INTERNATIONAL JOURNAL OF COMPUTERS COMMUNICATIONS & CONTROL
Online ISSN 1841-9844, ISSN-L 1841-9836, Volume: 18, Issue: 2, Month: April, Year: 2023
Article Number: 5299, https://doi.org/10.15837/ijccc.2023.2.5299

CCC Publications

AGORA UNIVERSITY PRESS

A new belief entropy and its application in software risk analysis

X.Y. Chen, Y. Deng

Xingyuan Chen*
School of Information Engineer
Kunming University, Kunming, China
*Corresponding author: chenxingyuan@kmu.edu.cn; stargarden.chen@hotmail.com

Yong Deng**
Institute of Fundamental and Frontier Science
University of Electronic Science and Technology of China, Chengdu, China
**Corresponding author: dengentropy@uestc.edu.cn; prof.deng@hotmail.com

Abstract

The measurement of uncertainty has been an important topic of research. In Dempster's framework, Deng entropy serves as a reliable tool for such measurements. However, it fails to consider more comprehensive information, resulting in the loss of critical data. An improved belief entropy is proposed in this paper, which preserves all the merits of Deng entropy. When there is only a single element, it can be degraded to Shannon entropy. When dealing with multiple elements, the partitioning method employed for mass functions makes it more responsive and efficient than alternative measures of uncertainty. Some numerical examples are given to further illustrate the effectiveness and applicability of the proposed entropy measure. Additionally, a case study is conducted on software risk analysis, demonstrating the practical value and relevance of the proposed method in real-world scenarios.

Keywords: Dempster-Shafer evidence theory, Deng entropy, Uncertainty measure, Belief entropy, Software risk analysis

1 Introduction

Uncertainty exists everywhere in our daily life. Due to the large amount of useful messages contained in uncertainty information, uncertainty information processing is widely used in many fields. It plays an important role in the target recognition [13, 16], complex network [31], artificial intelligence [17, 23], fault diagnosis [40], decision-making [51] and risk analysis [22], etc. In uncertainty information processing, it is necessary to measure uncertainty effectively and reasonably. Therefore, uncertainty measurement (UM) is widely studied as a hot topic. Several methodologies have been proposed, such as Dempster-Shafer evidence theory [5, 33], Bayesian probability theory [27], possibility theory [11], rough sets [30], Shannon entropy [34], fuzzy set [6, 50], and so on [29].

Within these approaches, Dempster-Shafer evidence theory (DST) is highly regarded because it needs fewer conditions and better deal with uncertainty than probability theory. It includes frame of discernment (FOD), basic probability assignment (BPA) and Dempster's rules of combination for data fusion. It provides a powerful method for the expression and fusion of uncertainty information [10, 36, 42]. DST is therefore widely applied in many fields [7, 43, 52].

Shannon entropy occupies a central position in information theory. It solves the problem of quantitative measurement of information and has received a lot of attention in practical applications [38, 41, 48]. Nevertheless, Shannon entropy is only useful in the probabilistic framework, which is why many scholars have made a lot of attempts to introduce entropy into evidence theory. Such as confusion measure [15], Hartley entropy [12], discord measure [21], strife measure [20], total conflict measure [14], dissonance measure [46] and so on. However, these methods are not ideal for measuring the uncertainty of BPA in some situations. For measuring uncertainty, dissonance and non-specificity are the two main aspects. These methods simply take either discord or non-specificity into account. In terms of discord, confusion measure, and strife measure were employed, while in terms of non-specificity, Dubois and Prade¡¯s weighted Hartley entropy was used. Therefore, how to combine both discordance and non-specificity measurement uncertainty of BPA is a question worth investigating.

Deng entropy [8] was proposed, which is more effective in some cases. First, both dissonance and non-specificity were considered by Deng entropy. Secondly, as a promotion of Shannon entropy, Deng entropy extends the probability to the mass function. Third, it is more reasonable that the Deng entropy increases monotonically with the size of the proposition. The greater the number of focus elements of the FOD, the higher the entropy. Due to its advantages, it is widely used in several fields [4, 18, 25]. However, Deng entropy did not consider the most possible states when splitting the mass function. There are limitations when it comes to dealing with certain issues. If all possible states are not taken into account, then the information is incomplete and lost.

A novel measure of belief entropy founded on Deng entropy is proposed. The proposed entropy inherits all the benefits of Deng entropy. At the same time, the way the mass function splits makes the proposed entropy more sensitive and effective than other uncertainty measures. Some examples and a risk assessment case demonstrate the effectiveness of the new entropy for uncertainty measurement.

The structure of this paper is as follows: An summary of the Dempster-Shafer evidence theory, Shannon entropy, and Deng entropy principles is given in 2.

Section 2 provides a brief overview of the concepts of Dempster-Shafer evidence theory, Shannon entropy, and Deng entropy. Section 3 proposes a new measure of belief entropy based on Deng entropy and provides several numerical examples. Section 4 illustrates an application of the proposed belief entropy to demonstrate its validity. Finally, the paper concludes in Section 5.

2 Preliminaries

In this section, some essential basics are briefly introduced.

2.1 Dempster-Shafer Evidence Theory

D-S evidence theory, also called as the dempster-Shafer evidence theory (DST) [5, 33], is an extension of Bayesian theory and is a potent tool for handling uncertain information. Some fundamental concepts are outlined below.

