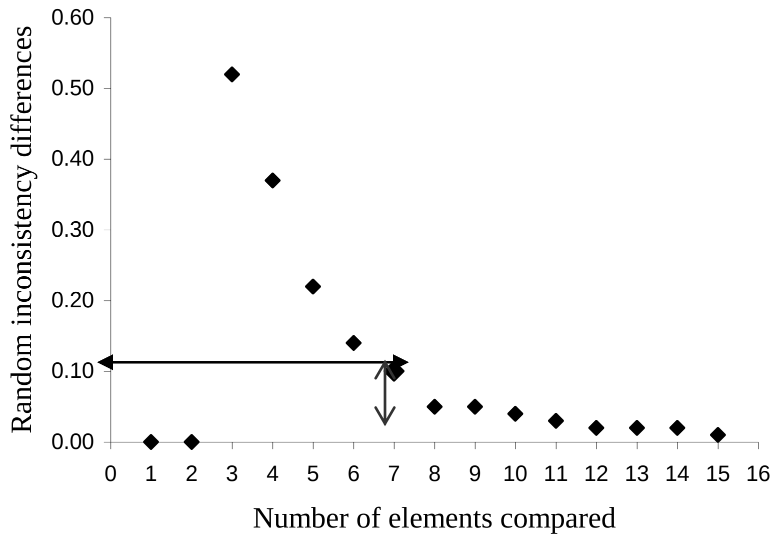

Figure 2. Plot of first differences in random inconsistency

3. Conclusions

Saaty's paper on "Magic Number Seven" concludes that, "The consistency of judgments is necessary for us to cope effectively with experience but it is not sufficient. We need redundancy of informed judgments to improve validity. However, redundancy gives rise to inconsistency. Therefore, we need to make a tradeoff between consistency and redundancy that implies validity." (Saaty, 2003, p.243) Our measure of random inconsistency reveals that as the number of elements being compared is increased the measure of inconsistency decreases so slowly that there is insufficient room for improving the judgments and therefore also consistency. We conclude that to serve both consistency and redundancy, it is best to keep the number of elements seven or less.

ISSN 1936-6744
https://doi.org/10.13033/ijahp.v9i3.552

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