CONSTRUCTION AND ANALYSIS OF THE ADVERTISING DISTRIBUTION MODEL

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Abstract: The study investigates the construction characteristics and evaluates the advertising distribution model by utilizing the case of beauty products as an example.

Keywords: *advertising, distribution model, promoting campaign, customer analytics;*

1. Research context

It is widely acknowledged that advertising plays a crucial role in the success of trade and often proves decisive in promoting a product. The advertising industry attracts highly talented individuals, including top-notch professionals who create their own unique styles. Advertising has become an integral part of our society's culture, influencing cinema, literature, and theater. Additionally, advertising shapes the mentality of the country, ultimately leading to changes in people's characters, desires, and ways of thinking. This, in turn, results in an accelerated pace of life for society as a whole [2].

As advertising is a form of information, we can create a mathematical model based on the classic model of information dissemination [] within a given society. The intense competition between manufacturers compels them to seek novel and innovative approaches to promote their products. Therefore, building such a model and analyzing its behavior based on varying parameters (such as advertising intensity and a person's propensity to change their mindset due to advertising) can undoubtedly prove useful for both a theoretical understanding and practical application.

2. Literature Review

The mathematical models studied in this paper are based on differential equations, including stochastic ones. Many well-known scientists, including A. V. Skorokhod, Y. I. Gikhman, M. M. Bogolyubov, devoted their work to the study of evolutionary systems in the form of stochastic equations. An extensive bibliography on this issue can be found in the book by V.S.

Monographs of Korolyuk. In particular, in [7], the approaches used in this article to the study of the asymptotic behavior of an evolutionary system with a diffusion perturbation were presented for the first time. The question of the asymptotic behavior of limit generators is of paramount importance. Similar problems were also solved earlier using qualitatively different methods (see, for example, [17]). The methods presented in our study allow us to explore a model consisting of Markov transitions that fit a random environment. In addition, these methods make it possible to reveal an additional diffusion component and significant jumps in the perturbation process in the boundary equation. The authors examine a social community of N0, potentially exposed to several types of advertising through two different channels. N1(t), N2(t), … denote the number of "followers" who have adopted the new ad product over time. In the presented article, a more natural generalization of the model is presented and the generator of limit processes is explicitly formulated, and an interpretation of the model is also proposed.

3. Purpose and aims of the study

The purpose of this article is to present the advancements in the asymptotic theory for stochastic evolutions, which are solutions of stochastic differential equations. The focus is specifically on those influenced by impulse disturbances and non-classical approximation schemes.
Furthermore, the research aims to develop and scrutinize a model for advertising distribution. The effectiveness of this model will be evaluated using a small business as an example.

4. The study

In this study, we will utilize the classic model of information dissemination [1] to examine the efficacy of advertising the company's new beauty product to consumers.
Let us revisit the fundamental assumptions underlying the model. Consider a community of consumers comprising N_0 individuals who are targeted with advertising for our product. In other words, this community is receptive to advertising, which implies that the probability of modifying their attitude towards the product can be increased through the dissemination of relevant information. At a specific time point $t = t_0$, the advertising distribution source initiates its operations, leading to the proliferation of the product's advertisement within the community.
The primary measure of the extent of advertising distribution is the quantity $N(t)$ which varies with time *t* and represents the number of individuals who have embraced the new product. For

ease of analysis, we will assume that at the onset $t = t_0$, $N(t_0) = N(0) = 0$, indicating that there are no initial supporters of the new product.

1. Advertising will be disseminated within the community through two distinct information channels.

a) The first channel is external to the community and may involve an advertising campaign conducted via social media, as well as seminars and meetings. The intensity of this channel, measured by the number of equivalent information actions per unit of time, is denoted by the parameter $\alpha_{11} > 0$.

b) The second channel is the internal channel, which involves interpersonal communication among members of the community. The intensity of this channel, which is measured by the parameter $\alpha_{12} > 0$, is determined by the number of equivalent informational contacts. During communication, consumers who have been exposed to advertising (whose number is $N(t)$), influence other consumers who have not yet been influenced (whose number is $N_0 - N(t)$, thereby making an additional "personal" contribution to the process of generating interest in the new product. We will assume that any consumer who has not yet been exposed to advertising will always have the chance to receive the information that is distributed through external channels, with a certain probability of perceiving it. As a result, the rate at which consumers are recruited through external channels is determined not only by the factor $N_0 - N(t)$ and the value of α_{11}, but also by the value of α_{12}, which represents the probability or tendency to perceive the information. This probability may depend on factors such as the level of trust in the information. It is important to note that although the internal channel is local and operates on a person-to-person basis, unlike the external channel, the speed of recruitment is still directly proportional to the number of consumers who have not been exposed to advertising yet. This value is denoted as $N_0 - N(t)$, and is also determined by the intensity of contacts α_{21}, and the tendency of consumers to perceive information through the second channel, which is represented by the value α_{22}.