The set X represents the framework of discernment (FOD), which contains all possible mutually exclusive and exhaustive hypotheses [33], defined as follows:

$$X = \{\theta_1, \theta_2, ..., \theta_n\} \tag{1}$$

The power set 2^X contains all possible subsets of X, as defined below:

$$2^X = \{\emptyset, \{\theta_1\}, \{\theta_2\}, ..., \{\theta_n\}, \{\theta_1, \theta_2\}, ..., \{\theta_1, \theta_n\}, ..., \{\theta_1, ..., \theta_n\}\} \tag{2}$$

where $\varnothing$ is an empty set.

For FOD X, the mass function can also be referred to as Basic Belief Assignment (BBA) or Basic Probability Assignment (BPA). It is mapping from 2^X to [0, 1], and description is as follows:

$$m : 2^X \to [0, 1] \tag{3}$$

constrained conditions as follows,

$$\begin{cases} \sum\limits_{A \in 2^X} m(A) = 1 \\ m(\varnothing) = 0 \end{cases} \tag{4}$$

A is called the focal element when $m(A) > 0$. $m(A)$ indicates the belief value that supports the proposition A. There are some studies about mass function [9, 44, 47], which has been used in many fields [2, 24].

For $A \subseteq X$, the belief function $Bel : 2^X \to [0, 1]$ is following:

$$Bel(A) = \sum_{B \subseteq A} m(B) \tag{5}$$

The plausibility function $Pl : 2^X \to [0, 1]$ is defined as,

$$Pl(A) = 1 - Bel(\bar{A}) = \sum_{B \bigcap A \neq \varnothing} m(B) \tag{6}$$

Apparently, $\forall A \subseteq X, Bel(A) \leq m(A) \leq Pl(A)$. Where $Bel(A)$is the lower bound of proposition A and $Pl(A)$ is the upper bound of Proposition A. $[Bel(A), Pl(A)]$ indicates the confidence interval of A.

Considering two bpa, the Dempster combination rule is denoted as[33]:

$$m(A) = \begin{cases} \frac{1}{1-k} \sum\limits_{B \bigcap C = A} m_1(B) m_2(C), & A \neq \varnothing \\ 0, & A = \varnothing \end{cases} \tag{7}$$

where A, B, C $\in 2^X$, k is a normalization factor,

$$k = \sum_{B \bigcap C = \varnothing} m_1(B) m_2(C) \tag{8}$$

The conflict coefficient, denoted by k, is a measure of the degree of conflict between two basic belief assignments (BPAs) m_1 and m_2. A value of $k = 0$ indicates that m_1 is consistent with m_2, while a value of $k = 1$ signifies that m_1 and m_2 are in total contradiction. However, when combining highly contradictory evidence, a counterintuitive conclusion may arise from Dempster's combination rule [45, 49]. Numerous research are devoted to solving the problem [39].

2.2 Shannon Entropy

Shannon entropy, also called information entropy, holds significant role in the field of information theory. Shannon entropy is defined as [34],

$$E_s(m) = -\sum_{i=1}^{N} P_i log_b P_i \tag{9}$$

where N represents the number of basic states, while P_i is the probability associated with state i. The probabilities P_i must satisfy the condition $\sum\limits_{i=1}^{N} P_i = 1$. When the unit of information is the bit, then $b = 2$. Shannon entropy can be defined as:

$$E_s(m) = -\sum_{i=1}^{N} P_i log_2 P_i \tag{10}$$

Within the framework of probability theory, Shannon entropy has proven to be a successful approach for measuring information uncertainty. Nevertheless, the measurement of information uncertainty within the DST framework is also a subject worth studying.

2.3 Uncertainty Measure in the DST Framework

Table 1 lists several measures of uncertainty in the DST framework.

Table 1: Some uncertainty measures

Literature	Expression
[15]	$C_h(m) = -\sum_{A\subseteq X} m(A) log_2 Bel(A)$
[46]	$E_y(m) = -\sum_{A\subseteq X} m(A) log_2 Pl(A)$
[12]	$E_{dp}(m) = -\sum_{A\subseteq X} m(A) log_2 \lvert A\rvert$
[21]	$D_{kr}(m) = -\sum_{A\subseteq X} m(A) log_2 \sum_{B\subseteq X} m(B) \frac{\lvert A\cap B\rvert}{\lvert B\rvert}$
[20]	$S_{kp}(m) = -\sum_{A\subseteq X} m(A) log_2 \sum_{B\subseteq X} m(B) \frac{\lvert A\cap B\rvert}{\lvert A\rvert}$
[14]	$TC_{gp}(m) = -\sum_{A\subseteq X} m(A) log_2 \sum_{B\subseteq X} m(B)[1 - \frac{\lvert A\cap B\rvert}{\lvert A\cup B\rvert}]$

2.4 Deng Entropy

The Deng entropy is an appropriate metric for the BPA in the DST. It can be described as [8],

$$E_d(m) = -\sum_{A\subseteq X} m(A) log_2 \frac{m(A)}{2^{|A|}-1} \tag{11}$$

Equation (11) defines the Deng entropy, where m is a BPA defined on FOD, and A is a focal element of m, with $|A|$ representing the cardinality of A. Through a simple transformation, the equation can be expressed by:

$$E_d(m) = \sum_{A\subseteq X} m(A) log_2(2^{|A|}-1) - \sum_{A\subseteq X} m(A) log_2(m(A)) \tag{12}$$

It can be divided into two components: $\sum_{A\subseteq X} m(A) log(2^{|A|}-1)$, which measures nonspecificity, and $-\sum_{A\subseteq X} m(A) logm(A)$, which measures discord.

