THEORIES ON COEFFICIENT OF VARIATION SCALES TRIANGLE AND NORMALIZATION OF DIFFERENT VARIABLES: A NEW MODEL IN DEVELOPMENT OF MULTIPLE CRITERIA DECISION ANALYSIS

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ABSTRACT

This paper is an attempt to solve various problems by the two factors of mean and standard deviation (SD) of variables, introducing coefficient of variation (CV) of data as the best option for prioritization, scaling, pairwise comparison and normalization of quantitative and qualitative variables. An algorithm was built based on a coefficient of variation scales triangle (CVST) consisting of natural numbers with coefficients of binomial expansion for each line, followed by new and independent grading and scaling. In view of the existing factors, the theory provides higher generalization and maximum reliability rates in comparison to other methods for multiple-criteria decision analysis (MCDA). On the other hand, in the normalization process of different variables (i.e. de-scalarization), a precise and infinite model was presented based on coefficient of variation scale triangle (multipurpose triangle), in such a way that decision makers could work with the software in a more convenient and precise manner. Therefore, the proposed theories may be considered as a new approach and definition in the valuation and progress of MCDA.

Keywords: theories; coefficient of variation scales triangle (CVST); multiple-criteria decision analysis (MCDA)

1. Introduction

The Analytic Hierarchy Process (AHP) is a multi-criteria decision making (MCDM) method that helps the decision-maker who is facing a complex problem with multiple conflicting and subjective criteria. Several papers have compiled AHP success stories in many different fields (Vargas, 1990; Ho, 2008; Golden, Wasil, & Harker, 1989; Harker & Vargas, 1990; Shim, 1989; Saaty, & Forman, 1992; Forman, & Gass, 2001; Kumar, & Vaidya, 2006; Omkarprasad & Sushil, 2006; Liberatore, & Nydick, 2008; Zahedi, 1986). The oldest reference found is Saaty (1972a). After this, Saaty (1977b) precisely described the AHP method. The vast majority of applications still use the AHP as described in this first publication and are unaware of successive developments. The Analytical Hierarchy Process (AHP) technique, developed by Thomas L. Saaty, is an MCDA method that helps decision makers make the best decisions in the face of complex problems consisting of multiple conflicts and internal criteria. The method has already been tested and established in many various fields of work (Saaty, 1980c). Recently, the application of AHP in animal science for

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the selection of the best dairy cows was applauded by researchers and scholars (Alitaneh, Naeeimipour, & Golsheykhi, 2015). Though the AHP covers many of the existing issues in this area, it loses most of its functionality of correlation between various factors, since the assumption is not valid for studying the effects of the interior and exterior environment. This major limitation led the developer of the AHP method to work on and present the Analytic Network Process (ANP). This new method takes into account the intertwined relations of decision making elements by replacing the hierarchical structure with a network one. The Analytic Network Process is considered a more expanded version of the AHP (Saaty, 1999d). In general, in the AHP method the question that is asked in the pairwise comparisons is which of the 2 elements is more effective. For the same purpose, Saaty proposed that the intensity detected for inter-factor comparisons be graded on a scale of 9. Thus, based on relative significance the scale of 9 shows that a factor is more significant than another while the scale of 1 shows no difference or equal significance. In AHP, once the relative weight vectors are calculated, Saaty suggests a consistency rate (CR) of 0.1 for reliability and acceptance of a judgment on the pairwise comparisons matrix. Otherwise, further study of the problem and re-evaluation of the matrices is recommended.

Briefly, Harker, and Vargas (1987) evaluated a quadratic and a root square. Lootsma (1989) argued that the geometric scale is preferable to the 1–9 linear scale. Salo and Hamalainen (1997) point out that the integers from 1-9 yield local weights, which are unevenly dispersed, so there is lack of sensitivity when comparing elements, which are preferentially close to each other. Based on this observation, they proposed a balanced scale where the local weights are evenly dispersed over the weight range [0.1, 0.9]. Earlier Ma, and Zheng (1991) calculated a scale where the inverse elements x of the scale 1/x are linear instead of the x in the Saaty scale (see Table 1).

