using concepts, and so forth.

I believe that the end result of such an assignment is that students are better able to appreciate the kinds of moves that do typically occur in everyday argument and better able to put them into perspective and construct alternative arguments, precisely because they have a better sense of how arguments develop in relation to each other and so in relation to a broader "perspective". Furthermore, I also believe that such assignments give the student more practical insight into the "motivated" nature of argument "flaws". He is therefore better able to "anticipate" them and more sensitive to the special probing moves that need to be carried out. Finally, he is much more sensitive (than I believe he would be under most "weak sense" approaches) to the profound "ethical" consequences of "ego-serving" reasoning, and to the ease with which we can fall prey to it. If we can indeed accomplish something like these results, then there is much to be said for further work and development of "strong sense" approaches. What I have described here is, I hope, the beginning of such work.

The Diversity of Proof

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I

The Classical Sceptics demanded a lot from a proof. For Sextus Empiricus, a proof must be an argument ("a system consisting of premises and a conclusion") which is conclusive (valid) and true (sound), and which is such that the conclusion is non-evident (adèlon) though its plausibility is discovered through the "power" of the premises (**Outlines of Pyrrhonism**, II 134-6). This account differs from that popular among most modern logicians in two respects: (i) it requires that a proof be both a valid and a sound argument; and (ii) it requires that a proof have a special sort of conclusion, namely one which is contentious, or at least not manifestly true, and, moreover, one our sole evidence for which comes from the argument itself. Of course Sextus goes on to show that such proofs are impossible, so there is some reason to suspect ulterior motives for the stringency of the account. Still, there are important questions raised by the Sceptical definition of proof: Are we correct in viewing valid, but unsound arguments as proofs? If deductive reasoning is supposed to provide us with a tool for expanding our knowledge, is it

really inappropriate to required that whatever is proved by a deductive proof must arise out of the "power" of the proof itself and not be something which can be otherwise known directly? Is the Sceptic's account of proof really too stringent?

If we focus on the first difference between the Sceptic's account and the modern, received account, two, conflicting, comments can be made. First, it seems clear that when we are serious about reasoning, when the conclusion of our reasoning matters to us, then we require that the premises stand firm for us. After all the point of reasoning is to make a justifiable move from what we know (believe) already to what we wish to know. Any proposition can be validly proved if we take care to select the right premises. But this is an odd sense of "proof" — it is a deductive game, the logician's equivalent of a parlour trick. Reasoning, when it is action-guiding, is typically quite different. Soundness in such contexts may be as important a feature of proofs as validity; in non-deductive cases it can be considerably more important.

On the other hand, it is not at all correct to say that for an argument to be a proof it must have premises which are known to be true. We often argue from a position of relative uncertainty, fully aware that the premises we are using may not be the whole truth, or, for that matter, fully aware that they are false. So in some sense the truth or falsity of premises is not directly relevant to the issue of when an argument is a proof. But by admitting this we need not be committing ourselves to one of the dogmas of modern logic—the dogma that in order to understand the nature of reasoning one must sharply distinguish between questions about validity of arguments and questions about the truth, falsity or epistemic status of premises. This dogma must be modified if we are going to have any hope of arriving at a philosophically unobjectionable account of what it means to reason. The point here is rather that deductive reasoning, like other forms of reasoning, can be exploratory. Indeed one of the principal virtues of deductive reasoning is that by means of it it is often possible to explore the consequences of hypothetical claims so as to consider whether those claims ought to be of interest to us. We often need to reason from premises the truth or falsity of which is indeterminate. We reason counterfactually; we reason from premises which we strongly suspect to be false or absurd. And these ways of reasoning are valuable to us. To require proofs be sound, then, is generally speaking a very unfortunate restriction on the notion of a proof.