2. The change rate in the number of followers $N(t)$, which refers to the number of new members joining per unit of time, is determined by two factors.

a. Firstly, the speed of external recruitment, which is directly proportional to the product of the recruitment intensity α_{11}, the recruitment probability α_{12}, and the number of potential members who have not yet been recruited $((N_0 - N(t))$.

Mathematically, this can be expressed as

$\alpha_{11}\alpha_{12}(N_0 - N(t)) = \alpha_1(N_0 - N(t))$, where $\alpha_1 = \alpha_{11}\alpha_{12}$

b. Secondly, the speed of internal recruitment, which is directly proportional to the product of the recruitment intensity α_{21}, the recruitment probability α_{22}, the number of active followers $N(t)$ and the number of potential members who have not yet been recruited $N_0 - N(t)$). This can be mathematically expressed as $\alpha_{21}\alpha_{22}N(t)(N_0 - N(t)) = \alpha_2 N(t)(N_0 - N(t))$, where $\alpha_2 = \alpha_{21}\alpha_{22}$.

By summarizing assumptions 1 and 2, we can derive the differential equation that constitutes the advertising distribution model (1).

$\frac{dN(t)}{dt} = (\alpha_1 + \alpha_2 N(t))(N_0 - N(t)),\ N(0) = 0, t > 0$ (1.1)

The simplest macro model under consideration does not account for various factors that could be significant for the researched process, such as the heterogeneity of the social community, the effects of information "forgetting," the potential interdependence of $\alpha_1$and α_2, values, the possibility of deliberate opposition to the advertising campaign, among others. Nonetheless, under these assumptions, the model permits an analytical solution. Specifically, the solution $N(t)$ of problem (1.1) $t \geq t_0 = 0$, with the initial condition $N(0) = 0$, takes the following form:

$$N(t) = \frac{N_0 exp\{\alpha_1 + \alpha_2 N_0\}t - \alpha_1}{(\alpha_2 - \frac{\alpha_1}{N_0})exp\{\alpha_1 + \alpha_2 N_0\}t}$$

An analysis reveals that the solution, excluding the time t=0, is invariably positive, monotonically increasing, and gradually converges to the value of N_0 over time. This model characterizes the diffusion of advertising when, ultimately (after a sufficiently long time, specifically $t \to \infty$), all individuals within the group have been "recruited." To facilitate understanding, we will explore the discrete-time equivalent of model (1). By applying the definition of the derivative, we obtain:

$$\frac{dN_i}{dt} = \frac{\Delta N}{\Delta t} = \frac{N_i^{(n+1)} - N_i^{(n)}}{1},$$

where $n \epsilon (0, \infty)$ – time.

We have obtained a discrete version of the model of competitive advertising struggle (1):

$N^{(n+1)} = (\alpha_1 + \alpha_2 N^{(n)})(N_0 - N^{(n)}) + N^{(n)}$ (1)

The primary limitation of the classical model is twofold: first, it assumes a constant intensity of information influence, and second, it fails to account for sudden and unpredictable events that may significantly impact the consciousness of information consumers. Given the current state of the world, where information spreads rapidly and reaches a broad audience, it is clear that rare yet highly influential factors must be considered. In this study, we develop and analyze a model of competitive advertising struggle in the following form:

$$dN^{\varepsilon}(t) = C\left(N^{\varepsilon}, x\left(\frac{t}{\varepsilon^2}\right)\right)dt + d\eta^{\varepsilon}(t),\ N^{\varepsilon}(t) \in \mathbb{R}. \tag{22}$$

where

$$C\left(N^{\varepsilon}, x\left(\frac{t}{\varepsilon^2}\right)\right) =$$

$$= \begin{pmatrix} -\alpha_1(x) + \beta_1(x)N_0 - \beta_1(x)N_1^{\varepsilon}(t) & -\alpha_1(x) - \beta_1(x)N_1^{\varepsilon}(t) \\ -\alpha_2(x) - \beta_2(x)N_2^{\varepsilon}(t) & -\alpha_2(x) + \beta_2(x)N_0 - \beta_2(x)N_2^{\varepsilon}(t) \end{pmatrix} \times$$

$$\times \begin{pmatrix} N_1^{\varepsilon}(t) \\ N_2^{\varepsilon}(t) \end{pmatrix} + \begin{pmatrix} \alpha_1 N_0 \\ \alpha_2 N_0 \end{pmatrix}$$

