# The Study of Solar Incident on Planar Surface for Kinetic Art

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## ABSTRACT

In future, renewable energy will become mainstream global energy source because it flows all the time, also the solar energy. The sun always directly emits light through the earth, depending on how we utilize it. The study will fill the gap that the most of photovoltaic panel facing to the same direction against solar incident for maximum efficiently. It would be better if photovoltaic appliance to have aesthetic and capability in the same time. But changing planar surface angle for artistic purpose also need to consideration of efficiency for solar energy. The purpose of this study is to explore the most appropriate angle of solar incident that impacts directly to the planar surface, which will leads to gain maximum electric energy from photovoltaic cell. The variation angle of planar surfaces will be discovered could possibly produce energy to be applied for any purposes. The additional sources of other renewable energy gadgets may install to generate more electric power. These planar surfaces can be composited together into many new shapes and forms for researcher to create more impressive artwork. In this study, the chosen material was Monocrystalline solar cell (mono-Si) to generate energy, wood and glass for kinetic sculpture. The procedures of study were as follow: 1. Specific location by Latitude will be achieved I. Altitude II. Seasons III. Sunlight direction IV. Sunrise and Sunset time 2. Calculate Solar radiation from each Photovoltaic cell for energy gain. 3. Compares output energy and mechanic usage for Kinetic Art 4. Increases capability of solar incident by improving angle of planar surface or multiply. Results of this research will be the search of artistic identity in art forms, it will be guideline for designers to develop potential Kinetic arts and or further creative works.

Keywords: geometric, solar incident, planar surface, kinetic art, photovoltaic

#### INTRODUCTION

Geometric is a basically design that can be create artwork very simply and also can adapt to enigmatic visual. Geometric shapes is the math of shapes made of points and lines. Some shapes are simple, such as the triangle, square, and circle. Other shapes are more complex that it takes math in order to create them. These shapes are the opposite of organic shapes. While geometric shapes are more precise, organic shapes are natural. Geometry is nothing but numbers made visible.

Geometric shapes are 2D and can be explain in geometry in 3D, which mean the points and lines are forming to become mass volume. It deals with the construction and representation of free-form curves, surfaces, or volumes. (Gerald Farin, 2002, pp. 1)

In recent years, the world economic growth and population increase need more energy, is an essentially important for the socio-economic development of developing, as well as developed countries. (Hrayshat ES., 2007 pp. 1873). Renewable energy is a sustainable and clean source of energy derived from nature. (Huang YH, Wu JH., 2007, pp. 345). Sunlight is one of renewable energy comes directly from sun. And with this, we can collect energy every day for free and waste no more non-renewable energy. A great amount of energy from sun is absorbed by the earth's surface itself, as well as by oceans, plants, and buildings. The energy that plants absorb is potentially convertible into food energy and material for combustion (logs, etc.) or fossil fuel (long-term decay). The energy absorbed by the oceans causes water evaporation and motivates the complete water (hydrologic) conversion cycle. (Richard Crowther, 1983, pp. 44)

The earth is a giant spacecraft, supporting life as it orbits the sun. The sun provides the earth with the energy it needs to sustain and maintain life. The earth with its thin atmosphere (a layer only 160 kilometers, or 100 miles, deep) supports more than four billion people. Smaller spacecraft in many ways duplicate the conditions and functions of the earth by utilizing the sun's energy and shielding their occupants as they travel through space. (Richard Crowther, 1983, pp. 44)

All visual art involves light. There is, however, a growing interest in light art, works where light is a primary medium. It is a hazy boundary that separates light art from other visual art. But new lighting technologies, creative artists, and the dynamic nature of light are producing more and more pieces that are clearly on the light art side of the border. Permanent and temporary light art pieces now appear in many venues. (Russell P. Leslie, 2004, pp. 28).

