INDONESIAN JOURNAL OF INNOVATION AND APPLIED SCIENCES (IJIAS)
Journal Homepage: https://ojs.literacyinstitute.org/index.php/ijias
ISSN: 2775-4162 (Online)
Research Article

Volume 3	Issue 1	February (2023)	DOI: 10.47540/ijias.v3i1.729	Page: 74 – 85

The Multi-Objective Transportation Problem Solve with Geometric Mean and Penalty Methods

K.P.O.Niluminda[1], E.M.U.S.B.Ekanayake[1]
[1]Department of Physical Sciences, Rajarata University of Sri Lanka, Sri Lanka
Corresponding Author: K.P.O. Niluminda; Email: kponiluminda@gmail.com

ARTICLE INFO

Keywords: Best Solution, Geometric Mean, Linear Programming, Multi-Objective Transportation Problem, Penalty Method.

Received : 27 November 2022
Revised : 20 February 2023
Accepted : 25 February 2023

ABSTRACT

The traditional (classical) Transportation Problem (TP) can be viewed as a specific case of the Linear Programming (LP) problem, as well as its models are used to find the best solution for the problem of predetermined how many units of a good or service need to be shipped from one source to multiple locations, with the goals being to reduce time or expense. Classical TP has one objective but when there are two or more objectives to be optimized for in a TP, the strategies used to optimize a single objective are inapplicable. The term "Multi-Objective Transportation Problem (MOTP)" refers to situations in which there are two or more objectives in a TP. The specific extension of the transportation problem is the MOTP. This work provided a novel alternative algorithm that uses geometric means along with the penalty technique to address MOTP. Specifically, analyzed data by comparing our method with numerical examples and presenting the results in a line graph. Our analysis shows that our approach yields better solutions than existing methods, demonstrating the novelty and effectiveness of our approach. The comparison with numerical examples provides a clear and intuitive way of presenting the superiority of our method, making it accessible to practitioners and researchers in the field. These findings have important implications for improving the accuracy and reliability of solutions to the problem at hand. Overall, our study contributes to the advancement of the field by providing a novel and effective method for solving the problem.

INTRODUCTION

Transportation of products and services from several supply locations to several demand centers is an important field in which linear programming is used. The simplex approach may also be used to resolve a TP that is formulated in terms of an LP model. It requires a lot of time to solve a TP using simplex methods, even though it has many variables and constraints. There are several shipping routes from various supply locations to various demand locations that make up the structure of the TP (Kankanam Pathiranage Oshan Niluminda, 2022; Niluminda & Ekanayake, 2022). The goal is to establish shipping routes between supply and demand hubs to fulfill the demand for a certain amount of products or services at each destination location with the supply of those same goods or services at each supply location at the lowest possible transportation expense.

There are several types of TP with different cases. One of the special types of TP is called MOTP. It is referred to as a multi-objective transportation issue when it incorporates numerous objective functions (Ekanayake et al., 2022). When dealing with real-world issues, every business aims to convey commodities while also achieving multiple objectives such as minimizing cost, time, distance, risk, etc. The first TP model was created by Hitchcock (Hitchcock, 1941) in 1941. In real-world scenarios, classical TP can be reformulated as MOTP models due to the complexity of the social and economic context, which necessitates explicit consideration of factors other than expense. Various methods for resolving management-level issues with many competing objectives were initially described by Charnes and Cooper (Tjalling C. Koopmans, 1941) in 1961. M. Zangiabadi (Zangiabadi & Maleki, 2007) use fuzzy goal programming to address MOTP in 2007. Lushu Li (Lohgaonkar & Bajaj, 2009) proposed a fuzzy compromise programming technique for MOTP. For the linear MOTP, Lakhveer Kaur (Ahmed et al., 2016) suggests a straightforward method for obtaining the optimal compromise solution. Osuji George (George A., 2014) proposed a method using a fuzzy programming algorithm. The three objective linear TPs were solved by Doke D.M. (Doke, 2015) using the arithmetic mean of the global assessment. Evolutionary algorithms were used for the MOTP by K. Bharathi (Bharathi & Vijayalakshmi, 2016) in 2016. Khilendra Singh (Singh & Rajan, 2020) used geometric means to address MOTP under a fuzzy environment. Using an S-type membership function, M.A.M. Khan (Khan & Kabeer, 2015) examines the multi-objective transportation issue. Kavita (Goel, 2021) suggested the novel row maxima method to MOTP utilizing c-program and fuzzy methodology. To solve a MOTP using Pareto Optimality Criteria, Khilendra Singh (Singh & Rajan, 2019) created a novel technique called the Matrix Maxima Method. The geometric mean method and ant colony optimization algorithm were proposed by E.M.U.S.B.Ekanayake (Ekanayake, 2022) in 2022 to address MOTP in fuzzy environments. Moreover, many researchers proposed several approaches to solve MOTP. M. Afwat A.E. (M. Afwat et al., 2018), T. Karthy (Karthy & Ganesan, 2018), Ekanayake E.M.U.S.B. (E. M. U. S. B. et al., 2021; Ekanayake, 2022; Ekanayake et al., 2020, 2021, 2022) Rakesh Verma (Zangiabadi & Maleki, 2013), Abouzar Sheikhi (Pandian & Anuradha, 2011), Sanjay R. Ahir (College, 2021), Kirti Kumar Jain (Jain et al., 2019), and M.A.Nomani (Nomani et al., 2017) were proposed a different type of algorithms to resolve MOTP.

