Singular Value Decomposition-Based ARMA Model Parameter Estimation of Non-Gaussian Processes

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Abstract—Autoregressive moving average (ARMA) modeling has been used in many fields. This paper presents an approach to time series analysis of a general ARMA model parameters estimation. The proposed technique is based on the singular value decomposition (SVD) of a covariance matrix of a third order cumulants from only the output sequence. The observed data sequence is corrupted by additive Gaussian noise. The system is driven by a zero-mean independent and identically distributed (i.i.d.) non-Gaussian sequence. Simulations verify the performance of the proposed method.

Index Terms— Time series forecasting, singular value decomposition, ARMA model, non-Gaussian process, parameters estimation.

I INTRODUCTION

In statistical signal processing, parametric modeling of non-Gaussian process experiencing noise interference has been a very important research area. The use of time series models is very effective techniques to model the parameters of non-Gaussian systems. Among different time series methods, one of the most often used procedures is ARMA model. The use of ARMA model identification has been applied in several areas such as seismic data processing, adaptive filtering, and communication systems [1]. There are several papers that have been written in the literature to estimate the parameters of a general ARMA process using a variety of second and higher-order statistics. The second order statistics (SOS) measures work fine if the signal under study has a Gaussian probability density function (PDF). That is because all of its statistical properties are completely determined by the first and second moments. Gaussian distribution is tractable and fairly realistic model [2]. However, many real-life signals are non-Gaussian. For example, the electromagnetic environment encountered by receiver systems is often non-Gaussian in nature. However, the receiving systems are designed to perform in white Gaussian noise [3]. Also, acoustic noise is in many cases highly non-Gaussian. Hence, in practice, there are situations where we must look beyond the autocorrelation of the available data to suppress additive noise and extract phase information. While the Gaussian random process still plays a great and significant role in stochastic signal processing, non-Gaussian random processes and higher order statistics (HOS), or cumulants, are of increasing importance to the researchers. HOS is currently an area of intense research and new results are constantly being reported.

In non-Gaussian process identification, the corrupted cumulants can be used as important information. This is because cumulants are generally asymmetric functions of their arguments, as such carry phase information about the ARMA transfer function. Therefore cumulant statistics are capable of determining the order of ARMA model [4, 5]. In addition, they are suitable for order selection when the ARMA process is corrupted by Gaussian noise of unknown covariance function [6]. Furthermore, HOS can capture the non-minimum phase information in the available signal.

Singular value decomposition (SVD) plays an extremely important role in engineering and science problems from both a theoretical and a practical point of view. For example, SVD is one of the most important tools of numerical signal processing. It is employed in a variety of applications in scientific computing, signal processing, automatic control, and many other areas. For example, SVD techniques have been used in spectrum analysis, filter design, system identification, and for solving linear equations. In linear systems of equations, SVD provides robust solution of both overdetermined and underdetermined least–squares problems. It allows one to diagnose the problem in a given matrix and provides numerical answers as well. It works also for singular matrices; for example, SVD can be used to find a solution of a set of linear equations corresponding to a singular matrix that has no exact solution by locating the closest possible solution in the least square sense. In addition, some systems of equations are sensitive to small changes in the values. In such systems, the SVD can help with the solution of ill-conditioned equations by identifying the direction of sensitivity and discarding that portion of the problem.

In system identification, SVD methods have been used for autoregressive (AR) and moving average (MA) model order determination of general autoregressive moving average (ARMA) models. Cadzow [7] proposed an algorithm that uses the SVD of an extended autocorrelation matrix for extracting the AR model order. Giannakis and Mendel [8] proposed a method that uses the SVD of a cumulant matrix for non-Gaussian processes. Zhang and Zhang [9, 10] proposed two techniques for MA model order determination. The first approach [9] uses the SVD of an autocorrelation matrix, while the second approach [10] uses the SVD of a cumulant matrix of non-Gaussian process. Reddy and Biradar [11] proposed an information theoretic approach to model selection using SVD. Al-Smadi [17] used SVD in ARMA model parameters estimation.

Several methods have been proposed to estimate the ARMA model parameters using HOS such as the methods in [12, 13]. In [12] Giannakis and Mendel (GM) developed a residual time series method for estimating the ARMA parameters using second and third order cumulants. The method starts by estimating the AR parameters using 1-D slice of the third order cumulants. Then using the AR parameters and the observation measurements, a residual time series is computed. Finally, the MA parameters are estimated using the residual time series in which case the residual satisfies an MA model. They developed a special structured matrix that contains both second and third order statistics of the output to satisfy this condition. The RTS method was described in [14] as being one of the best-known methods for estimating the coefficients of ARMA models. However, the estimation goes through three stages, and any error in the estimation will carry to next stage, which will result in an inaccurate estimation. In [13] Swami and Mendel developed a method for estimating the ARMA parameters using q 1-D slices of the cumulant of the observation measurements. Hence, this algorithm is called the "q-slice" (QS) solution. The method assumes that the AR parameters are available. The method starts by obtaining the impulse response of the system. It requires q-slices of the cumulants to estimate the first q coefficients of the impulse response. Then, the MA parameters are estimated using the AR parameters and the impulse response. Swami and Mendel stated that their method usually works for moderate SNR.