Since I want to avoid the dogma just mentioned, while taking account of the possibility of an argument being a proof where the premises are not true, I need to propose a non-standard notion of argumentative soundness. What we need is something along these lines: The premises of a proof must either be true, self-evident, warranted or what have you, or else they must fit into a consistent pattern, a pattern which we as reasoners have a particular interest in investigating in a particular case. Considerations of soundness, that is, have to do with the reasons we have for treating certain propositions as premises. Thinking that certain claims are true is one very good reason for treating them as premises: our interest is to see what follows deductively from what for us is not in question. But there are other reasons: We may be interested in exploring the consequences of a set of propositions which, say, provide a partial picture of the ways things might have been. Or we

might want to assure ourselves that a claim is true or very plausible by exploring the consequences of the assumption that it is neither true nor plausible, seeing if logical or material oddities follow from the opposite of what we believe to be true once that is embedded in a context which, for the purposes of the argument, is not in question. In a word, an argument might on this account be said to be sound just when the premises are acknowledged to be claims not in question for the purposes of the argument—not questioned in the context of **that** argument, but possibly the very heart of the controversy for other arguments. Speaking in this manner, we may say that an argument is an attempt to move in some rational way from what is, for the purposes of the argument, not in question to what is in question.

Looking at the second difference between the Sceptic and modern accounts of proof, more significant problems arise. The two accounts are difficult to reconcile. And the reason for this is that the modern conception has lost touch with the ordinary understanding of reasoning and argument, notions which the machinery of formal deductive logic was, at least originally, designed to elucidate. As often happens when a device for simplifying a jungle of interrelated concepts is introduced, it becomes intrinsically interesting to see what that device can do. Sometimes, as we explore the ins and outs of our new devices, the phenomena they were designed to clarify are pushed out of the picture. This is what has happened in the case of formal deduction.

Consider how odd it sounds to the modern ear to say that a proof must be an argument for a conclusion which—and I take it this is Sextus Empiricus's point—would not have been known had it not been for the argument. Sextus Empiricus gives the following example of what he has in mind by a proof: If motion exists, then the void exists (that is, a necessary condition for something being said to have moved is that, at the various, successive stages it must have occupied from here to there it must have occupied a space which was not already occupied, namely an empty place, a void); motion exists; therefore the void exists. This argument has all the required features of a proof. It is valid, has true premises, and its conclusion is a piece of new information, information which, ex hypothesi, could only have come to our attention indirectly, via a proof. On the other hand, Sextus Empiricus claims that, if it is presently night, then it could not be proved by means of an argument that it is night, since its now being night is, in his terminology, pre-evident (prodèlon), that is, something manifest, agreed upon by all and admitting of no (genuine) dispute.

Is there anything to this? If there is, understanding why these requirements concerning the nature of the conclusion of a genuine proof make sense requires an understanding of what reasoning is good for. In the tradition the Sceptics found themselves in—a tradition which, for the most part, did not acknowledge the difference between explanation and justification—the grounds for these requirements are very plausible. In that tradition there are things that are pre-evident and things that are not. In the former case no argument would provide us with any better grounds for thinking that such things are the case. But in the latter case, where what is non-evident is by definition that about which there is some controversy or dispute, the suggestion that argumentation plays its proper role only with regard to what is rebuttable is hardly objectionable. It would seem that for the Sceptics,

building upon then current conceptions of argument and proof, there was a closer connection between abstract reasoning and practical argumentation than is usually acknowledged today. The proper arena for argument and proof, for the Sceptics, is the epistemic arena; the proper pursuit of reasoning is that of uncovering new facts about the world. This would hardly be something modern logicians would reject, but they would nonetheless insist that formal reasoning is one thing, questions about truth and falsity, acceptability and unacceptability something quite different.

But the important question I want to pursue here is whether formal accounts of reasoning, accounts more or less removed from the kinds of context where they had their original home, can be satisfactory models of reasoning. The Sceptic, I suggest, correctly assumed that an adequate philosophical account of the nature of reasoning and of proof requires an understanding of the epistemological status of the ingredients of arguments.

2

The Sceptic, however, did not stop there. The Sceptic's argument against the possibility of proof builds on his view of the epistemological character of the ingredients of arguments. Can the Sceptic's insight about the proper approach to theories of reasoning and proof be detached from his scepticism?

For Sceptics like Sextus Empiricus, non-evident claims are those which have not been ascertained by direct apprehension in the past, and are not presently being apprehended. They are claims, moreover, which we have reason to suspect will not be directly apprehended in the future. Non-evident propositions must be eternally unknowable (Sextus Empiricus's example: whether the stars are even or odd in number). Given this epistemological characterization of non-evident propositions, the argument moves swiftly to the sceptical conclusion. Since the nature and existence of proofs is questionable, open to rational controversy, proof is non-evident. Hence, the question of the existence of proof remains a matter for the suspension of judgment. The same argument can be rearranged to produce the more familiar Sceptical argument: Since proofs must be conclusive, since their conclusions must be non-evident and since proof itself is non-evident, were proofs possible, then something non-evident could be shown conclusively. But nothing non-evident could be shown conclusively by means that are themselves non-evident. Thus, proof is impossible.