This work focus on building a new alternative algorithm to solve multi-objective transportation problem using geometric mean combined with the penalty method. In the end, the proposed method compares with existing different methods using illustrative examples of MOTP with several objectives.

METHODS

Geometric mean

By calculating the multiplication of the values of a collection of numbers, the Geometric Mean (GM) is the average value or mean that denotes the central tendency of the data. The n^{th} root of the multiplication of n numbers is another way to determine the geometric mean. GM average formula can be shown below in equation (1):

$$GM = \sqrt[n]{x_1.x_2.x_3 \dots x_n} = \left(\prod_{i=1}^{n} x_i\right)^{1/n} \quad (1)$$

Mathematical Formulation of MOTP

Amount x_{ij} in the MOTP is to be carried from sources i to destinations j at an expense c_{ij}. Here $i = 1, 2, \dots$ m and $j = 1, 2, \dots$, n and also ${c_{ij}}^k$ might represent shipping costs/times/ distances, transport risk, or power consumption, among other things. If the problem has a "t" number of objectives to minimize transport expenses, then it can be written as $Z^1(x), Z^2(x), Z^3(x), \dots, Z^t(x)$. The mathematical formulation of MOTP with constraints is given in below:

Objective functions:

$$\text{Obj. no. } 1 = Z^1(x) = \sum_{i=1}^{m} \sum_{j=1}^{n} {c_{ij}}^1 . x_{ij} \quad (2)$$

$$\text{Obj. no. } 2 = Z^2(x) = \sum_{i=1}^{m} \sum_{j=1}^{n} {c_{ij}}^2 . x_{ij} \quad (3)$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$\text{Obj. no. } t = Z^t(x) = \sum_{i=1}^{m} \sum_{j=1}^{n} {c_{ij}}^t . x_{ij} \quad (4)$$

Constraints:

Supply constraint:

$$\sum_{j=1}^{n} x_{ij} = a_i, \quad i=1, 2, \dots, m \quad (5)$$

Demand constraint:

$$\sum_{i=1}^{m} x_{ij} = b_j, \quad j=1, 2, \dots, n \quad (6)$$

$$x_{ij} \geq 0 \; ; \; \forall \;\; i=1, 2, \dots, m \;\&\;\; j=1, 2, \dots, n$$

Table 1. General Tabular representation of the multi-objective transportation problem with notations

Destination → / Source ↓	D_1	D_2	…	D_n	Supply (a_i)
S_1	$C_{11}^{\,1}, C_{11}^{\,2}, \dots C_{11}^{\,t}$	$C_{12}^{\,1}, C_{12}^{\,2}, \dots C_{12}^{\,t}$	…	$C_{1n}^{\,1}, C_{1n}^{\,2}, \dots C_{1n}^{\,t}$	a_1
S_2	$C_{21}^{\,1}, C_{21}^{\,2}, \dots C_{21}^{\,t}$	$C_{22}^{\,1}, C_{22}^{\,2}, \dots C_{22}^{\,t}$	…	$C_{2n}^{\,1}, C_{2n}^{\,2}, \dots C_{2n}^{\,t}$	a_2
⋮	⋮	⋮	…	⋮	⋮
S_m	$C_{m1}^{\,1}, C_{m1}^{\,2}, \dots C_{m1}^{\,t}$	$C_{m2}^{\,1}, C_{m2}^{\,2}, \dots C_{m2}^{\,t}$	…	$C_{mn}^{\,1}, C_{mn}^{\,2}, \dots C_{mn}^{\,t}$	a_m
Demand (b_j)	b_1	b_2	…	b_n	

Here;

C_{ijMAX} = Each column's or row's highest C_{ij} value

C_{ijMIN2} = Each column's or row's second-minimum C_{ij} value

The Proposed Novel Algorithm

This section presents the proposed novel algorithm with steps. This method can apply both balanced and unbalanced multi-objective transportation problems and the best solution or near-best solution can be obtained. The steps of the proposed method can illustrate as follows:

Step 1: Verify whether your MOTP is balanced. If the MOTP table is unbalanced, add a dummy row or column to make it balanced.