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Table 1
Different scales for comparing two alternatives

Scale type	Values								
Linear (Saaty,1977b)	1	2	3	4	5	6	7	8	9
Power (Harker, and Vargas, 1987a)	1	4	9	16	25	36	49	64	81
Root square (Harker, and Vargas,1987b)	1	1.41	1.73	2	2.23	2.45	2.65	2.83	3
Geometric (Lootsma,1989)	1	2	4	8	16	32	64	128	256
Inverse linear (Ma, and Zheng, 1991)	1	1.13	1.29	1.5	1.8	2.25	3	4.5	9
Asymptotical (Dodd, and Donegan,1995)	0	0.12	0.24	0.36	0.46	0.55	0.63	0.70	0.76
Balanced (Salo and Hamalainen, 1997)	1	1.22	1.5	1.86	2.33	3	4	5.67	9
Logarithmic (Ishizaka, Balkenborg, and Kaplan, 2006)	1	1.58	2	2.32	2.58	2.81	3	3.17	3.32

In this research and based on balanced applied theories of grading, some sort of a 9 grade scale was proposed for scalar triangles of CV (CVST), and a type of new approach for de-scalarization of quantitative variables (normalization). The resulting triangular analysis has some special features, including:

1. The ability to grade in higher scales based on the researcher’s decision.
2. Reasonable and orderly mathematical structure (binomial expansion).
3. Achieving a smaller consistency rate in comparison to Saaty’s CR.
4. No limitation in terms of number of variables (*n* variables) through de-scalarization of measurement units based on a range of 1-9 scales.
5. Easy registration of analyzed data in software applications based on 1-9 scales.

Generally, the relative scales triangles are based on CV of natural numbers, and since mean and SD parameters are taken together, the decision maker is able to choose the best criteria and target based on pairwise comparisons. Saaty (1977b) showed that the geometric mean is the most suitable mathematical rule for combining AHP judgements. In scale triangle theory however, a scale is used that takes not only the mean of natural numbers but also the variance and SD. Thus, it seems CV is a better and more precise method for normalization of numbers, followed by pairwise comparisons with the geometric mean or other methods. On the other hand, the AHP method uses nominal mean and scales (linear de-scalarization) for prioritization of the

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normalization process and pairwise comparisons. In the present paper, normalization and de-scalarization of quantitative variables is based on a scale triangle (multidimensional triangle).

2. Data and methodology

2.1 Coefficient of Variation (CV)

When calculating the dispersion of data, one always faces data measured with various scales. Thus, to compare the dispersion of data collected from the statistical population measured with various scales, the use of only mean or variance does not provide accurate results as both are dependent on measurement scales. For the same reason, a more reliable scale such as coefficient of variation (CV) is used, which is calculated by dividing the standard deviation (SD) on the mean according to the following equation.

$$CV = \frac{\sigma}{\mu} \tag{1}$$

As can be observed, mean (μ) and standard deviation (σ) lack the required level of precision on their own, and may not be considered as the target criteria. The CV is free of dimensions and for the same reason it is suitable for comparison of statistical data with various measurement units. It is therefore quite good for normalization and de-scalarization of different variables and factors *(*Everitt, 1998).

This paper attempts to explain the application of the scalar triangle in coordination and comparing various decisions. The scalar triangle helps decision makers conceptualize their decision options in an integrated manner using one scalar structural model, such that the optimal decision includes the ideas of all members and the decision maker. It may then be used as the best tool for making decisions in the shortest possible time. This paper provides an overview of the various stages of decision making and normalization of quantitative and qualitative variables using a scalar triangle in comparison to the hierarchical tree.

2.2 Scales triangle

The first step includes the creation of a triangular structure of natural numbers (*n*). In order to have a 9-step grading system the range of numbers was defined as 1-54 and the position of each number in the triangle was called a "cell" ($0<n\leq54$). The purpose of this triangle is to express the studied problem in view of each topic and it needs to have easy software navigation in order to enable quick and optimum decision making. The triangle is graded in 9 lines. However, there is room for expanding the scaled spectrums and the structure of the scalar triangle theory based on n increased number of cells and lines (expansion of the definition of natural numbers in cells, lines and scales). Basically, each horizontal axis (x) of the triangle is a set of several numeral cells expressing one line of decision making (see Table 2). Thus, based on the triangular structure of the table below and the related descriptions, the triangular scales are calculated and analyzed in 9 lines.

First, as a can be seen in the Table 2, for each line a mathematical function is defined based on the coefficients of binomial expansion, such that the computational potential of each line is an ordinal number specific to the same line.