Some of the more vulnerable premises in the Sceptic's argument are, fortunately, also some of the premises that are crucial to the entire enterprise—namely the claim that what is questionable is ipso facto non-evident, and that which is debatable is eternally unknowable. Claims like these make the sceptic the disreputable character he is. They possess that maddening mixture of properties, verisimilitude and tenacity.

In this case, however, we might be able to outmanoeuver the Sceptic. It seems clear, for a start, that to have reasons to question the truth of a claim is quite different from having reasons to think that it is eternally unknowable. It is, indeed, a fairly optimistic knowledge claim that any contentious claim is eternally unknowable, since it amounts to the prediction that human knowledge on this matter will remain static on this question for all

time. The Sceptic will respond that what is non-evident for us is a proposition the truth and falsity of which are equally open to doubt: we either have no reason for thinking it is true or false, or we have as much reason for thinking it true as for thinking it false. But, the Sceptic will continue, this is just what it means to say that a claim is questionable. If questionable, we cannot say a proposition is true or false. The proper posture to assume is one of suspended judgment.

But this is an exceptionally bad argument, resting as it does on an extremely peculiar approach to the question of when one is justified in doubting a claim. Sextus Empiricus's example—whether the number of stars is even or odd—is a good example for his argument, but it is hardly paradigmatic of the sort of claim which is debatable. Moreover, this stance on doubting—roughly, any doubt makes a proposition non-evident—ignores the ethics of doubting. There are cases, notably in the law, where we give people the benefit of the doubt. We may have very good reason for thinking that Jones murdered Smith, but lacking reasons beyond a reasonable doubt, we conclude that he did not murder Smith. Here we do not—since we cannot—suspend judgment. In other cases, some doubts are ignored: they are silly, out of place, or not in issue: "If I make an experiment I do not doubt the existence of the apparatus before my eyes. I have plenty of doubts, but not **that**." (Wittgenstein, **On Certainty**, s. 337.) Sometimes doubts matter, sometimes they do not.

The Sceptic asks us to consider an argumentative context where doubts always matter. If that is the rule defining the context he is interested in, then scepticism follows. But only if we do not doubt that rule. Doubting that rule is doubting scepticism, since the **only** reason the Sceptic can give for the plausibility of the rule is that scepticism is true. Suspending judgment on the rule that all doubts matter undercuts the reason for thinking all doubts should matter.

If we move to other argumentative contexts we will assume other standards of doubting. In the criminal law, considerations of fairness to the accused dictate that some doubts be taken very seriously; so seriously that they are deemed to negate the doubts we have on the other side of the question. In the experimental sciences, what is questionable is what, for the purposes of the enterprise, ought to be questioned. The standard of credulity shifts with the context. And reasoning is always reasoning within an argumentative context. Each context has its rules, none is epistemologically prior to all the others.

What Sextus Empiricus failed to see is that his argument for the impossibility of proof requires an argumentative context—defined by the rule that all doubts matter—that itself needs justification. Justifications are arguments, pieces of reasoning. But what reason is there for thinking that the sceptical argumentative context should have our attention? Again, we cannot suspend our judgment here; we need to be persuaded.

This suggests that for all its stringency, Sextus Empiricus's characterization of a proof leaves out an important element: proofs should be persuasive. Suppose we say that Sextus Empiricus's definition of a proof, although it fails as a definition, succeeds in providing us with important clues of what we should consider when looking at the question of the nature of a proof. We shall return to these

requirements later, but for the moment I want to focus on the requirement Sextus Empiricus omitted: persuasiveness.

3

Flattery, puffery and fallacy persuade, so at most persuasiveness is a necessary condition of a proof. But is it even that? Could a proof fail to persuade and yet be a proof? I want to argue that this in fact cannot be so since part of what it means for an argument to be a proof is that is is **seen** (in the argumentative context in which it appears) as a proof; and if it is seen as a proof, then it must persuade.