As an extension of Shannon entropy, Deng entropy is formally similar to it. The difference is that the belief of each focal element A is divided by the number of possible states in A, which is $2^{|A|-1}$. If BPA degenerates to probability, Deng entropy can deteriorate to the Shannon entropy.

$$E_d(m) = -\sum_{A\subseteq 2^X} m(A) log_2 \frac{m(A)}{2^{|A|}-1} = -\sum_{A\subseteq 2^X} m(A) log_2(m(A)) \tag{13}$$

Some of the uncertainty measures within the framework of Deng entropy can be found in [4, 26, 28, 53].

3 The Proposed Belief Entropy

This section presents an approach to measuring uncertainty in DST. Specifically, Section 3.1 introduces a novel belief entropy that builds upon Deng entropy. In Section 3.2, the efficacy of the proposed method is illustrated through numerical examples and a comprehensive discussion.

3.1 The Proposed Belief Entropy

$$E_n(m) = -\sum_{A \subseteq X} m(A) log_2 \frac{m(A)}{\sum_i (2^{|A_i|} - 1)} \tag{14}$$

For each $i = 1, 2, ..., 2^X - 1$, where X is FOD, and m is a BPA, A is a focal element, the belief value of A is normalized by dividing it with a term $\sum_i (2^{|A_i|} - 1)$, where $|A|$ indicates the cardinality of A. This term represents the total number of possible states that A can assume.s

Assuming there are two balls inside a box, all the possible state are displayed in Fig. 1. Let FOD X = {a, b}, the power set is $A = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$. The most possible state in A is obtained by calculating $\sum_i (2^{|A_i|} - 1) = (2^0 - 1) + (2^1 - 1) + (2^1 - 1) + (2^2 - 1) = 0 + 1 + 1 + 3 = 5$. The figure 3 represents the total number of states that comprise components belonging to both {a} and {b}, namely, {a}, {b}, and {a, b}. Alternatively, FOD = X = {a, b, c }, then $\sum_i (2^{|A_i|} - 1) = 19$, as depicted in Fig. 2.

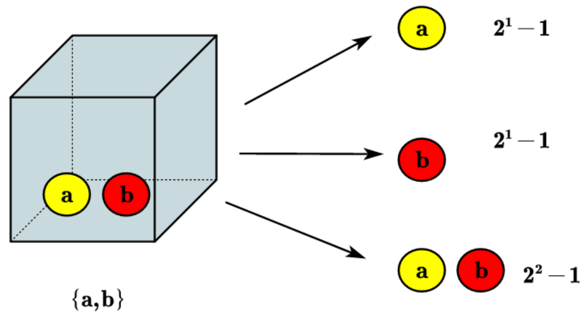

Figure 1: All the possible states when there are two balls inside a box

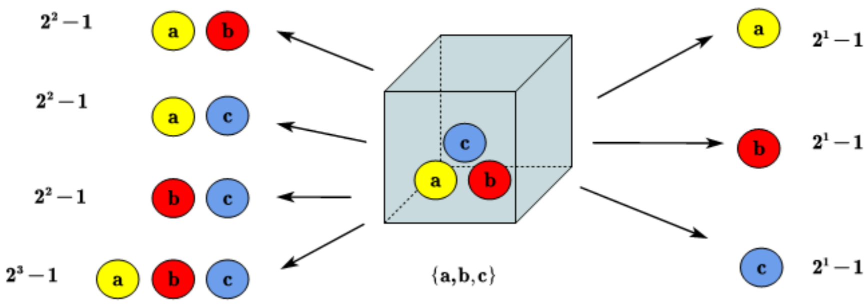

Figure 2: All the possible states when there are three balls inside a box

One simple conversion is as follows:

$$E_n(m) = \sum_{A \subseteq X} m(A) log_2 (\sum_i 2^{|A_i|} - 1) - \sum_{A \subseteq X} m(A) log_2 m(A) \tag{15}$$

where the first parameter measures the total non-specificity, whereas the second one measure of discord.

An example is presented.

Example 1. Suppose X = {A, B, C}, and the following are the two BPAs,

$$m_1 : m_1(\{A\}) = 0.6, m_1(\{A, B\}) = 0.4$$
$$m_2 : m_2(\{B\}) = 0.6, m_2(\{X\}) = 0.4$$

According to Eq. (14), the calculations are as follows,

$$E_n(m_1) = -0.6 \times log_2\frac{0.6}{2^1-1} - 0.4 \times log_2\frac{0.4}{(2^1-1)+(2^1-1)+(2^2-1)}$$
$$= 1.8997$$
$$E_n(m_2) = -0.6 \times log_2\frac{0.6}{2^1-1} - 0.4 \times log_2\frac{0.4}{3\times(2^1-1)+3\times(2^1-1)+(2^3-1)}$$
$$= 2.6701$$

It is shown that the entropy of m_2 is larger than that of m_1. which means the uncertainty of m_2 is greater.