Solar cells, also called photovoltaic (PV) cells, convert sunlight directly into electricity. PV gets its name from the process of converting light (photons) to electricity (voltage), which is called the PV effect. When scientists at Bell Telephone discovered that silicon (an element found in sand) created an electric charge when exposed to sunlight. Soon solar cells were being used to power space satellites and smaller items like calculators and watches. The direct conversion of sunlight to electricity occurs without any moving parts or environmental emissions during operation. The electricity from photovoltaic cells can be

used for a wide range of applications, from power supplies for small consumer produce large power stations feeding electricity into the grid. (Pearsall NM, Hill R, 2001, pp.:)

Normally, in visual art, the term kinetic art, derived from the Greek word kinesis, ref works that apparent movement, or art that depends on movement for its effects. It's r how things look like when they are moving. Although ostensibly fascinated by machi some Kinetic artists developed a profound interest in analogies between machines and human bodies. The Kinetic art utilizing mechanical or natural motion to bring about relationship between art and technology. The movement introduced Kineticism acros several forms of art, including any kind of arts such as painting, drawing, and sculptu and many of artists aspired to work with something newer resource for ideas in order bring Kinetic art to a wide audience.

According to reasons above, a Geometric have some significant to study for base desig combine with Photovoltaic cell because of both are Geometric shape. This can be me between art and science together which represent the stylishness of renewable energy object. Furthermore, this can be typify of renewable energy product of photovoltaic fbuilding whether interior or exterior decoration.

With all these reasons, this purpose of the research is to study the Geometric Design 1 Kinetic Art Creation with Renewable Energy: Photovoltaic which is so much importa And it also inspire to develop kinetic art. The research methodology is practice base, 1 process try to create kinetic art by using different kind of local natural materials. The stage will be access into the identity characteristic in art form. The researcher expects the results of this study will be a guideline for any designers to understand an inspirat Kinetic art, how to create artistic art form that touching any moving hearts, build up value of crafts and promote their gorgeous imagine. Finally, this study will be the mo useful for the next generation designers and any researchers who need to develop thei knowledge potentiality in Kinetic arts.

# METHODOLOGY

The mainstream of this research were consist to 2 issues; solar geometry and sculpture design. Finalize, they will lead to design development after data corrected according t Figure 1.

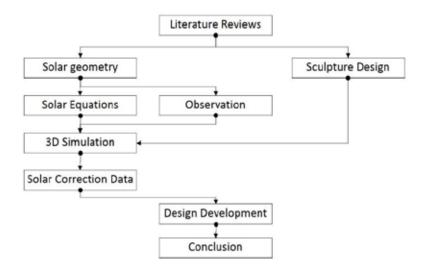


Figure 1. Research methodology

## **EXPERIMENTATION**

Figure 2. Shows the 4-steps study process used to assess the expected effects development of model appearance on specific site. The study process can be summarized as follows. In the first step, a site survey was performed using visual observation and a fish-eye lens camera to assess the area suitable for model situate. In the second step, the location for the model situate was selected and the shape of form was designed taking into for observation. In the third step, the possible light incident was recorded and estimated by entering the meteorological data and system design parameters into calculation software. Based on the appropriated angle of light incident of planar surface, the solar incident feasibility and development processes were assessed in the fourth step.

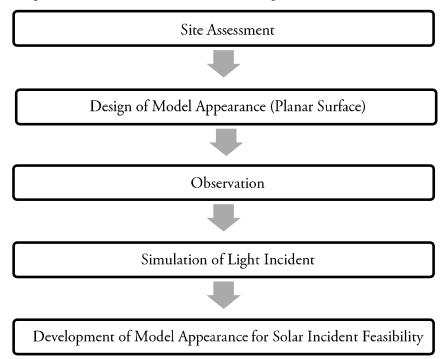


Figure 2. Overall procedure to Development of model appearance in the study site

## 1. Site assessment

The planar surface's orientation (azimuth, the direction it faces) or tilt varies from optimal solar incident's touch. The optimal combination of array tilt and azimuth will depend on a few variables, including geographical location, seasonal weather patterns. Azimuth and tilt are both required data from equation.