Logicians will object that the ability to persuade cannot be a mark of a proof because persuasion is a psychological, not a logical, phenomenon. The study of persuasiveness is a psychological or sociological investigation, and as such is of little interest to the logician who is trying to canonize modes of reasoning. Persuasiveness, the argument goes, may be the key to understanding the efficacy of proofs, but it cannot be a criterion of proof. Forging a link betweeen proof and persuasiveness, therefore, is obliterating the crucial distinction between reasoning and effective reasoning—between, that is, logic and rhetoric.

There is little doubt that modern logicians insist that their logic be "pure". The subject matter of formal logic are the logical systems themselves. Canonical forms and the like must be agreed upon and set aside before, what is for the logician, the more intriguing work can begin. It is only after the groundwork is finished that questions about systematic completenesss, consistency, decidability and other issues can be framed. The definition of a proof must be settled before the meta-logic can begin. Formal logicians thus stipulate a definition of proof, with an eye to the work it must do later on: A proof is a finite sequence of well-formed expressions such that each step in the sequence is either a premiss, an axion (or instance of an axiom), or follows from one or more previous steps by the rules of inference allowed. Although admirably brief and precise, this stipulative definition relegates to the sphere of the non-logical, indeed the non-rational, several aspects of proof that are of great concern when we actually engage in reasoning. This is purity at a cost.

My concern to link proof with persuasiveness is motivated by the belief that proof is properly a part of rhetoric; it is a notion we use to distinguish epistemically successful arguments from those which, for various reasons, are not successful. Yet, in the face of the formal definition of "proof" just given—a definition which seems to lack any hint of rhetorical concerns of efficacy or persuasiveness—I must proceed with caution.

Formal deductive logic presupposes an argumentative context for which the syntactically characterized notion of proof is admirably suited. That context is, roughly, mathematical demonstration. Mathematical reasoning, as computer technology amply demonstrates, is essentially a process of iterating extremely simple, algorithmic moves. The resulting concatenations can be, of course, unimaginably complex. But every demonstration is composed of moves of such simplicity that to fail to be persuaded, at

this level, is to be perverse. In the end, the persuasiveness of formal logic is captured in the faith we have in our formal devices, be they soft or hardware devices. Mathematical demonstration is an argumentative context of the value, power and importance of which is not open to serious doubt. But it is merely one of many contexts. Formal logicians tend to succumb, in other words, to the same temptation which overcame the wits of the Calssical Sceptics: to assume that the single argumentative context that they are particularly interested in exhausts the sphere of the rational. What is needed is a broader conception of reasoning.

Despite his many confusions, Aristotle provided us with such a broader conception. For Aristotle apodeitic reasoning, dialectic and rhetoric are all aspects of reasoning—distinguishable to be sure, but nonetheless intimately related. On this account, rhetoric is said to be the counterpart of dialectic. (The Greek here is antistrophe—that part of the choral ode that alternates with and answers the strophe. In the Greek, then, we are asked to view dialectic and rhetoric as working together to fill out and complete the argumentative context). For Aristotle effective reasoning in the end serves the function of making the true and the just apparent. Hence, a proof is an argument which ought to persuade (everyone, or at least the wise). Aristotle thus seemed content to recast what seems on the face of it to be a psychological concern as a normative-epistemological concern.

It is sometimes said of Aristotle that his comments about the nature of reasoning were made with an eye towards the Assembly, the open court where every citizen was both juror and judge. Since that institution was both viable and vital it is quite natural that he should treat the reasoning which took place there—the pleading and the defending—as paradigmatic. Aristotle also realized formal reasoning and attempted to account for it—inadequately, as it turns out. But more importantly, he saw that not all reasoning can be formal or systematic since such reasoning presupposes archoi, first principles or axioms which are not open to controversy or argument. In the **Topics**, where he identifies the salient features of dialectic, he carefully notes that dialectic is the reasoning appropriate to questions and problems which lack this sort of grounding. But he also argues that dialectic is the sort of reasoning which is useful in determining what these first truths are, so that dialectic is epistemologically prior reasoning. At the same time, the Aristotelean model of argumentation is not heirarchical in the way the model of modern formal logic is. Aristotle did not see formal reasoning at the apex of the argumentative pyramid, with low-grade approximations of formal reasoning falling below it. Rather he envisioned a more democratic arrangement with a different kinds of reasoning (and different kinds of proof) having the same rational status, although applicable to different subject-matters.

