AT THE JUNCTION OF MATHEMATICS AND PSYCHOLOGY: COGNITIVE ORIENTATION OF THE AHP/ANP AND NEW PERSPECTIVES OF STRUCTURING COMPLEXITY¹

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ABSTRACT

This paper aims to draw attention to the interdisciplinary research of the AHP/ANP methodology by emphasizing how it can be studied from a cognitive perspective. We provide an overview of the main cognitive approaches in decision-making, and consider different heuristics that lie at the basis of pairwise comparisons. We emphasize that the AHP/ANP must be considered at the junction of mathematics and psychology, and for further development of the methodology, we should examine the AHP/ANP from the cognitive point of view. We review the recent experimental studies of the AHP/ANP that test human behavior in real decision problems. We also discuss the future applicability of the AHP/ANP methodology in the Experience Age - the age of not only digital information and knowledge, but also behavior. This article is just a small step on the way to discovering the cognitive aspects and future extensions of decision making with the AHP/ANP.

Keywords: cognitive decision making; AHP; ANP; heuristics; heuristic decision making; cognitive psychology; experience age

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1. Introduction

Decision making is one of the fundamental cognitive processes of human beings. How are decisions made? Gigerenzer and Gaissmaier (2011) proposed three major answers to this question. They proposed that the mind applies logic, statistics, or heuristics. Each of these is suited to a particular kind of problem. While rational thinking is common in studying complex problems, analytical approaches lose their relevance when the probabilities of uncertain events are not adequately computed in formal models. In that case, heuristics is often the only practical method for decision making under uncertainty.

The term "heuristics²" implies an efficient cognitive process, conscious or unconscious, that ignores part of the information (Gigerenzer & Gaissmaier, 2011). The classical view of heuristics states that heuristic decisions imply greater errors than do "rational" decisions as defined by logic or statistical models. On the other hand, many studies show that a good heuristic can be better than a complex strategy when used in the proper environment (Mousavi & Gigerenzer, 2014).

In spite of different views on the nature of heuristics, the uncontestable fact is that in a world of uncertainty heuristics is an indispensable tool. Continuing the idea that our decisions are governed more by heuristics than by logic, how we use heuristics conforms to the natural human practice of making comparisons. Regarding Saaty's concept of decision processes, the AHP/ANP are psychophysical theories of measurement where comparisons are made based on feelings and judgments (Saaty, 2008). Thus, heuristics lies at the basis of the AHP/ANP pairwise comparisons.

For all that has been written about the AHP/ANP, much misunderstanding still exists (Whitaker, 2007). Moreover, the AHP/ANP concepts have been both highly praised and strongly criticized. This dichotomy is largely due to the difficulty of testing the methods because the AHP/ANP incorporates both quantitative and qualitative criteria (Ishizaka, et. al., 2011). From our point of view, in order to give convincing support of the applicability of the AHP/ANP in the decision making process, we should follow the new way of cognitive orientation of the methodology and use the methods of experimental economics that help understand human behavior while making decisions with the AHP/ANP.

The AHP/ANP is not just a mathematical theory, but an interdisciplinary approach that lies at the junction of mathematics and psychology, conscious and unconscious, tangible and intangible, subjective and objective truth. It is this interdisciplinary approach that we need to adopt in the new reality, which is the post-digital³ age. Increasingly, we are no longer in a world where digital technology and media are separate from a 'natural' human and social life (Jandrić, et al., 2018). According to Simon Jenkins (The Guardian, 2017), "We are now heading for "post-digital", the age of experience". The Experience Age is

² Heuristics - a way of solving problems by discovering things yourself and learning from your own experiences (Cambridge Dictionary).

³ The term postdigital has in recent years been applied across a broad range of disciplines, often with contradictory meanings. The term postdigital is also entering the academic discourse. To map the various definitions, deployments and discussions see Taffel (2015), Jandrić, et.al. (2018).

the age of not only digital information and knowledge, but also behavior. Today, information about our activities, feelings, movements and thoughts is often collected without our consent, and our perception of reality can be manipulated by nudging⁴. However, applying the same measure to the entire population (even to clusters of the population) would not be good, but far too little is known to take appropriate individual measures (Helbing, 2019).

The aim of this paper is to draw attention to the interdisciplinary research of the AHP/ANP methodology by emphasizing how it can be studied from a cognitive perspective.