For the site assessment monitoring for light incidents on planar surface On Thursday 11th August 2016 (rainy season) at the top of a building close to Bangkoknoi canal in Bangkok, Thailand (13° 46' 44.3136'' N; 100° 28' 9.012'' E). The procedures were calculated for sunrise and sunset time from the date on particular day at the study site, using a digital topographic map to locate the specific site. In order to assess the effects of shadows on the model, researcher analyzed the daily sunshine hours on site using the solar radiation analysis method on the period of autumnal equinox (The moment when the Sun appears to cross the celestial equator, heading southward. Due to differences between the calendar year and the tropical year. At the equinox, the Sun rises directly in the east and sets directly in the west, day and night are of approximately equal duration all over the planet. They are not exactly equal, however, due to the angular size of the sun and atmospheric refraction.) To measure the shadow effects caused by small obstacles, such as vegetation, small building and the skyline was recorded and analyzed using a fish-eye lens camera. This enabled field analysis of the skyline at the solar site. The results of shadow-effect were recorded in the form of a shading data for shading elements surrounding the solar site, as expressed from approximately time and angle calculation by values between "0.0" (complete shielding of direct radiation reaching the PV system) and "1.0" (no shading effects). (Melo, E.G, 2013, pp. 15)

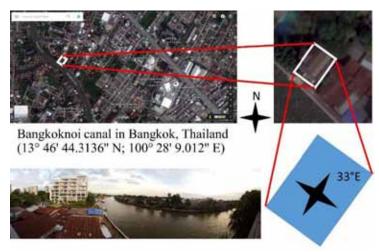


Figure 3. Shows observation site

# 2. Design of model appearance (planar surface)

For the purpose of the study, it was hypothesized that planar surface of model constitute a tilt array for the highest value of solar incident on the surface was designed with the following parameters: number of planar per model (if there are separated), and array spacing.

## a. Model Surfaces

Practical modeling of spatial surfaces is more convenient by means of transformation of their flat developments made as topologically connected kinetic structures. According to topology, any graph on a surface consists of three types of elements: planar facets (F), linear edges (E) and point vertexes (V). There are two well-known types of transformable structures, based upon the structural units corresponding to the planar (F) and linear (E) elements of surface division. Those two types of surface models can be represented respectively by the folding structures with hinged flat faces (figure. 4a), and the kinematical nets with non-triangle meshes (figure 4b).

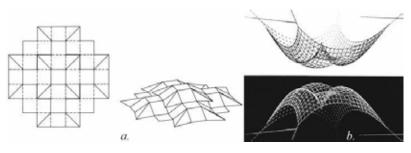


Figure 4. x.Transformable surface models:

- a) folding structure (Vranka 1990: 9);
- b) kinematical net (Otto, Burkhardt and Hennicke 1974: 130)

In addition to the planar and the linear types of the flat developments of surfaces there is a third possible type of flat transformable structures with vertexes (V) as structural units. Approximations of surfaces by ordered sets of points is the common method in mathematics and computer graphics. In these cases a separate point is represented by a dot in the virtual space, and its numerical value relates to the three Cartesian coordinates. The physical model of a point may be represented as the contact of two physical 3D bodies such as tangent solid spheres or tangent cylinders with non-parallel axes. A flat or spatial system of the contact points produces the vertex surface model, that is, the point approximation of real continual surfaces. (Dmitri Kozlov. 2013, pp 242)

In combining the geometric terms of the supplementary textbooks with the terms from the basic textbook a logical sequence of instruction results from placing "space"- the set of all points - after "sets of points." "Paths (straight and curved)" logically follows "points." "intersecting lines" should follow "lines." "Curved lines" should follow "straight lines," and "regions" - a limited part of a plane - comes after "plane." (Shirley Jo Probert. 2014, pp 24-25).