Abstracting from Aristotle's treatment, an important picture of argumentation emerges. Fundamentally, reasoning resists formalization, the permanent detachment from the countless argumentative contexts in which reasoning actually takes place. Moreover, reasoning has an end, epistemic advance, which is concretized in the particular case by the surrounding context. Both the appropriate manner of reasoning and the standard of proof are functions of the context. Rhetoric deals with the efficacy of reasoning, as manifested in a particular context. When an argument persuades—or, as Aristotle would have it, when it ought to persuade—that in part determines its candidacy as a proof. The concept of proof is thus-textured, although neither vague nor ambiguous.

4

The formal logician may agree that, in ordinary language, the notion of "proof" is open-textured. But this is, he would argue, a defect of the ordinary use of the term; it is a defect remedied by the sort of stipulative definition that one finds in texts on symbolic logic. Obviously, no one could could object to the endeavour of firming up a open-textured notion for particular purposes. What is objectionable is setting up the stipulative definition as the standard against which other characterizations of proof must be measured for adequacy. I have been suggesting that characterizations of proof can only be assessed by looking to broader issues. Aristotle's approach is therefore instructive: he viewed reasoning teleologically and made his account of argumentation, and so of proof, reflect the purpose of reasoning, the point of arguing.

C.S. Peirce also took talk of the purpose of reasoning seriously and attempted at various times to come to grips with the **motive** of reasoning. In one place, Peirce makes the following remark:

> Facts are hard things which do not consist in my thinking so and so, but stand unmoved by whatever you or I or any man or generations of men may opine about them. It is those facts that I want to know, so that I may avoid disappointments and disasters. Since they are bound to press upon me at last, let me know them as soon as possible, and prepare for them. That is, in the last analysis, my whole motive in reasoning. (**Collected Papers,** 2.173: "The Criterion of Validity in Reasoning")

Although pessimistic in tone, we have here the motivation for trying to understand the nature of proof. Indeed, Peirce is giving us the basis for the terms of reference of logical theory. The logician has two tasks, Peirce believed: "1st, to bring out the amount and kind of **security** (amount of certainty) of each kind of reasoning, and 2nd, to bring out the possible and esperable uberty, or value in productiveness of each kind" (**ib.**, 8.384). One of the ways of satisfying the first of these goals is to come up with an account of the nature of proof which will provide us with the sort of confidence we need to prepare for all the evil things which can happen to us if we do not reason. We want our reasoning to be secure, and it would seem that one very important way of providing this is by getting straight about when it is correct to say that some argument is a proof.

Against **this** standard, how well does the formal logician's stipulative definition fare? Not requiring soundness for proof, it would seem that this conception of proof does not provide us with much security at all: pick your

premisses properly and any proposition can be proved. In practice, however, this is not a fair objection. The most important application of the formal definition of proof is the generation of theorems in a formal system, and if the system is consistent, we have very good reasons for having faith in the theorems. Admittedly, most theorems of a formal system are uninteresting. But there is no denying that formal proof, in the argumentative context in which it is the appropriate conception of proof, does provide us with Peircean security.

But what of Peirce's other requirement, the value of productiveness? Peirce's remarks seem directed to what has come to be called ampliative reasoning, reasoning that allows one to go beyond the content of the premisses to a conclusion which provides, in some appropriate sense, new information. Reasoning of this sort—induction for example—has a central place in our lives and demands respect. Unfortunately, inductive proofs, or more generally, non-deductive proofs hold out the possibility of providing new information only because they fail to count as deductive proofs. The security of deductive proofs—if the argument is valid, then if the premisses are true the conclusion **must** be true as well—seems incompatible with the productiveness of non-deductive proofs. In the face of this dilemma one could either say, non-deductive arguments cannot be proofs because they are insecure, or one could say, deductive arguments cannot be proofs because they are non-productive.

As is well-known, J.S. Mill seemed to be committed to the latter. Following Sextus Empiricus, Mill took seriously the metaphor that in a deductive inference the conclusion must be "contained in" the premisses. On the basis of this metaphor, Mill argued that deductive reasoning is non-productive since every deductive argument is a **petitio principii**: in order to have come to be convinced of the truth of the premisses one must also have considered and been convinced of the truth of the conclusion. Were we to carry out the investigations required to assure ourselves that all men are and have been mortal, we must also have convinced ourselves of the fact that Socrates was mortal. The conclusion that he is provides no new information. The security of a deductive argument undercuts its productivity. Deduction is no more than a way of re-arranging what we know already, an uninteresting and epistemologically useless endeavour. What follows from this realization is either scepticism, as in Sextus Empiricus's case, or Mill's view that only ampliative reasoning is useful reasoning. And by implication, Mill is committed to the view that, say, inductive proof is the only sort of proof worthy of the name: deductive security is not a possible characteristic of proof.