Step 2: Calculate the Geometric Mean (GM) of every cell using the following equation (7) and create a new table using GM values.

$$\mathrm{GM}=\left(\prod_{i=1}^{n} x_i\right)^{1/n} = \sqrt[n]{x_1 \times x_2 \times x_3 \times \ldots \times x_n} \quad (7)$$

Here; x_i is the unit cost of each objective in each cell

Step 3: Use the formula below (8) to determine the penalty value for each row and column:

$$|C_{ij_{MAX}} - C_{ij_{MIN_2}}| \quad (8)$$

Step 4: Assign the relevant min (Supply, Demand) to the minimum C_{ij} value cell, selecting the maximum penalty value for each row and column.

Step 5: If there is a tie in the maximum penalty value for a column or row, choose the penalty value that corresponds to the minimum C_{ij} value of that rows or columns.

Step 6: If the allocation in the previous row meets the supply at the origin, cross out the corresponding row. If it fulfills the requirement there, cross out the corresponding column.

Step 7: The procedure should be terminated if demand is satisfied at each destination and supply is enough at each origin. If not, repeat the preceding steps.

Step 8: Utilizing a MOTP allocation table, determine the relevant effective cost values for each objective.

RESULTS AND DISCUSSION

This section will investigate both balanced and unbalanced MOTP and compare the newly recommended method to an optimal solution. Example 1 (Ekanayake et al., 2022). This example represents a multi-objective transportation problem with three objectives such as cost, time, and distance.

Table 2. Step 1 (Initial multi-objective transportation table with cost, time, and distance)

(Cost, Time, Distance)	D_1	D_2	D_3	D_4	Supply
S_1	21	16	15	13	11
	1	2	1	4	
	11	13	17	14	
S_2	17	18	24	23	13
	3	3	2	1	
	16	18	14	10	
S_3	32	27	18	41	19
	4	2	5	9	
	21	24	13	10	
Demand	6	10	12	15	

Table 3. Steps 2 – 3 (Geometric mean of objectives (Cost, Time, Distance))

Geometric Mean	D_1	D_2	D_3	D_4	Supply
S_1	6.13	7.46	6.34	8.99	11
S_2	9.34	9.90	8.75	6.13	13
S_3	13.90	10.90	10.53	15.45	19
Demand	6	10	12	15	

Table 4. Steps 4 – 7 (Penalty method for example 1)

	D_1	D_2	D_3	D_4	Supply	P1	P2	P3	P4	P5	P6
S_1	6.13 **(6)**	7.46	6.34 **(3)**	8.9 **(2)**	11	2.65	2.65	1.53	1.12	-	-
S_2	9.34	9.90	8.75	6.13 **(13)**	13	1.15	-	-	-	-	-
S_3	13.90	10.90 **(10)**	10.53 **(9)**	15.45	19	4.55	4.55	4.55	0.37	0.37	10.53
Demand	6	10	12	15							
P1	4.56	1.00	1.78	6.46							
P2	7.77	3.44	4.19	6.46							
P3	-	3.44	4.19	6.46							
P4	-	3.44	4.19	-							
P5	-	10.90	10.53	-							
P6	-	-	10.53	-							

Table 5. Step 8 (Final allocation table of example 1)

(Cost, Time, Distance)	D_1	D_2	D_3	D_4	Supply
S_1	(21, 1, 11) **[6]**	(16, 2, 13)	(15, 1, 17) **[3]**	(13, 4, 14) **[2]**	11
S_2	(17, 3, 16)	(18, 3, 18)	(24, 2, 14)	(23, 1, 10) **[13]**	13
S_3	(32, 4, 21)	(27, 2, 24) **[10]**	(18, 5, 13) **[9]**	(41, 9, 10)	19
Demand	6	10	12	15	

Minimum Cost = (6 × 21) + (3 × 15) + (2 × 13) + (13 × 23) + (10 × 27) + (9 × 12) = 874
Minimum Time = (6 × 1) + (3 × 1) + (2 × 4) + (13 × 1) + (10 × 2) + (9 × 5) = 95
Minimum Distance = (6 × 11) + (3 × 17) + (2 × 14) + (13 × 10) + (10 × 24) + (9 × 13) = 632

Table 6. Comparison analysis of example 1 with different methods

Comparison Analysis	Minimum Cost	Minimum Time	Minimum Distance
New Row Maxima Method (Goel, 2021)	938	117	457
Product Approach (M. A. E. Afwat et al., 2018)	938	132	552
Geometric Mean Method (Singh & Rajan, 2020)	904	107	587
Ekanayake's Method (Ekanayake, 2022b)	904	107	587
Proposed Method	874	95	632
LINGO	796	89	527