## b. Shape

The shape of architectural objects in general can be treated as envelopes: 2D surfaces embedded into 3D space. (Dmitri Kozlov. 2013, pp 253)

## c. Proportion

Proportioning systems have always been among the most essential elements of visual order and an important consideration of aesthetics. At the same time, they constitute the basis for dimensional coordination in building design, fabrication, and construction. The golden ratio, which is not merely a human invention but the most common ratio in the geometry of natural forms as well, is included in since it alludes to planar and spatial tilling and tessellations that have often been encountered in architectural history. (Katherine A. Liapi. 2002, pp. 80).

# 3. Observation

The observation of solar geometry consist about 4 main factors; solar incident, time, date and atmosphere.

a. Solar Incident

A simple logic drives the whole design. The relative position of each facade component with the different seasonal sun paths defines its geometry (openness, deepness and panelling inclinations). Each component is defined globally yet resolved locally, and calculated individually, module by module, following the same shared rules. (Areti Markopoulou. 2011, pp. 131)

## b. Time

The solar day is exactly 12 hours long at the equator during the equinoxes. During the summer solstice (June 21), the sun is at its northern extreme position and the declination is at its positive maximum. (Edward Anderson, 1983, pp. 25)

## c. Date

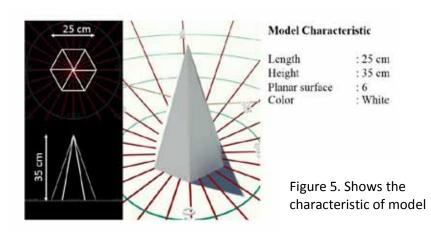
As the earth progresses along its orbit about the sun, solar declination changes

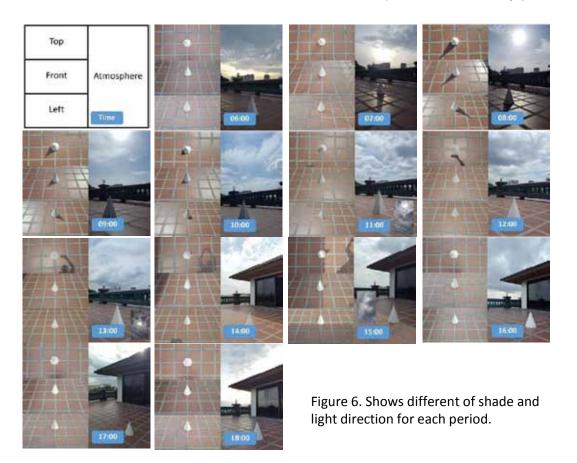
due to the tilt of the earth's polar axis with regard to its orbital plane. Solar declination then depends on the earth's position in its orbit, that is, on the day of the year. (Edward Anderson, 1983, pp. 24)

d. Atmosphere

Solar radiation incident on any given surface can be decomposed into two components, the direct or beam component emanating from the sun and a diffuse component that results from multiple reflections and scattering because of particles in the atmosphere. The diffuse component may also include reflections from the ground and local surroundings, where the surface in question is sloped rather than horizontal. Differentiating between the two components is vital for accurate calculations in most solar energy applications; however, a number of steps may be required to arrive at realistic estimates at an appropriate level of detail for a given location depending on the basic data available. (Yasser Aldali, 2014, pp. 9)

The model was putted on the roof of the building, see figure 5. For the characteristic and some details. The camera was set for 4 point align to the wall; front view (toward 33°NE), left view (toward 123°SE), and top view and perspective view observe atmosphere for clouds status. The time was assigned for 1 hour period started at 06:00 (Dawn) until 18:00 (Dusk) o'clock as shown in Figure 6. The model Edged Cone shape situated at the top of a building and set to point to Solar Zenith Angle.