Table 2
Natural numbers triangle

Line(l)	Equation	Numbers									
1	$(X_i + Y_j)^{l_1}$					1	2				
2	$(X_i + Y_j)^{l_2}$				3	4		5			
3	$(X_i + Y_j)^{l_3}$				6	7	8	9			
4	$(X_i + Y_j)^{l_4}$			10	11	12		13	14		
5	$(X_i + Y_j)^{l_5}$			15	16	17	18	19	20		
6	$(X_i + Y_j)^{l_6}$		21	22	23	24		25	26	27	
7	$(X_i + Y_j)^{l_7}$		28	29	30	31	32	33	34	35	
8	$(X_i + Y_j)^{l_8}$	36	37	38	39	40		41	42	43	44
9	$(X_i + Y_j)^{l_9}$	45	46	47	48	49	50	51	52	53	54

Thus, if L and K are normal numbers in the presumed triangle, for $(X+Y)^L$ we have:

$$L=1,2,\dots,n \qquad K=1,2,\dots,m$$

$$(x+y)^L=\sum_{K=1}^{L}\binom{L}{K}X^{L-K}Y^{K} \rightarrow \binom{L}{K}=\frac{L!}{K!(L-K)!} \tag{2}$$

The above equation is a fixed binomial coefficient. Hence, the setting of the binomial expansion triangle for each line was based on this. In the next step, the X and Y axes must be defined for each cell and the whole line numbers triangle. The basic assumption of the scalar triangle is that each cell is formed by the horizontal X vector (addition of all numbers on x axis) and vertical Y vector (multiplication of numbers on the Y axis).

$$A = \{(x, y) | x, y \in \{1,2, \dots, n\}, x = y\} \tag{3}$$

Moreover, to determine the exponent of each line, and if N_n is a set of natural numbers with a finite range, for each line we have the following condition:

$$N_n = \{1,2, \dots, n\} \qquad f: N_n \rightarrow L \qquad f(n) = l_n \tag{4}$$

Besides taking the 9 assumed lines in the triangle, the paper goes on to define the L_n condition of a set of natural numbers up to L_9. Thus, the basics for the exponent of each line equation were defined according to L_n condition. Now, the final equation for each line of the triangle may be defined as follows:

$$T_{ij} = \sum_{i=1}^{L}\left(X_i + Y_j\right)^{l_n} \tag{5}$$

Now, whenever a total of X values and the numeric value of each cell is equal to the last line of the assumed triangle, we have, the X value on the horizontal axis of each cell of the triangle is equal to:

$$X_i \times C_i = L_9 \rightarrow X_i = L_9 \times C_i \tag{6}$$

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On the other hand, if the result of multiplication of Y in the numerical value of each the cell is assumed to be equal to the last line of the triangle, we have, the Y value on vertical axis of each triangle cell is equal to:

$$Y_j + C_i = L_9 \rightarrow Y_j = L_9 - C_i \quad (7)$$

To prevent dispersion, and in favor of the normal distribution of data and variables, a base 10 logarithm condition was added to the equation.

$$T_{ij} = \log_{10}[(L_9 \times C_i + L_9 - C_i)^{l_n}] \quad (8)$$

T_{ij}: the final number for each cell from numbers triangle equation, where each cell includes a presumed number (T_{ij}).
C_j : the assumed number for each cell in the scales triangle is between 1 and 54
L_9 : the assumed number for the last line of the triangle is the natural number 9
l_n : the exponent value assumed for each line, which is between natural numbers 1 and 9 in the scales triangle.

Finally, in order to make a unified and outstanding curve of the balance of the x and y vectors, each natural number existing in each cell (1 to 54) is calculated through the overall T_i equation, therefore creating a set of numbers with normal and balanced distribution (see Table 3).