Mill is usually attacked on the grounds that it follows from his account that mathematical axioms are (or, have no more security than) empirical generalizations. This is a problem, but there is another aspect of his discussion that needs to be challenged.

Bishop Whateley, Mill notes, originated the metaphor of "containment" of conclusion in premisses in order to account for the legitimacy—and so, the security—of deductive arguments. Mill took this seriously, making nonsense of deduction. One could point out that some deductive rules of inference allow us to create conclusions not all the component parts of which are found in the premisses; the rule of disjunctive addition operates in this fashion. But more importantly, it can be objected that

Mill was misled by the metaphor; he was misled into thinking that the argumentative context in which deductive reasoning operates was one in which it was always possible, or intelligible, to search for the conclusion among the premisses. Mill generalized from the syllogism about Socrates's mortality. But Mill's argument loses credibility as soon as the example of deductive reasoning is changed and the background argumentative context is given life.

As it happens, the theorem that every Cauchy sequence of rational numbers converges is provable from a relatively small number of set-theoretic axioms and definitions. And, in **some** sense, that conclusion is "contained in" those premisses. But whatever sense that is is not a sense of the metaphor which could plausibly persuade anyone familiar with number theory that on the way to understanding the set-theoretic axioms and definitions one has already considered, and become convinced of, the proposition that every Cauchy sequence of rational numbers converges.

In short, there is no real difficulty in reconciling Peirce's two requirements of security of inference and productiveness or epistemic advance in the case of formal deduction. The proof that every Cauchy sequence of rational numbers converges is both secure and useful. The new information provided, although not wholly unexpected, constitutes an epistemic advance about number theory. Moreover, formal deduction satisfied both of Peirce's requirements because of the stipulative definition of proof that is employed. Proof in the deductive sense is supremely suitable for those areas of human knowledge which are, or can be made to be, rigorously ordered. Deductive argumentation explores the links and connections in such areas, thereby yielding new information. Since the connections between the propositions in mathematics, and mathematizable disciplines, are clear and necessary, inferences here can be no weaker than these connections. The deductive definition of proof, because the subject matter appropriate to deductive reasoning allows for reasoning which is conclusive, must characterize proof in a manner in which the security captured is the security provided by conclusiveness.

Rescued from Mill's attack, the formal logician may feel the temptation to overstate his case. Not only, he may be tempted to say, is the formal definition of proof appropriate to deduction a guarantee of both security and productiveness, it is the only guarantee. Without deductive security, a security underwritten by conclusiveness, proof is impossible. However productive non-deductive or ampliative arguments are, their lack of deductive security vitiates their productiveness. Induction must be justified, deduction is justified by its conclusiveness.

We have seen this claim before; and the answer to it is the same as before: deductive proof is one sort of proof, there are others. Security provided by conclusiveness is possible, and essential, in certain argumentative contexts; it is neither possible, nor desirable in others. It is not as if there are occasions when productiveness or epistemic advance is so important to us that we, so to speak, take our chances and lower our demand for security. It is rather that, in some argumentative contexts, less-than-conclusive proof **is** security. We need no more, it would destroy the argument, the proof, if we got more.

5

Aristotle opens up the **Nicomachean Ethics** with a comment that sums up what I have been hinting at so far:

...it is the mark of an educated man to look for precision in each class of things just so far as the nature of the subject admits; it is evidently equally foolish to accept probable reasoning from a mathematician as to demand from a rhetorician demonstrative proof. (1094b 23-30)

On the face of it Aristotle's advice seems trivially true: you cannot use a microscope to study social organizations, you cannot investigate the properties of tachyons by conducting surveys. There is that advice, but there is also something more. Contained in this remark is the suggestion that the subject matter determines what is to count as a good argument. Admittedly, if this were the fragment of Aristotlean thought that survived, it might be tempting to say that Aristotle thought that demonstrative proof is better, stronger, more genuine than other possibilities (for, after all, we are foolish to "accept" probable reasoning from a mathematician and to "demand" demonstrative proof from a rhetorician). But we do have the rest of Aristotle, and in light of the **Topics** and the **Rhetoric** it is very unlikely that Aristotle was making that judgment in the **Ethics.**