## 4. Simulation of light incident

Equations was used for calculation for solar systems. It enables more accurate analyses of how to adjust planar surface to face most directly to sunlight, given the environment, all over model characteristics and the atmosphere conditions. For an explanation of the Excel formulas. In this study, after observation for primary data collection. The solar time and solar altitude were calculated by entering the meteorological data of the study site and any values which needed, such as angle of planar, time, date and etc. The shading data obtained by onsite solar assessment according to the tilt angle of the model surface array, into the formulas. The following equation were used to calculate for results;

#### a. Solar Time

Time based on the rotation of the Earth with respect to the Sun. Solar time units are slightly longer than sidereal units due to the continuous movement of the Earth along its orbital path. For example, by the time the Earth has completed one full rotation on its axis with respect to the fixed stars, it has also moved a short distance in its orbit and is oriented slightly differently to the Sun, so that it must turn slightly more on its axis to complete a full rotation with respect to the Sun. For finding solar time, the equation is Solar time = Standard time + 4 (Lst - Lloc) + E (3a.1)

Solar time will use to correct in solar declination equation (John A. Duffie, 2006: 11)

#### b. Solar Declination

The angle formed by the line from the center of the earth to the center of the sun on a particular day and the plane containing the earth's equator is called solar declination 8. As the earth progresses along its orbit about the sun, solar declination changes due to the tilt of the earth's polar axis with regard to its orbital plane. Solar declination then depends on the earth's position in its orbit, that is, on the day of the year. Solar declination for each day of the year is listed in The American Ephemeris and Naval Almanac for each year. (Cooper, P. I. 1969) suggests the approximate equation for calculating the declination.

$$\delta = 23.45 \left( \frac{360}{365} (284 + \pi) \right)$$
(3b.1)

In this equation, n is the day number with 1 being January 1 and 365 being December 31. This approximation is satisfactory for flat-plate and mildly concentrating collectors. (Edward Anderson, 1983, pp. 24)

c. Solar Zenith and Azimuth Angles

In order to relate the solar zenith and azimuth angles to latitude, date, and time of day, we need to consider two coordinate systems. The first is the local coordinate system that sues the horizontal plane and the vertical axis on the earth's surface. Taking i, j and k to be unit vectors pointing, respectively, south, east, and vertically up, we can write the solar unit vector S as. (Edward Anderson, 1983, pp. 24)

$$\begin{split} S &= i \sin \theta z \cos \gamma s + j \sin \theta z \cos \gamma s + k \sin \theta z \\ \text{In the earth-centered coordinate systems, S can be expressed as} \\ S &= i \cos \delta \cos \omega + j \cos \delta \sin \omega + k \sin \delta \\ \end{split} \tag{3c.1}$$

Where i, j and k are unit vectors along the meridional axis m, the easterly asix e, and the polar axis p, respectively.

The components of S in the two coordinate systems are then related by

$$SV = Sm \cos \emptyset + Sp \sin \emptyset, \qquad (3c.3)$$
  

$$SS = Sm \sin \emptyset - Sp \cos \emptyset, \qquad SE = Se$$

Substitution of the components of S given in Eqs. (3c.1) and (3c.2) into EQ. (3c.1) gives

$$\cos \theta z = \sin \phi \sin \delta + \cos \phi \cos \delta \cos \omega, \qquad (3c.4)$$

$$\cos \gamma s = \frac{\sin \phi \cos \delta \cos \omega - \cos \phi \sin \delta,}{\sin \theta z} \qquad (3c.5)$$

$$\sin \gamma s = \frac{\cos \delta \sin \omega}{\sin \theta z} \qquad (3c.6)$$

The solar zenith angle and azimuth angle can be calculated with these results for a given latitude, date, and time of day. (Edward Anderson, 1983, pp. 26-27)

## d. Sunrise and Sunset

Neglecting the refractive bending of the sun's rays by the atmosphere, we can say that the local sunrise and sunset occur when the sun is in the horizontal plane at the point of observation. Then the zenith angle for either event is 90°. Hence, for sunrise or sunset, the corresponding hour angle:

$$\omega s = \pm \cos \left[ -\tan \phi \tan \delta \right]$$
(3d.1)

with the positive sign corresponding to sunrise and the negative sign to sunset. When  $\gamma$  exceeds 90°, the sun rises north of east, and when  $\gamma$  is less than -90° (say, it is -100°), the sun sets north of west. In the mid and lower latitudes of the northern hemisphere this occurs during the summer. (Edward Anderson, 1983, pp. 28)

According to the equation above, solar data can be collected as follows: On Thursday 11th August 2016, Sunrise time was 06:04 at 74° and Sunset time was 18:43 at 286° from north. The site point to 33°E from N. And because of Rainy season, the atmosphere status was scatter clouds. So, all data such as time, date and location has been insert into 3D application for simulating the solar incident on model for check the correction of shade and light. See figure 7.