3.2 Numerical Examples and Discussion

A. Probabilistic consistency

When there is only a single element in a BPA or $|A| \equiv 1$, the proposed entropy can be calculated as follows,

$$E_n(m) = -\sum_{A\subseteq X} m(A)log_2\frac{m(A)}{\sum_i 2^{|A_i|}-1}$$
$$= -\sum_{A\subseteq X} m(A)log_2\frac{m(A)}{2^1-1}$$
$$= -\sum_{A\subseteq X} m(A)log_2\, m(A)$$
$$E_n(m) = E_d(m) = E_s(m)$$

Example 2. Suppose FOD X={A} and m_3 (A)=1, Shannon entropy (E_s), Deng entropy (E_d) and the new entropy (E_n) are calculated according to (10), Eq. (11) and Eq. (14),

$$E_s(m_3) = -1 \times log_2 1 = 0$$
$$E_d(m_3) = -1 \times log_2\frac{1}{2^1-1} = 0$$
$$E_n(m_3) = -1 \times log_2\frac{1}{2^1-1} = 0$$

As shown in Example 2, it can be seen that m_3 assigns a belief of one hundred percent on the proposition A, which means there is no uncertainty in an information system. It is clear from the results that all entropies are equal to 0, which is reasonable.

Example 3. Suppose that in FOD X = A, B, C, D, the BPAs are ascribed as below,

$$m_5 : m_5(A) = m_5(B) = m_5(C) = m_5(D) = \frac{1}{4}$$

In this example, A, B, C, D have the same belief values with the same level of support. (E_s), (E_d) and (E_n) can be calculated as below,

$$E_s(m_5) = -\frac{1}{4}\times log_2\frac{1}{4} - \frac{1}{4}\times log_2\frac{1}{4} - \frac{1}{4}\times log_2\frac{1}{4} - \frac{1}{4}\times log_2\frac{1}{4}$$
$$= 2$$
$$E_d(m_5) = -\frac{1}{4}\times log_2\frac{\frac{1}{4}}{2^1-1} - \frac{1}{4}\times log_2\frac{\frac{1}{4}}{2^1-1} - \frac{1}{4}\times log_2\frac{\frac{1}{4}}{2^1-1} - \frac{1}{4}\times log_2\frac{\frac{1}{4}}{2^1-1}$$
$$= 2$$
$$E_n(m_5) = -\frac{1}{4}\times log_2\frac{\frac{1}{4}}{2^1-1} - \frac{1}{4}\times log_2\frac{\frac{1}{4}}{2^1-1} - \frac{1}{4}\times log_2\frac{\frac{1}{4}}{2^1-1} - \frac{1}{4}\times log_2\frac{\frac{1}{4}}{2^1-1}$$
$$= 2$$

From the above results in Example 3, the proposed belief entropy is the same with both Shannon entropy and Deng entropy if the mass value is only assigned one element.

B. Superiority

Example 4. Suppose $X = \{1, 2, ..., 14, 15\}$ with 15 elements, and the following are the BPAs [8],

$$m(\{3,4,5\}) = 0.05, m(6) = 0.05, m(A) = 0.8, m(X) = 0.1$$

The range of propositions denoted by the symbol A spans from 1 to 14. Table 2 presents the results obtained from various uncertainty metrics applied to both new entropy and Deng entropy, while Fig. 3 provides a visual comparison of the findings. Notably, the new belief entropy grows monotonically with the increase in the size of A, which is the same as Deng entropy. Nevertheless, a key distinction arises in that the proposed entropy significantly surpasses Deng entropy. The reason is that more available information contained in BPA is taken into account in the proposed entropy.

Table 2: Results of a variable number of elements in A

Cases	Deng entropy	The new entropy
A={1}	2.6623	3.6114
A={1, 2}	3.9303	5.3286
A={1, 2, 3}	4.9082	6.86941
A={1, ..., 4}	5.7878	8.2890
A={1, ..., 5}	6.6256	9.6479
A={1, ..., 6}	7.4441	10.9728
A={1, ..., 7}	8.2532	12.2773
A={1, ..., 8}	9.0578	13.5689
A={1, ..., 9}	9.8600	14.8524
A={1, ..., 10}	10.6612	16.1306
A={1, ..., 11}	11.4617	17.4053
A={1, ..., 12}	12.2620	18.6778
A={1, ..., 13}	13.0622	19.9487
A={1, ..., 14}	13.8622	21.2187