This paper is organized as follows: Section 2 provides an overview of studies that aimed to verify the practicality of AHP/ANP using laboratory experiments. Section 3 describes common heuristic approaches in decision-making. Section 4 summarizes the cognitive aspects of decision making with the AHP/ANP. Section 5 provides the discussion of future applicability of AHP/ANP in the Experience Age. Section 6 presents our conclusions.

2. Experimental validation of the AHP/ANP

Despite the fact that the AHP/ANP has been applied in a diverse range of areas, there is no clear evidence that the methods provide its users with their 'best' choice and not an arbitrary one (Ishizaka et. al., 2011). It seems difficult to assess whether a satisfying choice has been made by the decision makers because feedback on the decision may be very slow.

The reason that the AHP/ANP methodology is underestimated is probably because there are not enough controlled laboratory experiments testing the approach with nonmeasurable decision criteria and analyzing participants' level of satisfaction after the experiment. The authors believe that accumulating controlled laboratory tests on basic everyday decisions would lead to growth in the popularity of the AHP/ANP in more important problems. Experimental validation with subjective results is more convincing than the techniques with verifiable objective results because they deal with problems where the AHP/ANP is more likely to be applied (Ishizaka et al., 2011).

As a first step, we provide an overview of experimental studies that test the AHP on elementary decision problems.

Huizingh and Vrolijk (1997) asked 180 participants to solve the hypothetical problem of choosing a room to rent. The authors observed that participants were more satisfied with the AHP result than with a random selection. In Brugha (2000), two groups of 10

⁴ Nudging – a concept that proposes indirect suggestions as ways to influence the behavior and decision making of groups or individuals. Examples of Nudging are personalized prices, special offers, different advertisements, and etc.

students were asked to solve the hypothetical problem of a career and a car selection. It was observed that the participants preferred to use Scoring With Intervals (scoring with respect to a reference) over relative measurement (as in the AHP), but relative measurement was preferred when intervals were difficult to identify. The results calculated by the methods were not compared, probably because it was a fictitious problem.

Korhonen and Topdagi (2003) empirically investigated the performance of the AHP, when the utility of the objects cannot be evaluated on the same ratio scale. This kind of problem occurs when a decision maker is asked to compare, for instance, the objects (s)he likes to the objects (s)he hates. In that case, the authors expected very poor performance of the AHP because each object the decision maker likes is presumably "absolutely better" than any object (s)he hates. To test the hypothesis, four vegans and four non-vegans used the AHP to rank meals described on paper. However, the results demonstrated that the AHP is able to estimate the reasonable utility values for objects very well. In another experiment, Brugha (2004) asked 53 students to choose what they would do next year. It was observed that they preferred to use simple methods for screening and more elaborate methods for ranking (SMART, MAUT and AHP). The results calculated by the methods were not analyzed, probably because it was a fictitious problem.

Ishizaka et al. (2011) tested how well the AHP fares as a choice support system in a real decision problem. The authors tested the problem of selecting a box of five chocolates. The ranking provided by the AHP was statistically compared with three additional rankings given by the subjects in the experiment, one at the beginning, one after providing the AHP with the necessary pairwise comparisons and one after learning the ranking provided by the AHP. While the rankings varied widely across subjects, the authors observed that for each individual all four rankings were similar. Hence, subjects were consistent and the AHP was, for the most part, able to replicate their rankings. Furthermore, while the rankings were similar, the authors found that the AHP ranking helped the decision makers reformulate their choices by taking into account suggestions made by the AHP.

Thus, by reviewing actual laboratory experiments it is shown that the AHP is useful in assisting the decision making process. However, to give convincing support of the applicability the AHP/ANP in the decision-making process, we should provide more laboratory experiments that stress the decision-making process with the AHP/ANP (note that we could not find any experimental studies that tested the applicability of the ANP).

3. Heuristics overview

As we move from the laboratory to reality, it is obvious that our life is full of choices that are too unique to lend any useful data for analysis except heuristics. Therefore, we will provide an overview of common heuristic approaches for decision making under uncertainty.

Simon (1957) was the first to propose an alternative basis for the mathematical modeling of decision making by introducing a theory of "bounded rationality" that states when

International Journal of the	113
Analytic Hierarchy Process	

individuals make decisions, their rationality is limited by the tractability of the decision problem, the cognitive limitations of their minds, and the time available to make the decision. Simon suggests that people use heuristics to make decisions rather than a strict rigid rule of optimization. In this view, decision makers seek a satisfactory solution rather than an optimal one (Simon, 1956).