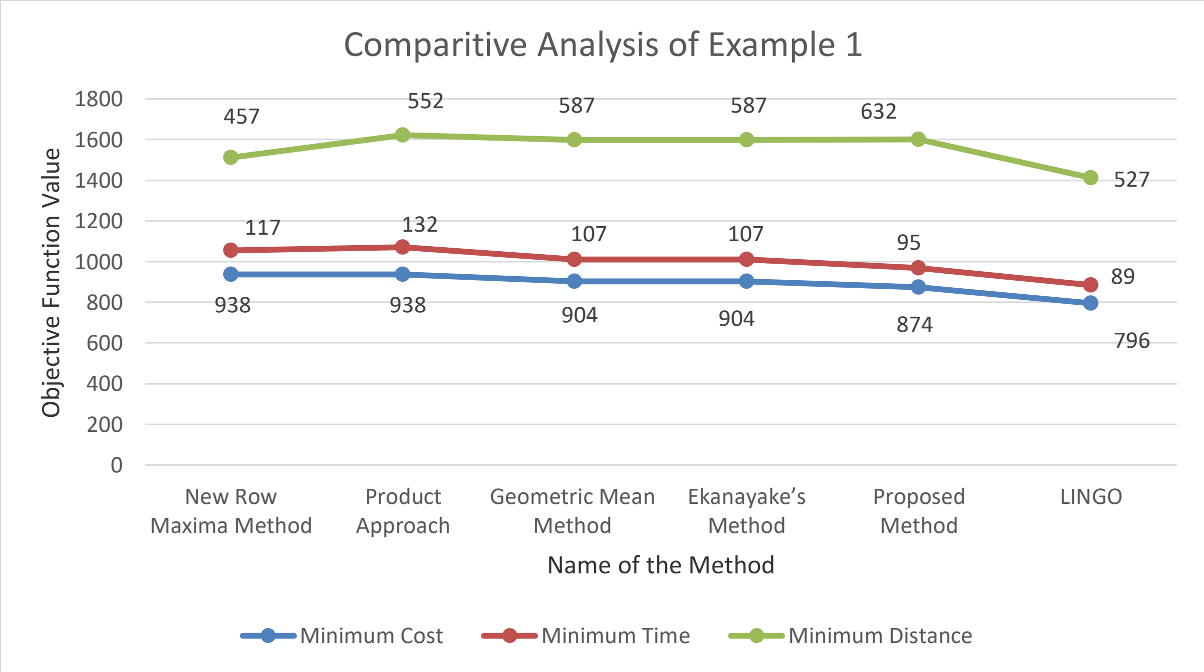

Figure 1. Comparison Analysis of example 1 with different methods

Example 1 shows the MOTP, which has three objectives. The objective of that problem is to minimize the cost, time, and distance. Table 6 represents a comparison analysis of the proposed method with the New Row Maxima Method (Goel, 2021), Product Approach (M. A. E. Afwat et al., 2018), Geometric Mean Method (Singh & Rajan, 2020), Ekanayake's Method (Ekanayake, 2022b), and Optimum solution obtained by LINGO. Figure 2 shows the line graph representation of that comparison. The proposed method gives a better solution in cost, time, and distance objectives by comparison to the other existing methods.

Example 2 (Singh & Rajan, 2019)**.** This example represents a multi-objective transportation problem with two objectives such as cost and time.

Table 7. Step 1 (Initial multi-objective transportation table with cost and time)

(Time, Cost)	D_1	D_2	D_3	Supply
S_1	(13, 14)	(15, 15)	(16, 10)	17
S_2	(7, 21)	(11, 13)	(2, 19)	12
S_3	(19, 17)	(20, 26)	(9, 9)	16
Demand	14	8	23	

Table 8. Step 2 – 3 (Geometric mean of objectives (Cost, Time))

Geometric Mean	D_1	D_2	D_3	Supply
S_1	13.49	15.00	12.65	17
S_2	12.12	11.96	6.16	12
S_3	17.97	22.80	9.00	16
Demand	14	8	23	

Table 9. Steps 4 – 7 (Penalty method for example 2)

	D_1	D_2	D_3	Supply	P1	P2	P3	P4	P5
S_1	13.49 **(14)**	15.00	12.65 **(3)**	17	1.51	0.84	0.84	0.84	12.65
S_2	12.12	11.96 **(8)**	6.16 **(4)**	12	0.16	5.96	5.96	-	-
S_3	17.97	22.80	9.00 **(16)**	16	4.83	8.97	-	-	-
Demand	14	8	3						
P1	4.48	7.80	3.65						
P2	4.48	-	3.65						
P3	1.37	-	6.49						
P4	13.49	-	12.65						
P5	-	-	12.65						