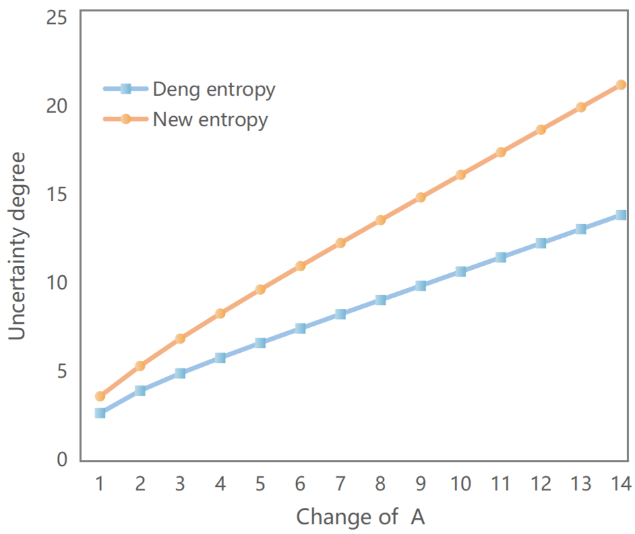

Figure 3: Comparison between the new entropy and Deng entropy

Furthermore, to provide a more comprehensive comparison, six additional uncertainty measures are introduced and depicted in Fig. 4. With the increasing number of elements in proposition A, measures can be divided into three different categories. The first category entails a constant level of uncertainty, exemplified by Hohle's confusion measure. The second category is characterized by a decreasing level of uncertainty, which includes Klir & Ramer's discord measure, Yager's dissonance, Klir & Parviz's strife measure, and George & Pal's conflict measure. Notably, both categories one and two appear counterintuitive as they disregard nonspecific uncertainty and only account for discordant uncertainty. In contrast, the third category, which encompasses Dubois & Prade's weighted

Hartley entropy, Deng entropy, and the proposed entropy, exhibits a corresponding increase in uncertainty as A expands. However, Dubois & Prade's weighted Hartley entropy exclusively considers non-specificity while ignoring discord uncertainty, a perspective that is equally unsound. Ultimately, the proposed entropy outperforms other uncertainty measures by offering a more reasonable and effective approach.

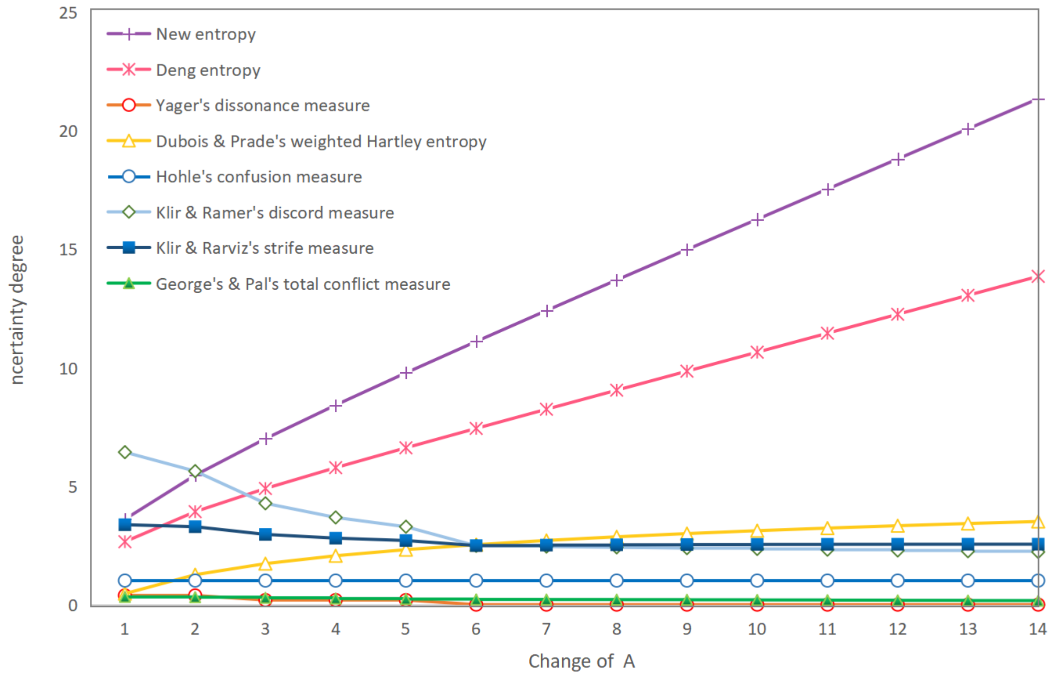

Figure 4: Comparison between the new entropy and other uncertainty measures

4 Applicaiton

Numerous techniques have been proposed to determine the weight of software risk [32, 35, 37]. This paper presents a straightforward risk evaluation model that utilizes the proposed entropy. As we know, it is crucial to establish a rational weight for risk evaluation. The proposed entropy is used to measure the uncertainty of risk. The uncertainty of risk increases with entropy, so the weight decreases with entropy. Whereas, the larger the weight, the smaller the risk uncertainty and the lower the entropy.

4.1 A Simple Software Risk Assessment Framework Based On The New Belief Entropy

Software risk encompasses two important aspects. The probability of risk occurrence and the severity of loss are affected. In accordance with Boehm's seminal work on software engineering [1], risk may be explicated as follows:

$$Risk = Probability(P) \times Severity(S) \tag{16}$$

As shown in the Fig. 5, software risk assessment can include the following steps,

Step 1. Expression of risk assessment.

Step 1.1. Establishing assessment criteria.

Since most experts and decision-makers (DMs) usually use linguistic information such as low, high, very high, etc to assess risk. In this paper, we provided the following evaluation criteria, as shown in Table 3, and some explanations are given in Table 4 and 5. Notably, the criteria are not fixed, and can be adjusted according to the actual situation.

Step 1.2. Assign and translate assessments into BPA.