Table 3
Balance numbers for the CVST

$(X_i + Y_j)^2$	3.037	3.226	3.380							
$(X_i + Y_j)^3$	5.268	5.439	5.590	5.725						
$(X_i + Y_j)^4$	7.798	7.947	8.085	8.212	8.331					
$(X_i + Y_j)^5$	10.553	10.684	10.807	10.923	11.034	11.139				
$(X_i + Y_j)^6$	13.488	13.603	13.713	13.819	13.921	14.019	14.113			
$(X_i + Y_j)^7$	16.571	16.674	16.773	16.870	16.963	17.053	17.141	17.226		
$(X_i + Y_j)^8$	19.782	19.874	19.964	20.052	20.138	20.221	20.303	20.382	20.460	
$(X_i + Y_j)^9$	23.103	23.187	23.269	23.350	23.428	23.506	23.581	23.656	23.728	23.800

Now, in one of the most significant steps and according to Equation (8), the total of the normal data for each is calculated for each line using the CV equation and the resulting numbers are introduced as random indices of the scales triangle. By finding a random index equation, one can attain the final goal of this paper, which is prioritization and special scaling/grading by means of a triangular structure of natural numbers (see Table 4). It must be noted that a random index is needed for the calculation of an inconsistency rate. Nevertheless, in the description and definition of this theory there is no need for the use of a random index in the scales triangle. Another advantage of this model is that it can make use of a random index of hierarchical analysis in the calculation of inconsistency rates of numbers triangle.

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Table 4
Random index (RI) for the CVST

Size of matrix	1	2	3	4	5	6	7	8	9	10
RI	9	5.34	3.58	2.61	2	1.62	1.35	1.15	1	0.87

In the final step, after the CV is calculated and the values of random indices are set, the final scale (grading) is also calculated in the scales triangle model through the following equation:

$$\mathrm{W_{ij}} = \frac{\mathrm{X_i max}}{\mathrm{X_{ij}}} \tag{9}$$

2.3 For example (line 2)

$T_{ij} = \log_{10}[(9 \times 3 + 9 - 3)^2] = 3.037$
$T_{ij} = \log_{10}[(9 \times 4 + 9 - 4)^2] = 3.226$
$T_{ij} = \log_{10}[(9 \times 5 + 9 - 5)^2] = 3.380$
$CV_T = 3.037\,,\ \ 3.226\ ,\ \ \ 3.380 = 5.34$ (RI)
$\mathrm{W_{ij}} = \frac{9}{5.34} = 1.6$ (Equally to moderately)

Where, W_{ij} is the final scale or weight, $X_i max$ is the largest relative index, and X_{ij} is each of the relative indices. Therefore, the final scale was determined according to the analysis of a scales triangle of CV through formulation of natural numbers as described below (Table 5).

Table 5
Pairwise comparison scale for CVST preferences

Numerical rating ($\mathbf{W_{ij}}$)	**Verbal judgments of preferences**
9	Extremely preferred
7.8	Very strongly to extremely
6.6	Very strongly preferred
5.5	Strongly to very strongly
4.5	Strongly preferred
3.4	Moderately to strongly
2.5	Moderately preferred
1.6	Equally to moderately
1	Equally preferred

One of the most important advantages of using scale triangles is the lower inconsistency rate in comparison to AHP, which is an indication of its higher precision in the analysis of different variables. For instance, pairwise comparison A is related to AHP scaling and matrix B is related to scales triangle scaling (CVST).

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$$A=\begin{bmatrix} 1 & 2 & 6 & 5 & 1/2 \\ 1/2 & 1 & 4 & 3 & 1/3 \\ 1/6 & 1/4 & 1 & 1 & 1/6 \\ 1/5 & 1/3 & 1 & 1 & 1/5 \\ 2 & 3 & 6 & 5 & 1 \end{bmatrix} \quad \lambda_{max} = 5.083$$ CI=0.0208 CR=0.0186 < 0.1

Dividing all the elements of the weighted sum matrices by their respective priority vector element, we obtain:

(.2909, .1693, .0547, .0621, .4230)

$$B=\begin{bmatrix} 1 & 1.6 & 5.5 & 4.5 & \frac{1}{1.6} \\ \frac{1}{1.6} & 1 & 3.4 & 2.5 & \frac{1}{2.5} \\ \frac{1}{5.5} & \frac{1}{3.4} & 1 & 1 & \frac{1}{5.5} \\ \frac{1}{4.5} & \frac{1}{2.5} & 1 & 1 & \frac{1}{4.5} \\ 1.6 & 2.5 & 5.5 & 4.5 & 1 \end{bmatrix} \quad \lambda_{max} = 5.031$$ CI=0.0078 CR=0.0070 << 0.1 OK

(.2953, .1805, .0615, .0708, .3919)

Comparing vectors by compatibility:

G= 94.02% > 90% Thus, priority vectors A and B are compatible vectors (equivalent vectors for measurement purposes).