Table 10. Step 8 (Final allocation table of example 2)

(Time, Cost)	D_1	D_2	D_3	Supply
S_1	(13, 14) **[14]**	(15, 15)	(16, 10) **[3]**	17
S_2	(7, 21)	(11, 13) **[8]**	(2, 19) **[4]**	12
S_3	(19, 17)	(20, 26)	(9, 9) **[16]**	16
Demand	14	8	23	

Minimum Time = (14 × 13) + (3 × 16) + (8 × 11) + (4 × 2) + (16 × 9) = 470
Minimum Cost = (14 × 14) + (3 × 10) + (8 × 13) + (4 × 19) + (16 × 9) = 550

Table 11. Comparison analysis of example 2 with different methods

Comparison Analysis	Minimum Time	Minimum Cost
New Row Maxima Method (Goel, 2021)	656	652
Product Approach (M. A. E. Afwat et al., 2018)	440	583
Matrix Maxima (Singh & Rajan, 2019)	470	550
Proposed Method	470	550

Example 2 TP has two objectives (Time, Cost). The goal is to reduce the overall time and expense. Table 11 shows the comparative analysis of example 2 and figure 2 represents its graphical representation. By comparing the New Row Maxima Method (Goel, 2021), Product Approach (M. A. E. Afwat et al., 2018), and Matrix Maxima Method (Singh & Rajan, 2019) our proposed method gives a better solution. Both Matrix Maxima and the proposed method give the same results.

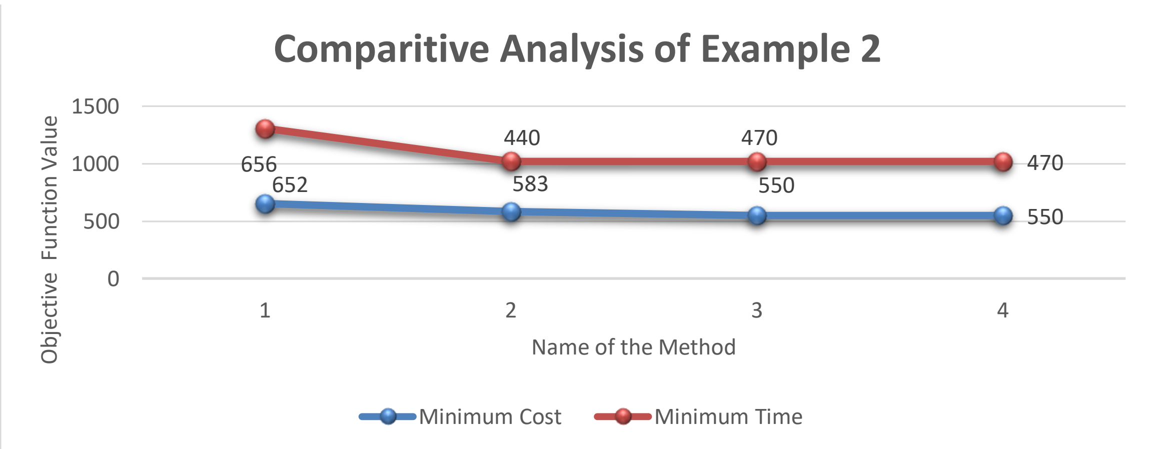

Figure 2. Comparison Analysis of example 2 with different methods

Example 3 (Singh & Rajan, 2019). This example represents a multi-objective transportation problem with two objectives such as cost and time.

Table 12. Initial multi-objective transportation table with cost and time

(Time, Cost)	D_1	D_2	D_3	D_4	Supply
S_1	(6, 1)	(4, 2)	(1, 3)	(5, 4)	14
S_2	(8, 4)	(9, 3)	(2,2)	(7, 0)	16
S_3	(4, 0)	(3, 2)	(6, 2)	(2, 1)	5
Demand	6	10	15	4	

Table 13. Comparison analysis of example 3 with different methods

Comparison Analysis	Minimum Time	Minimum Cost
New Row Maxima Method (Goel, 2021)	162	83
Product Approach (M. A. E. Afwat et al., 2018)	114	62
Matrix Maxima (Singh & Rajan, 2019)	115	57
Proposed Method	121	54

The MOTP with two objectives such as time and cost shown in example 3. The objective of that example is to minimize the total time and cost. This example compares with New Row Maxima Method (Goel, 2021), the Product Approach (M. A. E. Afwat et al., 2018), Matrix Maxima Method (Singh & Rajan, 2019), and the proposed algorithm. The proposed method gives the most minimum cost value compared to the other methods in this example. Table 13 shows the comparative results of that example and figure 3 represents the line graph analysis with the existing three methods.