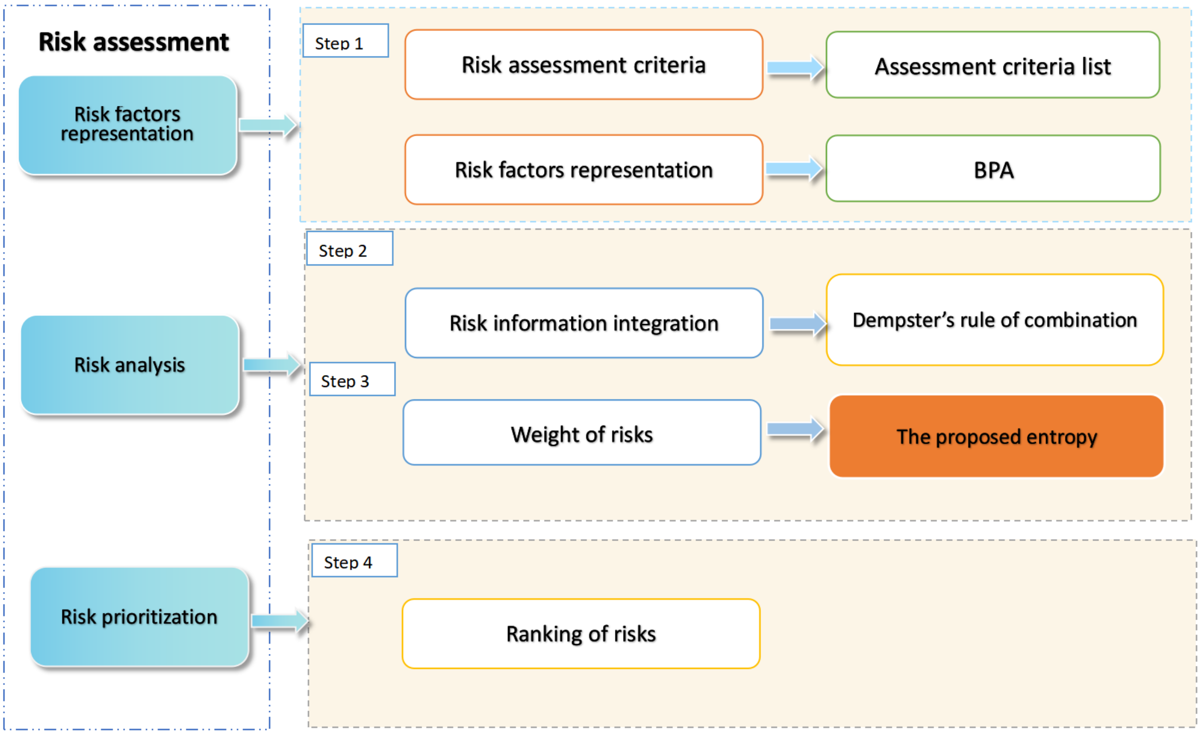

Figure 5: software risk assessment framework

Table 3: Linguistic terms

Level	Linguistic terms
−2	Very low
−1	Low
0	Medium
1	High
2	Very high

Table 4: The probability of risk

Linguistic terms	Possibility
Very low	The consequences are negligible
Low	The consequences are not overly severe
Medium	The consequences are moderate
High	The consequences are serious
Very high	The consequences are extremely severe

Table 5: The severity of risk

Linguistic terms	Severity
Very low	The consequences are negligible.
Low	The consequences are not too severe.
Medium	The consequences are moderate
High	The consequences are serious
Very high	The consequences are extremely severe

N experts are invited to assess both the probability and severity of risk using the criteria presented in Table 3. For example $\{p_{-1}(0.6), p_{-1}p_0(0.4)\}$ means the probability of "low" is 60% and the probability of hesitation between "low" and "medium" is 40%. Then the evaluation results should be translate into BPA. $\{p_{-1}(0.6), p_{-1}p_0(0.4)\}$ is equivalent to $m_1(\{p_{-1}\}) = 0.6, m_2(\{p_{-1}, p_0\}) = 0.4$. If $\{s_{-1}(0.5), s_0(0.5)\}$ means the severity of "low" is 50% and "medium" is also 50%. Translating the evaluation results into BPA is $m_1(\{s_{-1}\}) = 0.5, m_2(\{s_0\}) = 0.5$.

Step 2. Integrating Risk Information.

For each risk, the assessment values given by all experts are fused using the Dempster's combination rule.

Step 3. Calculating risk weights W_R.

Step 3.1. Calculating the uncertainty of the probability and severity.

According to Eq. (17), use the new entropy to calculate the uncertainty of the probability $E_n(P)$ and the uncertainty of the severity $E_n(S)$.

$$E_n(R) = E_n(P) + E_n(S) \tag{17}$$

Step 3.2. Calculating the credit value for each risk.

According to Eq. (18), $CredR$ are calculated as following,

$$CredR = \frac{E_n(R)}{1 + E_n(R)} \tag{18}$$

Step 3.3. Calculating the weight for each risk

According to Eq. (19), w_e are calculated as following,

$$w_R = \frac{CredR}{\sum_{y=1}^{n} CredR} \tag{19}$$

Step 4. Ranking of the risk.

Step 4.1. Calculate the value the probability and the severity.