Exploring the territory of bounded rationality that Simon (1957) had defined, Amos Tversky and Kahneman (2003) developed their own perspective on heuristics in human decision-making. A judgment is said to be mediated by a heuristic when the individual assesses a specified target attribute of a judgment object by substituting a related heuristic attribute that comes more readily to mind.

In an earlier work, Tversky and Kahneman (1974) described three judgmental heuristics: (1) representativeness, (2) availability, and (3) anchoring and adjustment. These heuristics underlie many intuitive judgments. We will briefly describe each judgmental heuristic.

Representativeness is usually employed when people are asked to judge the probability that object A belongs to class or process B. An individual thing has a high representativeness for a category if it is very similar to a prototype of that category.

Consider the following problem:

Bob is an opera fan who enjoys touring art museums when on holiday. Growing up, he enjoyed playing chess with family members and friends. Which situation is more likely?

A. Bob plays trumpet for a major symphony orchestra B. Bob is a manager

A large proportion of people will choose A in the above problem because Bob's description matches the stereotype we may hold about classical musicians rather than managers. In reality, the likelihood of B being true is far greater because managers make up a much larger proportion of the population.

Availability is employed when people are asked to assess the frequency of a class or the probability of an event by the ease with which instances or occurrences can be brought to mind. When an infrequent event can easily be brought to mind, people tend to overestimate its likelihood. For example, investors may judge the quality of an investment based on information that was recently in the news, ignoring other relevant facts (Tversky & Kahneman, 1974).

Adjustment from an anchor is usually employed in numerical prediction when people make estimates by starting from an initial value that is adjusted to yield the final answer. Different starting points yield different estimates, which are biased toward the initial values. For example, participants received comprehensive information regarding a property, including either a high or low list price, before touring the property. Finally, participants were asked to estimate the actual value of the property. When the list price was high, the final estimates also tended to be elevated. In their experiments, Tversky and Kahneman (1974, 2003) showed that people acting under uncertainty rely on a limited number of heuristic principles which reduce the complex tasks of assessing probabilities and predicting values to simpler judgmental operations. Nevertheless, Kahneman and Tversky (1996) argue that cognitive biases have efficient practical implications for areas including clinical judgment, entrepreneurship, finance, and management. Thus, according to Kahneman and colleagues, the usage of heuristics saves effort but at the cost of accuracy. In this view, humans and other animals rely on heuristics because searching for information and making computations costs time and effort; the trade off with heuristics is some loss in accuracy but with faster and more frugal cognition (Gigerenzer & Gaissmaier, 2011).

In contrast, Gigerenzer and Goldstein (1996) argue that heuristics are "fast and frugal" and refer to simple, task-specific decision strategies that can be used to make judgments that are accurate rather than biased. Unlike statistical optimization procedures, heuristics do not try to optimize (i.e., find the best solution), but rather satisfy (i.e., find a good-enough solution).

Gigerenzer and Gaissmaier (2011) review four classes of fast and frugal heuristics. The first class exploits recognition memory, the second relies on one good reason only (and ignores all other reasons), the third weights all cues or alternatives equally, and the fourth relies on social information.

The studies of fast and frugal heuristics have shown that less effort can lead to judgments that are more accurate. For example, heuristics from the one-reason heuristics group (e.g. take-the-best⁵, hiatus⁶) often predict more accurately than multiple regression, linear and Bayesian models, neural networks, and decision-tree algorithms, and show the same or better performance than Pareto/NBD model (Gigerenzer & Goldstein, 1996; Czerlinski et al., 1999; Martignon & Hoffrage, 2002; Brighton, 2006; Wübben & Wangenheim, 2008). As another example, tallying heuristics to estimate a criterion does not estimate weights but simply counts the number of positive cues. It has been shown that tallying often predicts with equal or greater accuracy than multiple regression (Czerlinski et al., 1999).

Nevertheless, a criticism of the "less is more effect" also occurs in the literature, possibly because the effect is predicted to be small (Pachur & Biele, 2007; Pohl, 2006; Katsikopoulos et al., 2010). In any case, the results of all research put heuristics on par with the standard statistical models of "rational" cognition (Gigerenzer & Gaissmaier, 2011).