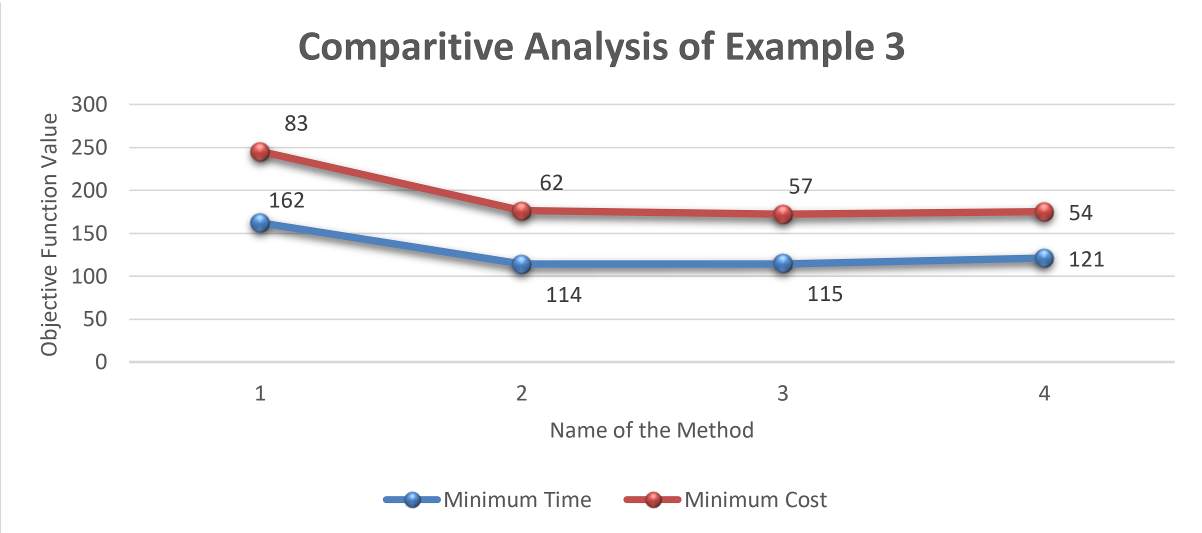

Figure 3. Comparison Analysis of example 3 with different methods

Example 4 (Ahmed et al., 2016). This example represents a multi-objective transportation problem with two objectives.

Table 14. Initial multi-objective transportation table with objectives Z_1 and Z_2

(Z_1, Z_2)	D_1	D_2	D_3	Supply
S_1	(3, 5)	(4, 2)	(5, 1)	8
S_2	(4, 3)	(5, 4)	(2, 3)	5
S_3	(5, 2)	(1, 3)	(2, 1)	2
Demand	7	4	4	

Table 15. Comparison analysis of example 3 with different methods

Comparison Analysis	Z_1	Z_2
Kaur's Method (Ahmed et al., 2016)	55	40
Proposed Method	56	39

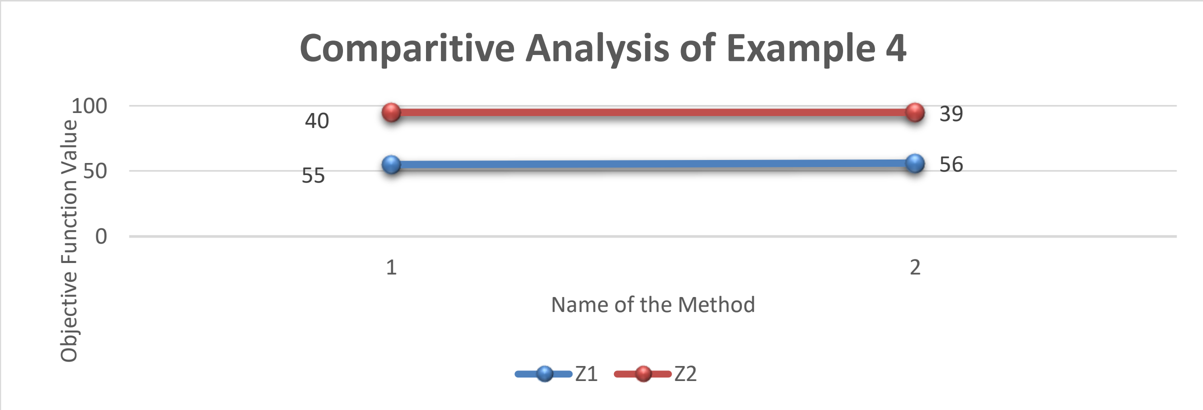

Figure 4. Comparison Analysis of example 4 with Kaur's method

The proposed method was compared with Kaur's Method (Ahmed et al., 2016) in this example. This TP has two objectives (Z_1 and Z_2). The comparative evaluation of that example is shown in table 15 and the graphical evaluation shows in figure 4. By looking at above table 15 and figure 4, the proposed method gives the most minimum value in the Z_1 objective.