Aristotle, in other words, is not arguing that a non-demonstrative argument is the best we can hope for when our subject-matter is ethics or politics. He is rather claiming that non-demonstrative proof is proof in ethics or politics, it is the epistemic assurance required to claim moral or political knowledge. Aristotle's doctrine of the mean, despite the hint that qualitative assessment is possible concerning human behaviour, does not make for the use of purely demonstrative reasoning in morality. The mean is used by Aristotle in a qualitative manner: "To feel [anger, pity, appetite, etc.] at the right times, with reference to the right objects, towards the right people, with the right motive, and in the right way, is what is both intermediate and best, and this is characteristic of virtue." (**Ib.** 1106b 21-4) Hence, one cannot **calculate** what one ought to do: morality does not admit of that sort of proof. But it does admit of proof, proof based on a consideration of cases designed to reveal the correct principle in the light of the mean. It is proof available to the **phronimos**, the man of moral and intellectual wisdom.

Aristotle was willing to allow a diversity of kinds of proofs, kinds that differ not only in form, but in the conceptions of security that are presumed. There is, unhappily a tendency among philosophers to react to the phenomenon of legitimate variety or genuine diversity in one of two ways: to demand an essence, or to insist that "anything goes". For a variety of reasons, this tendency to manufacture dilemmas is most pronounced in moral philosophy. To take one example, it seems intuitively the case that there are many sorts of lives that are virtuous, although these lives may differ wildly. But some moral philosophers oversteer, in the face of this diversity, and insist that there must only be one sort of virtuous life. The diversity we seem to see is an illusion. Other philosophers are so impressed by the diversity taht they are moved to argue that there cannot be anything precise e can say about virtuous lives. Any life is (or might as well be called) a virtuous life.

By claiming that there is a diversity of kinds of proof, I am opening the door to analogous responses. I have had some things to say about the first response, that there is an essence of proof, a definition that sets out necessary and sufficient conditions for an argument being a proof. Sextus Empiricus, Mill and modern logicians all have suggested essences of proof, and each has, I suggest, managed only to capture one of many kinds of proof. The search for the pure form of proof, the ideal, the standard, is illusory: proof is an epistemological notion which, in application, must fit into the various argumentative contexts in which it serves the role of standard of success: security and epistemic advance. The yearning for an essence of proof can be remedied by considering the approaches which Aristotle and Peirce took. Consider the motive for reasoning; keep in mind the point of reasoning.

But the opposite response to the claim that there is a diversity of kinds of proof has not been met. I have suggested that we have at our disposal clues for the identification fo proofs—Sextus Empiricus supplied a few, modern logicians and Mill have insisted on others, and I have made some remarks about persuasiveness standing a good chance of being yet another. But clues are not necessary and sufficient conditions. In the end the only thing that every proof has in common with every other proof is the property of being a proof. Proof in mathematics is not proof in a court of law.

Now, to argue diversity of proof in the manner in which I have here makes the response that I cannot set limits to what could conceivable count as a proof especially worrisome. The most I have been able to say about proof is that a proof is epistemic assurance, it is being sure, it is having good reasons for thinking so. But what is to count as a good reason depends on the argumentative context in which the putative proof is raised. In the end, the best argument for the diversity of proof is a presentation of examples of kinds of proof. But what is the argument against the challenge that, on this account, it is possible that for any sequence of propositions, there might be an argumentative context in which that sequence is a proof?

The response to this challenge is simply that fallacy exists. An account of the fallacious in reasoning serves two functions. The first is to provide a means by which to assess putative proofs and proffered arguments within a given argumentative context: a theory of fallacy helps to delimit the notion of proof relative to an argumentative context. The second is to delimit the range of possible argumentative contexts: a theory of fallacy sets the limits to what **could** count as a context against the background of which reasoning is possible. The fallacious is the countervailing consideration that limits the diversity of argumentative contexts, and so the diversity of proof. Fallacy, in short, is the failure to secure the point of reasoning: a failure of security or a failure of epistemic advance.