⁵ Take-the-best - to infer which of two alternatives has the higher value, (a) search through cues in order of validity; (b) stop search as soon as a cue discriminates; (c) choose the alternative this cue favors (for more details see for example Gigerenzer & Goldstein, 1996)

⁶ Hiatus - set an interval (the hiatus) and infer whether one selected criterion is active or not during the hiatus (for more details see for example Wübben & Wangenheim, 2008).

4. AHP/ANP as a cognitive process

Many comparative studies of heuristics and analytical approaches have been done, but we could not find any research about the comparative evaluation of heuristics and psychophysical methods such as the AHP/ANP. Could we apply heuristics without sufficiently reducing the complexity of the problem? Could we combine logic and heuristics?

Saaty (2015) emphasizes that the ANP mathematical approach aims at capturing the grand design and modularity of the brain and its workings. As the main functions of the AHP/ANP are to (1) structure complexity, (2) measure preferences, and (3) synthesize the results, we will consider them as three parts of the cognitive process that underlie decision making.

4.1 Structuring complexity

Structuring the complexity is the basis of decision-making. When facing a choice between options that differ on several decision-relevant attributes, one could rely on the most relevant attribute only to make the decision, or one could integrate information from several attributes and base the choice on that combined information. Understanding the cognitive processes underlying decision-making is one of the most important subjects of psychological research.

As the brain works through the functioning of its networks of neurons, where neurons are decision makers deciding to fire or not to fire, interactions of the neurons are similar to the flow of influences in decision-making. Thus, our perception of reality is better represented through a network that enables us to more accurately understand, control, and predict happenings in the world around us (Saaty, 2015)⁷.

However, if the considered problem is intricate, a designed network structure may be very large, which can lead to reduced concentration from the decision maker on the procedure of pairwise comparisons. Thus, it is the decision makers' responsibility to build a hierarchy/network structure that represents the reality and is not overburdened with unnecessary details. In order to reduce the problem of a high number of judgments, the traditional AHP/ANP procedure may be implemented using clusters and pivots, as presented by Ishizaka (2012). Objects are divided into several ordered clusters such that two adjacent clusters have one common object, the pivot. Then, pairwise comparisons are performed for each cluster and priorities are calculated. Final priorities are derived by using the pivot to link priorities of each cluster.

On the other hand, psychological experiments show that in order to simplify the choice between alternatives, people often disregard components that the alternatives share, and focus on the components that distinguish them (Tversky, 1972). This approach to choice

⁷ In this paper, we do not separate the AHP and ANP approaches because they imply the same methodology, but we should remember that ANP is recommended for studying complex decision problems.

problems may produce inconsistent preferences because a pair of prospects can be decomposed into common and distinctive components in more than one way, and different decompositions sometimes lead to different preferences (Kahneman & Tversky, 1979). To take into account these findings, in some decision problems we may assess criteria of possible alternatives before the alternatives are exactly defined (Weiss, 1987).

Thus, the main step of decision making, which is constructing a network (or hierarchy) structure of the problem, is a cognitive process where we should find the right balance between perception and reality. Although there are no standard recipes for building the "right" network (or hierarchy) structure, we should remember that in structuring complexity there is an upper limit on our capacity to process information on simultaneously interacting elements with reliable accuracy and validity. This limit is seven plus or minus two elements, as published by Miller (1956), as well as in the context of the AHP by Saaty and Ozdemir (2003).

4.2 Measuring preferences

According to natural human practice of making comparisons, Saaty suggested a scientific way of constructing a subjective scale that can be applied to a diverse range of issues being assessed (Saaty, 2008). Saaty (2015) also suggests that if there is adequate knowledge, one can compare anything with anything else that shares a common attribute or criterion, but priorities always depend on what other things are compared with.

Continuing the idea that our decisions are governed more by heuristics than by logic, we emphasize that the procedure of pairwise comparisons involves an inference in which one could rely on judgmental heuristics (such as availability or anchoring and adjustment). Reasoning this way, judgments in pairwise comparisons may be skewed due to attribute substitution. When an infrequent event can easily be brought to mind, we tend to overestimate its likelihood (Tversky & Kahneman, 1974). For example, a driver may estimate the safety of car A "Strongly better" than car B because yesterday he saw car B crash on the way to work. Or, an investor may judge the quality of an investment to startup B "better" than to startup A because he saw information about startup B in the news more often, ignoring other relevant facts. Moreover, due to the endowment effect, we can give higher judgements to things that we already own than to things that we do not own. It is also well documented that humans behave differently depending on their emotional states (Loewenstein & Lerner, 2003). For example, anger increases risk taking, fear is, in general, correlated with higher risk aversion, and it is not clear how sadness affects risky choices (Campos-Vazquez & Cuilty, 2013).