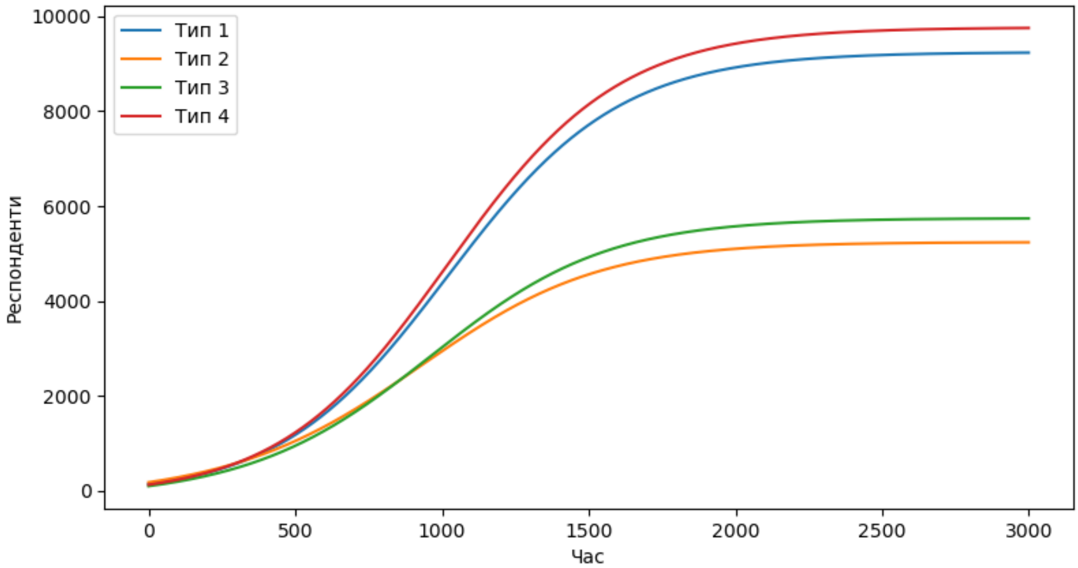
The proposed model considers both the stochastic impact of the environment on the level of information diffusion of an advertising campaign, as well as infrequent random fluctuations that cause significant short-term changes in the number of supporters of pertinent ideas. The primary finding of this study is that the impact of significant jumps is sustained in the limit process. These large jumps, for instance, represent high-impact events that immediately and substantially influence individuals' thoughts. While they are rare, their effect is substantial, which is not captured in any known models. In our problem formulation, the average model of competitive advertising struggle has the following form:

$$\mathbf{L}\varphi(w) = \hat{C}(u)\varphi'(w) + \Gamma\varphi(w),$$

where $\hat{C}(u) = \int_X \pi(dx)C(u,x)$.

If a switching process with "favorable" characteristics, such as the Ornstein-Uhlenbeck process, is known, the above equations can be explicitly computed while considering the type of potential operator.

Consider the example of four vectors and their initial conditions as follows: N_0 = 30000, $N_1(0)$ = 140, $N_2(0)$ = 180, $N_3(0)$ = 100, $N_4(0)$ = 129. The coefficients associated with these vectors are α_1 = 0.000012, α_2 = 0.000015, α_3 = 0.000018, α_4 = 0.000015, β_1 = 0.00000012, β_2 = 0.00000009, β_3 = 0.0000001, and β_4 = 0.00000012.



As depicted in the graph, although the first type of advertising had a larger number of supporters at the outset, it ultimately lost to the second type due to the latter's higher growth rate, as ensured by its corresponding parameters.

4. Summary and conclusions

This study proposes a novel model for advertising campaigns that incorporates randomness, which may be more suitable for today's context where breaking news can have swift and substantial effects on audiences via television and the Internet. Unlike the classical case, the behavior of this generalized model cannot be explicitly analyzed for a fixed moment in time. As is common practice for stochastic models, it is feasible to derive functional limit theorems that capture the behavior of the process over extended time intervals. This enables the averaging of the process's limiting characteristics and the construction of explicit solutions. Put differently, any functions dependent on the Markov process must be averaged over the stationary measure of its transitions.

6. References

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