Table 6: Values of score

BPA	score	BPA	score
$m(\{p_{-2}\})/m(\{s_{-2}\})$	$\frac{0}{9}$	$m(\{p_{-2}, p_{-1}\})/m(\{s_{-2}, s_{-1}\})$	$\frac{1}{9}$
$m(\{p_{-1}\})/m(\{s_{-1}\})$	$\frac{2}{9}$	$m(\{p_{-1}, p_0\})/m(\{s_{-1}, s_0\})$	$\frac{3}{9}$
$m(\{p_0\})/m(\{s_0\})$	$\frac{4}{9}$	$m(\{p_0, p_1\})/m(\{s_0, s_1\})$	$\frac{5}{9}$
$m(\{p_1\})/m(\{s_1\})$	$\frac{6}{9}$	$m(\{p_1, p_2\})/m(\{s_1, s_2\})$	$\frac{7}{9}$
$m(\{p_2\})/m(\{s_2\})$	$\frac{8}{9}$	$m(\{p_X\}/m(\{s_X\})$	$\frac{9}{9}$

For example, a severity of risk value after fusion is as follows: $(m\{s_0\}) = 0.2079, m(\{s_0, s_1\}) = 0.1386, m(\{s_1\}) = 0.5941, m(\{s_1, s_2\}) = 0.0594$. According to Table 6, the value of S is calculated as:

$$\begin{aligned} S &= \frac{4}{9} \times m(\{s_0\}) + \frac{5}{9} \times m(\{s_0, s_1\}) + \frac{6}{9} \times m(\{s_1\}) + \frac{7}{9} \times m(\{s_1, s_2\}) \\ &= \frac{4}{9} \times 0.2079 + \frac{5}{9} \times 0.1386 + \frac{6}{9} \times 0.5941 + \frac{7}{9} \times 0.0594 \\ &= 0.6117 \end{aligned}$$

Step 4.2. Calculate the risk using the Eq. (20) and proceed with ranking the results.

$$Risk = w_R \times P \times S \tag{20}$$

4.2 A Case Study

In this section, we will give a case study. There are six risk factors in a software project. And three experts are invented to the risks.

Step 1. Based on the assessment criteria in Table 3, three experts gave their assessment and converted into BPAs. The results are shown in Table 7.

Table 7: Experts assignment

Risks	Experts	P	S
RS1	$m_1(.)$	$\{p_{-1}p_0(0.6), p_0p_1(0.4)\}$	$\{s_{-1}(0.5), s_0(0.5)\}$
	$m_2(.)$	$\{p_{-1}(0.4), p_0p_1(0.25)\}$	$\{s_0s_1(0.8)\}$
	$m_3(.)$	$\{p_{-1}(0.65), p_1(0.35)\}$	$\{s_0(0.7)\}$
RS2	$m_1(.)$	$\{p_{-1}(0.7), p_0(0.2)\}$	$\{s_0(0.4), s_1s_2(0.6)\}$
	$m_2(.)$	$\{p_{-1}p_0(0.6), p_0p_1(0.4)\}$	$\{s_1(0.65)\}$
	$m_3(.)$	$\{p_0(0.5)\}$	$\{s_0(0.6)\}$
RS3	$m_1(.)$	$\{p_{-2}(0.3), p_{-2}p_{-1}(0.3), p_{-1}(0.4)\}$	$\{s_0s_1(0.7), s_1s_2(0.3)\}$
	$m_2(.)$	$\{p_{-1}(0.6), p_{-1}p_0(0.3)\}$	$\{s_0(0.6)\}$
	$m_3(.)$	$\{p_{-1}p_0(0.7)\}$	$\{s_1(0.75)\}$
RS4	$m_1(.)$	$\{p_{-1}(0.7)\}$	$\{s_1s_2(0.8)\}$
	$m_2(.)$	$\{p_0(0.9)\}$	$\{s_1(0.75)\}$
	$m_3(.)$	$\{p_{-1}(0.4), p_0p_1(0.25)\}$	$\{s_0s_1(0.6), s_1s_2(0.4)\}$
RS5	$m_1(.)$	$\{p_0(0.5), p_0p_1(0.4)\}$	$\{s_{-1}(0.5), s_0(0.5)\}$
	$m_2(.)$	$\{p_0(0.5)\}$	$\{s_0s_1(0.8)\}$
	$m_3(.)$	$\{p_{-1}p_0(0.7), p_0p_1(0.3)\}$	$\{s_{-1}s_0(0.6), s_1(0.4)\}$
RS6	$m_1(.)$	$\{p_1(0.5), p_1p_2(0.4)\}$	$\{s_{-1}s_0(0.8), s_1(0.2)\}$
	$m_2(.)$	$\{p_0p_1(0.8)\}$	$\{s_0(0.65)\}$
	$m_3(.)$	$\{p_0(0.6), p_1(0.25)\}$	$\{s_0s_1(0.9)\}$

Step 2. After using Dempster's combination rule, the fusion results of P and S are shown in Table 8.

Step 3. Based on Eq. (17), Eq. (18) and Eq. (19) the weights of risks are $w_1 = 0.1704, w_2 = 0.1870, w_3 = 0.1613, w_4 = 0.1879, w_5 = 0.1403, w_6 = 0.1531$.

Step 4. Based on Eq. (20), the risk values and rankings are shown in Table 9. We can easily obtain that $RS4 > RS6 > RS2 > RS1 > RS5 > RS3$. The $RS4$ is the highest risk and it should be focused on. While $RS6$ is the second highest one, and also needs to be given attention.