Measurement of judgment consistency plays a key role in the procedure of pairwise comparisons. While several authors still suggest that a consistency test is needed for adequate pairwise comparisons, psychological experiments show strong evidence of the irrationality in human decision-making (Zhang et al., 2018; Tversky & Kahneman, 1981). Moreover, in recent years a new way of thinking has evolved using psychology and economics that is trying to show that transitivity need not always be satisfied in order to make a rational decision (Moreno-Jiménez & Vargas, 2018).

Saaty (2013) argues that if humans were always perfectly consistent, they would not be able to learn new things that modify or change the relations among what they knew

International Journal of the	117
Analytic Hierarchy Process	

before. But, there is a level of tolerable inconsistency that we must allow beyond which the judgments would appear to be uninformed, random, or arbitrary.

Thus, while people rely on a limited amount of heuristic to make biased judgments, the AHP/ANP offers a judgmental structure for evaluation in a consistent manner without simplification of the problem (Tversky & Kahneman, 1974).

While the validity of using the fundamental scale in making comparisons has been validated by physical and decision problem experiments, critiques have been expressed about the distance of the AHP/ANP from the axioms of classical utility theory (Saaty, 2008; Dyer, 1990). Saaty (1990) and Forman and Gass (2001) have always rejected this criticism, arguing that the normative foundations of the methodology are not in utility theory, but in the theory of measurement.

Nevertheless, Bernasconi et al. (2010) reexamined the descriptive and normative foundations of the AHP in light of the modern theory of psychological measurement and gave empirical evidence to suggest that the method of ratio scaling mimics the cognitive process involved in decision-making.

4.3 Synthesizing

In Saaty (2015) it is noticed that our brain has a way of meshing together its parts and subparts using the matrix theory that arises from decision-making that has analogous complexity.). Saaty (2015) considers the brain to be a synthesizer of the firings of individual neurons into clusters of information, and these in turn into larger clusters and so on, leading to an integrated whole. Thus, Saaty's ANP methodology provides a mathematical way to synthesize the signals in the brain.

Any decision has several favorable and unfavorable aspects to consider. To make complex decisions we need to synthesize not only clusters of information but also different sides of the problem, e.g. benefits (and opportunities), costs and risks (BOCR). Each of these four concerns utilizes a separate structure for the decision (Saaty & Vargas, 2006).

Psychological experiments show that when making a choice under both risk and uncertainty, the subjective value of a specific loss is larger than the subjective value of an equivalent gain (Kahneman & Tversky, 1979). This phenomenon is called "loss aversion" and is supported by huge amounts of examples. Some examples include the endowment effect, the status-quo bias, and under-investment in the stock market (Morewedge & Giblin, 2015; Dean et.al., 2017; Benartzi & Thaler, 1995). On the other hand, there are studies that show that in some situations people are not loss averse, e.g. when the decision involves exchanging goods, like money, that are given up as intended (Novemsky & Kahneman, 2005). Similarly, the ownership of multiple units attenuates the endowment effect and the implied loss aversion (Rottenstreich et al., 2013).

In light of this discussion, we suggest that applying Saaty's BOCR-concept of structuring a problem serves to make the decision process transparent and understandable. As the last level of synthesizing, the AHP/ANP can be applied in the process of group decision making and in the negotiation process (Saaty & Vargas, 2012; Moreno-Jiménez &

Vargas, 2018; Vargas, 2017). Even though group decision making is critical at all societal levels, the main question today is the opportunity of applying the AHP/ANP methodology in the Experience Age. Today, people generate and store more data than ever before as they interact with both real and virtual environments. Mobile devices, Internet everywhere, micro-computers, mobile sensors and high-speed connectivity are all native to the new reality. Thus, information is moving toward experience, driven by the changing context of our online interactions.

Digital traces of behavior and cognition offer an unprecedented opportunity to test theories outside the laboratory. One of the most promising areas of future growth in the Experience Age may be achieved by analyzing naturally occurring real-world data sets that affect and reveal human behavior (Goldstone & Lupyan, 2016; Paxton & Griffiths, 2017). The question in this direction is the opportunity of converting digital traces of behavior to the ratio scale to measure social preferences.