Moreover, the inconsistency rates calculated by the Saaty method and scales triangle method demonstrated that they had much better results than pairwise comparison A and scale triangle matrix B coefficient of variation. This proved the high precision and efficiency of the proposed method. In this method each cell is a function of an equation, and eventually the total of the equation of each given line is governed by the CV of the same line.

Table 5 provides good insight into the comparisons and preferences (oral judgment) of a scales triangle containing the significance of various factors. In summary, the basis for a scales triangle may be described in a few stages:

1. Creating a triangular structure of natural numbers.
2. Creating 9 numeric lines for pairwise comparisons.
3. Defining binomial expansion functions for each line of the triangle $(X+Y)^L$.
4. Calculating or approximating numbers in each cell based on a logarithmic formula (formulation).
5. Calculating the CV of a set of numbers produced by the formulation of each line (e.g. in line 1).
6. Finding a random index for each line.
7. Finding the final rates through the random indices equation.
8. Calculating and achieving smaller inconsistency rates.
9. Better efficiency and precision in analysis of qualitative variables.

2.4 Normalization of variables

One of the most important issues in MCDM is the existence of various scales for quantity and quality indices. Initially, it was just a lack of a standard for measurement

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of quality indices, for which Saaty used a linear de-scalarization technique. Instead of his technique, we have applied the scales triangle technique. The variable normalization method is also based on the scales triangle. In general, each variable must be normalized and correlated if it is to be measured and weighed along with other variables so that its measurement and weighting is in relation to all other variables. One the biggest problems of a decision maker is how to enter quantity values into related software (data bigger than 9 and less than 1).

The “narrow” range of 1 to 9 should not be a problem when the model is correctly made as can be observed in research works related to medicine, biology and animal science. For instance, there are two dairy cows. Cow 1 has excellent body weight, a daily milk production of 110.27 lbs, a somatic cell count of 1459, and milk impurity of 0.83%. Cow 2 has medium body weight, a daily milk production of 95.08 lbs, a somatic cell count of 2610, and a milk impurity rate of 0.47%. The decision maker uses a software application to analyze quality variables based on existing scales, but is unable to make suitable analyses in a short time and with high precision in terms of milk production, somatic cells, and impurity rate. Also, the AHP/ANP software needs considerable time for pairwise comparisons between various groups due to the existence of various units and traits.

In order to address such problems, the present study focuses on making precise and actual estimations of comparisons and attempts to develop a suitable model for normalization of quantity variables (de-scalarization). As noted earlier, when using CV for calculation of the mean there is a need for more than one data or variable, and since this is not possible for data available on dairy cattle (each cow has a record of various factors), one must look for a model capable of properly normalizing all records and variables based only on the one reported data.

Let’s assume a set of rational numbers as a set whose numbers may be written in the following general format:

$$Q = \{X = a/b \mid (a,b)\ Z\ ,\ b > 0\} \qquad (10)$$

In this method, the range of normalization of a quantity variable is a set of numbers bigger than zero (rational numbers) which must be standardized if the primary variable is to be normalized. Thus, the first practical step is to calculate the square root and then the inverse power of the last line of the triangle is used to formulate and finalize the two equations. It must be emphasized again that to analyze the CV of a number there must be more than one data, variable, record, parameter, etc. For the same reason, two equations were extracted from the scales triangle with the maximum continuity and correlation, and their CV was analyzed with minimum error. Besides, in order to analyze two desirable and balanced equations T_q which is related to the mathematical model of scales triangle, the following steps were followed:

- At first, if Q is taken as a number bigger than zero and its square root is calculated, we have: $X_q = \sqrt{Q}$
- After that, using the equation of the last line of triangle and dividing the power of the first line by that of the last line, the first equation (T_{q1}) was produced, and then the second equation (T_{q2}) included the result of the first equation and the primary variable. At the end of this process, the two

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standard and balanced equations T_{q1} and T_{q2} were formulated with maximum correlation and reliability:

$$T_{q1} = \log_{10}[\left(L_9 \times x_q + L_9 - x_q\right)^{1/l_9}] \tag{11}$$

$$T_{q2} = \log_{10}[\left(L_9 \times x_q + L_9 - T_{q1}\right)^{1/l_9}] \tag{12}$$