Example 5 (Ekanayake, 2022)**.** This example represents a multi-objective transportation problem with three objectives such as cost, time, and distance.

Table 16. Initial multi-objective transportation table with cost, time, and distance

(Cost, Time, Distance)	D_1	D_2	D_3	D_4	Supply
S_1	(6, 13, 6)	(4, 11, 3)	(1, 15, 5)	(5, 20, 4)	14
S_2	(8, 17, 5)	(9, 14, 9)	(2, 12, 2)	(7, 13, 7)	16
S_3	(4, 18, 5)	(3, 18, 7)	(6, 15, 8)	(2, 12, 6)	5
Demand	6	10	15	4	

Table 17. Comparison analysis of example 5 with different methods

Comparison Analysis	Minimum Cost	Minimum Time	Minimum Distance
New Row Maxima Method (Goel, 2021)	112	461	130
Product Approach (M. A. E. Afwat et al., 2018)	114	425	128
Geometric Mean Method (Singh & Rajan, 2020)	114	425	118
Ekanayake's Method (Ekanayake, 2022b)	114	425	118
Proposed Method	114	425	118
LINGO	114	424	106

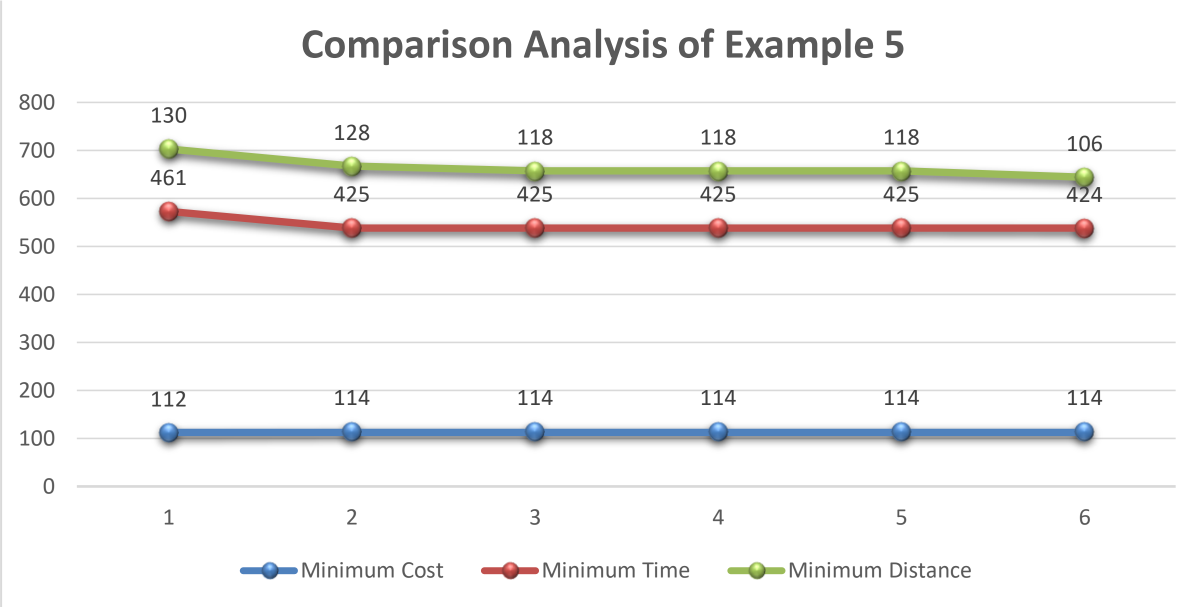

Figure 5. Comparison Analysis of example 5 with different methods

The TP with three objectives (Cost, Time, Distance) is represented in this example 5. The main objective of this example is to minimize the total cost, time, and distance. The comparison analysis of this example shows in table 17. Compare to the other existing method proposed method gives the optimal solution to the cost objective and a near-optimal solution to the time and distance. Figure 5 presents the comparison result of three objectives using a line graph. Compare to the Geometric Mean Method (Singh* & Rajan, 2020) and Ekanayake's Method (Ekanayake, 2022b), the proposed method also gives the same outcomes.

Example 6 (Doke et al., 2015)**.** This example represents a multi-objective transportation problem with three objectives.