Another direction of AHP/ANP applicability is the opportunity of building internet ratings of goods/services based on comparative assessment. According to neuroeconomic results that show people compare choices within a set rather than assigning separate utilities, we could use the system of online pairwise comparisons to collect and store individuals' judgments and then build appropriate ratings of choices (Camerer et al., 2004).

5. Discussion: AHP/ANP in the Experience Age

While the AHP/ANP methodology is intended to reflect an individual's decision process, it is not complete without accounting for social dynamics in the decision. This idea is supported by a wealth of evidence showing that people routinely base decisions on the choices of other people (Abrahamson & Rosenkopf, 1997; Delre et al., 2007). Moreover, living in the Experience Age, an increasing amount of information about us is exploited to manipulate our choices.

In Helbing (2019), Gigerenzer described an example about parliamentary elections in India in 2014. In a study, undecided voters could find out more information about the candidates using an Internet search engine. However, web pages had been manipulated. For one group, more positive items about Candidate 1 popped up on the first page and negative ones were shown later on. The other groups experienced the same for the other candidates. This and similar manipulative procedures are common practice on the Internet. It is estimated that for candidates who appear on the first page of a search (thanks to such manipulation), the number of votes they receive from undecided voters increases by 20%.

Helbing (2019) uses the term Big Nudging to describe when the increasing amount of personal information about us, which is often collected without our consent, reveals what we think, how we feel and how we can be manipulated. This insider information is exploited to manipulate us to make choices that we would otherwise not make, for example, buying some overpriced products or those that we do not need, or perhaps giving our vote to a certain political party.

Thus, living in the Experience Age, we need a new dynamic system of assessing the changing reality based on objective information, perception and subjective judgments. For the functioning of society, it is essential that people fill different roles, which are fitting to the respective situations they are in. From our point of view, as AHP/ANP has the instruments for measuring preferences and synthesizing different aspects of cognition, it could have a big future in the Experience Age. For example, since natural language is used to indicate a preference between different criteria, we can test the opportunity of using natural language processing for collecting human preferences from the web and translating them into numerical values for further integration into the AHP/ANP. Although the AHP/ANP is aimed at measuring individual preferences, in the era of huge amounts of data we should think about the opportunity of aggregation of human judgments for identifying global trends, changes and risks.

6. Conclusions

Considering the AHP/ANP in light of the modern theory of psychological measurement, it is suggested that the methods mimic the cognitive process involved in decision-making. As AHP/ANP incorporates both quantitative and qualitative criteria, it is difficult to assess the final decision. In some areas, slow feedback on the quality of the decision makes it impossible to adjust the AHP/ANP approach for experts in their decision processes. From our point of view, increasing the amount of controlled laboratory tests on a basic everyday decision would lead to growth in the popularity of the AHP/ANP in more important problems. We think that the experimental validation with subjective results is more convincing than the techniques with verifiable objective results because they deal with problems where the AHP/ANP is more likely to be applied (Ishizaka et al., 2011). Thus, for further development of the AHP/ANP we should follow the new way of cognitive orientation of the methodology and use laboratory experiments to understand human behavior.

A huge amount of studies concerning the role of heuristics in decision-making have been done. The "heuristics and biases" program, introduced by Kahneman and Tversky, showed that simple heuristics are efficient because they piggyback on basic computations that the mind has evolved to make. Heuristics of the "fast and frugal heuristics" program, developed by Gigerenzer and colleagues, ignore information to make decisions faster, more frugally, and/or more accurately than more complex methods.

However, heuristics as an efficient cognitive process may be used not only in fast decisions, but also in deliberate strategies. The AHP/ANP offers a judgmental structure for comparisons in a consistent manner without sufficiently reducing the complexity of the problem. Whether applying heuristics may help answer questions like "which of the two elements has more influence", pairwise comparisons give a more accurate answer on "how strong is this influence". Moreover, when people make decisions across the boundaries of different areas of information they need a way to synthesize priorities in addition to applying heuristics.

Living in the Experience Age, decision making is inseparable from social dynamics. This is especially relevant in an age where the Internet and mobile devices have transformed how people look for and communicate about different things. People generate and store

International Journal of the	120
Analytic Hierarchy Process	

huge amounts of data as they interact with both real and virtual environments. Thus, one of the most promising areas of future development of the applicability of the AHP/ANP may be achieved by analyzing naturally occurring real-world data sets that affect and reveal human behavior.

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122

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