In these two equations, due to reduced significant error and maximum reliability of the analysis result, the values of T_{q1} and T_{q2} showed better numerical correlation, as confirmed by the correct and relevant results obtained from calculation of their numerical value and CV (2 variables) through Equation 13. Interestingly, in analysis of numbers bigger than 0 the CV was on a 9-grade scale. It must be noted that this mathematical model is capable of numerical calculations up to *n* numbers. It must be emphasized that the obtained CV was also normal and lacked statistical units (de-scalarization).

$$CV_T = \sum_{q>0}^{n} T_{q1}\ T_{q2} \tag{13}$$

Generally, normalization by a scales triangle goes through the following stages:

1. Getting the square root of the related data or variable.
2. Formulating two equations based on the last line of the triangle.
3. Finding more than one data (at least 2) for the CV calculation.
4. Calculating the CV for the two data.
5. Normalizing several variables with various scales, grading and performing pairwise comparisons.

Based on this and a normalization procedure on scales triangle, for example the records of dairy cow production rates in Table 6, we have:

$X_q = \sqrt{110.27}$ = 10.5
$T_{q1} = \log_{10}[(9 \times 10.5 + 9 - 10.5)^{1/9}] = 1.0142$
$T_{q1} = \log_{10}[(9 \times 10.5 + 9 - 1.0142)^{1/9}] = 1.0564$
$CV_T = 1.0142\ , 1.0564 = 2.88$ (Normalized)

Table 6
Normalizing several variables with various scales

Milk impurity (%)	Somatic cell count(number)	Daily milk (lb)	Body weight (Quality trait)	Cow 1
0.83	1459	110.27	Excellent	Data
4.53	2.15	2.88	9	Normalized
Milk impurity (%)	Somatic cell count(number)	Daily milk (lb)	Body weight (Quality trait)	Cow 2
0.47	2610	95.08	Medium	Data
4.67	2.03	2.92	4.5	Normalized

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As shown above, in this stage all quantity variables are scaled by means of the scales triangle. Thus, the pairwise comparisons were performed with high speed and accuracy in comparison to other methods.

3. Conclusion

One condition for the efficiency of the above process is to prioritize research plans and projects using methods suitable to prioritization. Based on the present paper, the theory of scales triangle (CVST) and normalization of quantitative and qualitative variables shows a high potential for extensive use in various fields. This is due to its high capacity in modeling actual problems, high speed of analysis and the ease of learning for users. Though mathematical and statistical methods provide optimized results for planning and decision making, such techniques and models often require precise and definite data, which is a serious issue in actual conditions where it is difficult to collect data and takes a lot of time. Hence, it may be said with near certainty that this new proposed method may play a major role in grading, normalizing (descalarization) of different variables and making hard decisions. It seems the theories of scales triangle and normalization of numerical variables is an integrated model for analysis for quantitative and qualitative variables. Particularly, these theories are designed to show newer, better and faster paths with high usability potential and ease of software navigation. Since grading methods of other scholars are based on Saaty's 1-9 scale analysis, it is safe to claim that the new model is independent of any existing grading methods. The scales may be increased or decreased based on increased number of cells and lines in the scales triangle (noting that while solving problems the equations must be calculated according to the last line (l_n) of the triangle). The present paper attempted to ease the use of related software by prioritization into a 1-9 scale. Efforts were also focused on putting the scaling into a process that is dependent on data and final weights.

In the end and given the wide use of AHP/ANP, it seems that more future works will focus on the scales triangle method. It is imperative to note that such decision making techniques, like all other methods, only serve to convert data into information for decision makers, and it is the decision maker who has to make the best choice. The method proposed here may lead to better results and changes in pairwise comparisons. The paper demonstrated, on the other hand, that the structure of a numbers triangle is capable of including and analyzing several equations at once. This may lead more researchers into the field in the near future. The author hopes these results play a significant role in the expansion and clarification of comparisons and decisions, and finally creates a unified framework for proper selection and a new applied method for multiple-criteria decision making analysis.

Finally, the results of this research in relation to a new scale, known as the coefficient of variation scales triangle (CVST), suggest it can be used along with other scales. Also, the CVST model can be identified as a suitable and practical method.

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