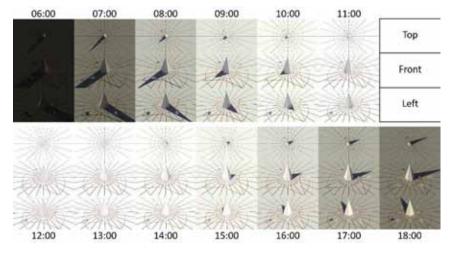


Figure 7. Shows solar incident in 3D simulation

# 5. Development of Model Appearance for Solar Incident Feasibility

The newly designed was estimated in accordance with the initial shape of model. In order to change the appearance, 3d modeler application were used for create simulated model into real scale and assume present period sunlight in approximately location to observation situated site. According to topology, the development of model shape form were consists of three types of elements: planar facets (F), linear edges (E) and point vertexes (V).







Figure 8. Shows three types of elements.

Planar facets (F)

Point vertexes (V)

Linear edges (E)

Firstly, original model was create to the same dimension of real model which used for observation, see figure 9. And after corrected from calculated data using equation, model have to be shorten because of it have to change its shape for contact more light from the sun. From the observation site, Bangkok. 15° is the average degree of sunlight direction in the sky, so planar surfaces have to point to 90°-15° (Solar zenith – average solar position) to face against the sun, see figure 10. So on, more FEV were added for more complex shape by simply method; two planar which have connected surface for

15°/15°. Now, just minus 5° for the first one and plus 5° for the other and then become  $10^{\circ}/20^{\circ}$ . Average of sunlight will share the incident on both planar surfaces as shown in figure 11.

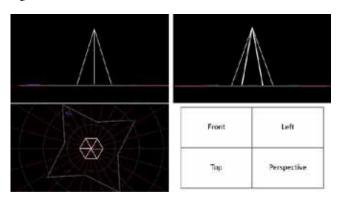
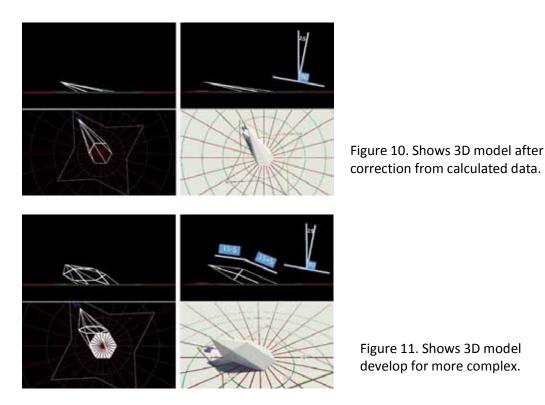


Figure 9. Shows 3D model in the same dimension of observation's model.



# **RESULTS AND DISCUSSION**

#### Site assessment

Thailand DMS latitude and longitude coordinates are 13° 44' 12.1812'' N; 100° 31' 23.4696'' E which means the sun will be located approximately 14° - 15° up in the sky from Solar Zenith Angle, toward south. So, for the most appropriate angle of planar surface for solar incident. The model should be change the angle of shape point up toward the sun for 75° (90° - 15°).

Different latitude could be cause of changes for following factors; season, atmosphere, daytime, nighttime, solar radiation and etc. The same original model shape could be change into various art forms up to the reasons above.

Design of Model Appearance (Planar Surface) Geometric design is kind of basic design to complex, simply art form can created by repeat action such as rotate, scale and move to FEV. The many of original art forms may such a simply design but after apply some movement to them, more new complex art form will be happen. Some designs were created from free thought but for some designs, it have to consider about uncontrolled factors. And the design have to change for more support.