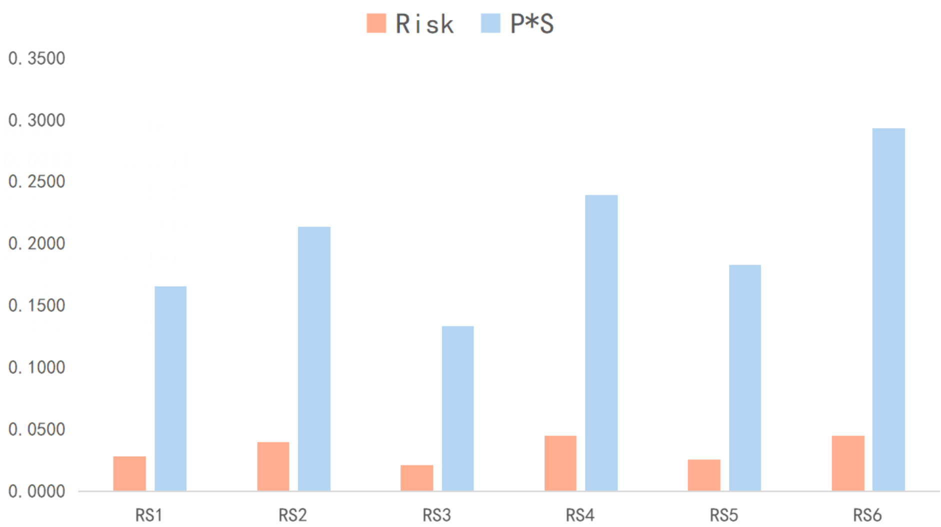

Figure 6: Comparison of risk values

In Table (9) and Fig 6, we can see that the results will be different by using the proposed method. And if we just use P*S, the risk values and rankings are different. This is because the uncertainty of

Table 8: Fusion Results

Risks	P	S
RS1	$\{p_{-1}(0.5821), p_0(0.2507), p_0p_1(0.1672)\}$	$\{s_0(0.7692), s_1(0.2309)\}$
RS2	$\{p_{-1}(0.4118), p_{-1}p_0(0.0588), p_0(0.4902), p_0p_1(0.0392)\}$	$\{s_0(0.3684), s_1(0.4105), s_1s_2(0.2211)\}$
RS3	$\{p_{-2}(0.0127), p_{-2}p_{-1}(0.0127), p_{-1}(0.9746)\}$	$\{s_0s_1(0.2079), s_0s_1(0.1386), s_1(0.5941), s_1s_2(0.0594)\}$
RS4	$\{p_{-1}(0.2638), p_0(0.6626), p_0p_1(0.0307), p_x(0.0429)\}$	$\{s_0(0.375), s_0s_1(0.0938), s_1(0.4688), s_1s_2(0.0625)\}$
RS5	$\{p_{-1}p_0(0.035), p_0(0.89), p_0p_1(0.075)\}$	$\{s_{-1}(0.1667), s_0(0.8333)\}$
RS6	$\{p_0(0.1304), p_0p_1(0.0261), p_1(0.8109), p_1p_2(0.0261), p_x(0.0065)\}$	$\{s_{-1}s_0(0.0322), s_0(0.8874), s_1(0.0805)\}$

Table 9: Risk values and rankings

Risks	P*S	Proposed method	P*S Rating	Proposed method Rating
RS1	0.1654	0.0282	5	4
RS2	0.2137	0.0400	3	3
RS3	0.1333	0.0215	6	6
RS4	0.2395	0.0450	2	1
RS5	0.1829	0.0257	4	5
RS6	0.2936	0.0449	1	2

the expert assessment value is measured using the proposed entropy. More information is considered. Also, avoiding subjective bias of experts and assigning weights from an objective point of view, it has a more fair and reasonable character.

To prove the validity of the proposed entropy, we also made some other comparisons. We analysed the example in Ref. [3]. We replaced the Deng entropy with the proposed entropy and calculated the uncertainty. The final rankings of the risks are the same as in Ref. [3]. Therefore, the proposed entropy is effective.

5 Conclusion

Measure uncertainty in the DST framework is still an open issue. In this paper, a new belief entropy based Deng entropy has been proposed. In the DST framework, the performance of uncertainty measures can be improved when using the new entropy. The new entropy can identify the uncertainty more effectively and provides a new way of the splitting of the mass function thinking about measuring uncertainty. The experimental results illustrate that the new entropy we proposed is better than other methods. A case study on risk assessment further highlights the effectiveness of the proposed entropy in practical applications. However, it is worth noting that the new entropy may not fully satisfy all the characteristics proposed by Klir and Wierman [19]. Whether or not it is necessary in Dempster-Shafer theory is still a topic for discussion.

In the future, based on the new belief entropy, we can do more research. According to the main ideas of BPA divisions in this paper, more divisions methods can be designed. Besides more application could be use this entropy.

Acknowledgment

The work is partially supported by National Natural Science Foundation of China (Grant No. 61973332), JSPS Invitational Fellowships for Research in Japan (Short-term).

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Cite this paper as:

Chen, X.Y.; Deng, Y. (2023). A new belief entropy and its application in software risk analysis, *International Journal of Computers Communications & Control*, 18(2), 5299, 2023.
https://doi.org/10.15837/ijccc.2023.2.5299