Table 18. Initial multi-objective transportation table with cost, time, and distance

(Z_1, Z_2, Z_3)	D_1	D_2	D_3	D_4	Supply
S_1	(3, 2, 8)	(2, 5, 4)	(5, 7, 3)	(7, 9, 2)	10
S_2	(4, 4, 5)	(3, 4, 3)	(3, 5, 4)	(5, 6, 2)	20
S_3	(2, 3, 7)	(1, 2, 2)	(4, 6, 6)	(3, 8, 8)	40
Demand	15	15	20	20	

Table 19. Comparison analysis of example 6 with different methods

Comparison Analysis	Z_1	Z_2	Z_3
Doke's Method (Doke et al., 2015)	205	335	305
Proposed Method	235	325	265

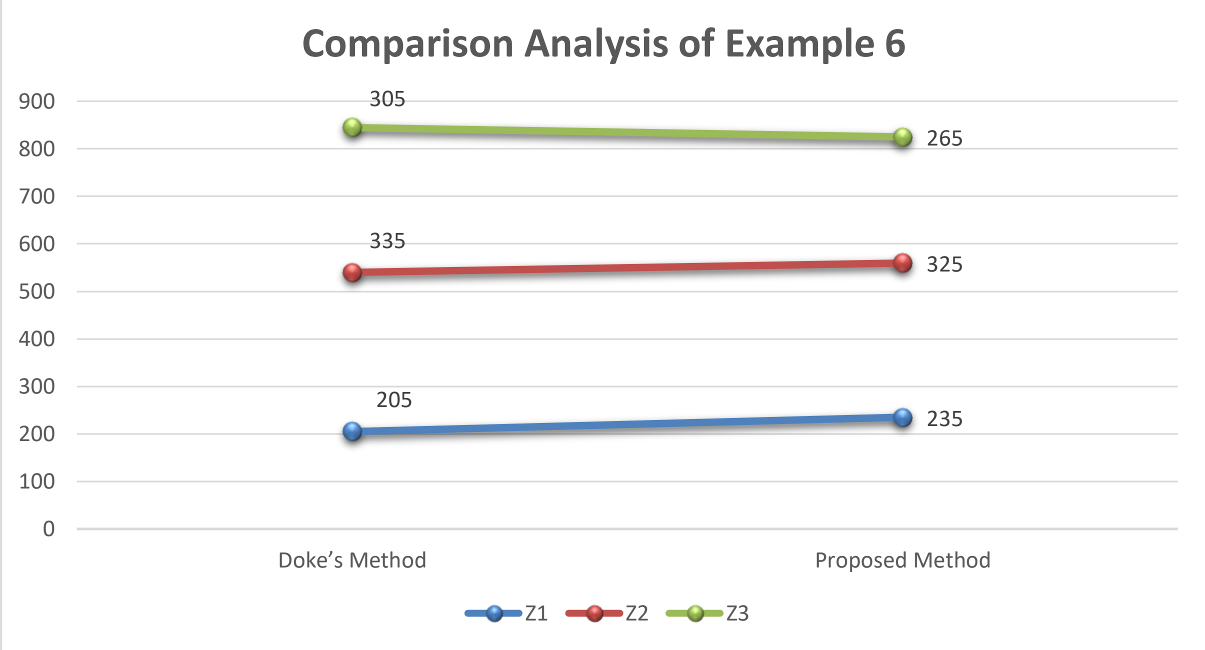

Figure 6. Comparison Analysis of example 6 with Doke's method

In example 6 TP has three different objectives (Z_1, Z_2, and Z_3). The main objective of this example is to minimize the objective function values Z_1, Z_2, and Z_3. The proposed method results compare with Doke's Method (*A Solution to Three Objective Transportation Problems Using Fuzzy Compromise Programming Approach*, n.d.) in this example. The comparative results between the proposed method and Doke's Method are represented in table 19 and it line graph evaluation shows in figure 6. By looking at the comparison table, both values of objective functions 2 and 3 (Z_2 and Z_3) are best when using our proposed method.

CONCLUSION

Multiple supply stations to multiple demand stations are connected by a massive number of transport routes in the structure of the TP. The goal is to figure out how many units of a given thing should be moved from one place to another so that there will be enough products or services at each destination point to meet demand. Multi-Objective Transportation Problems (MOTP) are those where more than one objective needs to be optimized. Several methods have been put forward in the literature to solve MOTPs. Instead of utilizing traditional methods, the geometric mean technique with the penalty method is applied in this study to solve a MOTP. This paper discussed six different instants which have two or three objectives that have been studied in the literature, and when compared to those other recent multi-objective transportation algorithms, the suggested technique performs the best results. As can be seen, this strategy is simple to comprehend and only requires a few steps to get a more effective result.

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