## Observation

The observation of solar geometry consist about 4 main factors; solar incident, time, date and atmosphere. All of the actions must be accurate and regularly done in assigned period. To observe sunlight in summer may overflow values too much and acquired data could mistake to the results because of solar radiation is too strong. In rainy or winter season, atmosphere could be unpredictable because of humidity or air pressure that affect to vapor or clouds. So, it would be better if the observation can set on the lowest period for less expectation.

## Simulation of Light Incident

It have to clone every environment and context from observation process for more accurate results. Even computer could be mistake by a little omit. 3D simulation takes the most similar to the real spectacle which happen in real. With this process, the modeler can develop some art form by refer the results.

## Development of Model Appearance for Solar Incident Feasibility

If model would be situated in the different location, the shape of model have to change for the most appropriately solar contact. Example, Iskenderun on the eastern Mediterranean coast of Turkey DMS latitude and longitude coordinates are 36°3501300 N; 36°10 0 2400 E which means the sun will be located approximately 36° up in the sky from Solar Zenith Angle, toward south. So, for the most appropriate angle of planar surface for solar incident. The model should be change the angle of shape point up toward the sun for 54° (90 - 36°).

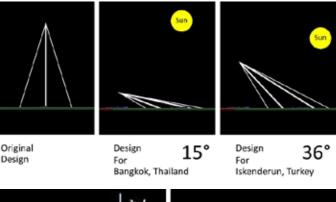


Figure 12. Shows the expected shape of model if it were built at the top of a building in Iskenderun (36°3501300 N; 36°10 0 2400 E), on the eastern Mediterranean coast of Turkey.

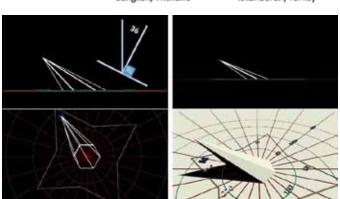


Figure 13. Shows comparison of different location cause of model changing.

# CONCLUSION

The development of model shape form could be change by rotate, scale and move the three types of elements: planar facets (F), linear edges (E) and point vertexes (V) by according to topology. The more complex, may turn simply model into aesthetical sculpture but this action have to improve under consideration.

Changed of model form will be differently from original because of sunlight. Different latitude, environment and period could be major factors for consideration of making arts.

Thailand DMS latitude and longitude coordinates are 13° 44' 12.1812'' N; 100° 31' 23.4696'' E which means the sun will be located approximately 14-15° up in the sky from Solar Zenith Angle, toward south. So, for the most appropriate angle of planar surface for solar incident. The model should be change the angle of shape point up toward the sun for 15°.

For other location on earth, latitude will be the first priority to check for. According to the earth geometry, angle of sunlight, season and atmosphere will change when latitude changes and longitude is not necessary to concern of.

All of the conclusion could be record the study result as guideline for inspire any artist making some design which intend to create artistic artwork.

## NOTATION

The following symbols are used in this paper:

- $\delta$  = Solar declination angle
- $\theta$  = Solar incident angle
- $\varphi$  = latitude
- $\gamma$  = Azimuth angle
- $\omega s = Sun's$  hour angle at sunrise or sunset
- Lst = standard longitude for the time zone in question (reckoned as positive west of the Greenwich meridian)
- Lloc = true longitude of the site for which the calculation is to be made
- E = Equation of Time correction

## INNOVATIONS

These are interesting solutions that require further studies in order to make some of them have other applications and the changes proposed give them new uses.

It is a facade system based on a set of mathematical rules and logics ready for export and adaptable to different locations or materials in response to local conditions and available technologies and materials. Based on adaptation and not on repetition, it is able to read context conditions and respond to them. (Areti Markopoulou. 2011, pp. 130-131)

It can be attach to the photovoltaic cell for benefits from energy issues because of these cells need to be facing to the sunlight for make some electric energy. Furthermore, maybe energy that generate from photovoltaic cell could be used for moving own model to make kinetic art.

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