In this paper, we present a novel technique to estimate the parameters of a general ARMA(p,q) process using the SVD of a special covariance matrix formed from the third order cumulants of the output sequence only. Section 2 presents the formulation of the problem. Simulation examples are discussed in Section 3. Section 4 draws some conclusion remarks.

II PROBLEM FORMULATION

Let $\{x(t)\}$ denote a stationary ARMA(p,q) system given by

$$x(t) = - \sum_{i=1}^{p} a_i x(t-i) + \sum_{i=0}^{q} b_i w(t-i) ; \quad a_0 = b_0 = 1 \quad (1)$$

where $x(t)$ is the observed time series, $w(t)$ is the input sequence which is not observed, a_i and b_i are parameters, p and q are the order of the model. By expressing the transfer function of the model in z-domain while the model is assumed free of pole-zero cancellations and exponentially stable, it can be formulated as follows.

$$H(z) = \sum_{k=0}^{\infty} h(k) z^{-k} = \frac{\sum_{k=0}^{q} b_k z^{-k}}{1 + \sum_{k=1}^{p} a_k z^{-k}} = \frac{B(Z)}{A(Z)} \quad (2)$$

Now, for identifying the time series with non-Gaussian process, a driving noise sequence $w(t)$ is assumed to add in the ARMA model, which is a zero mean, stationary and non-Gaussian independent identically distributed (i.i.d.) noise. The output signal, $y(t)$, becomes

$$y(t) = x(t) + v(t) \quad (3)$$

where $v(t)$ is a Gaussian noise independent of input $w(t)$, and hence of output $x(t)$. With this model, cumulants can be used.

Under the assumption that $x(t)$ is stationary, the third order cumulants of the noisy output $y(t)$ is

$$c_{3y}(n,m) = E[y(t) y(t+n) y(t+m)] \quad (4)$$

As the cumulants of higher than second-order found in a Gaussian process is identical to zero, then

$$c_{3x}(n,m) = c_{3y}(n,m) \quad (5)$$

That is, the cumulant of third order (and higher) is insensitive to the additive Gaussian noise of unknown covariance function. The cross-cumulant between $x(t)$ and $w(t)$ is

$$c_{wxx}(n,m) = E[w(t) x(t+n) x(t+m)] \quad (6)$$

Now, multiplying both sides of Equation (1) by $x(t+n)x(t+m)$, yields

$$x(t)x(t+n)x(t+m) + a_1 x(t-1)x(t+n)x(t+m) + \cdots + \\ a_p x(t-p)x(t+n)x(t+m) = \\ w(t)x(t+n)x(t+m) + b_1 w(t-1)x(t+n)x(t+m) + \cdots + \\ b_q w(t-q)x(t+n)x(t+m) \quad (7)$$

Taking the expected value for Equation (7), we obtain

$$c_{3x}(n,m) + a_1 c_{3x}(n+1,m+1) + \cdots + a_p c_{3x}(n+p,m+p) = \\ c_{wxx}(n,m) + b_1 c_{wxx}(n+1,m+1) + \cdots + b_q c_{wxx}(n+q,m+q) \quad (8)$$

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By stacking Equation (8) for several of n and m ranging from $-\Phi$ to Φ where Φ denotes the range of the third order cumulants to be used, the system in (8) can be expressed in matrix format as follows

$$\underline{c} = -C_{3x}\underline{a}_p + C_{wxx}\underline{b}_q \qquad (9)$$

The vector $\underline{c}$ contains third order cumulants at $n = m = 0$, the vectors $\underline{a}_p$ and $\underline{b}_q$ contain the parameters for the process in Equation (1). The matrix C_{3x} contains the cumulants of the output sequence and the matrix C_{wxx} contains the cross-cumulants of the input and output sequences given by

$$C_{3x} = \begin{bmatrix} C_{3x}(-\Phi+1,-\Phi+1) & C_{3x}(-\Phi+2,-\Phi+2) & \cdots & C_{3x}(-\Phi+1+p,-\Phi+1+p) \\ C_{3x}(-\Phi+1,-\Phi+2) & C_{3x}(-\Phi+2,-\Phi+3) & \cdots & C_{3x}(-\Phi+1+p,-\Phi+2+p) \\ \vdots & \vdots & \cdots & \vdots \\ C_{3x}(\Phi-1-\Omega,\Phi-1-\Omega) & C_{3x}(\Phi-\Omega,\Phi-\Omega) & \cdots & C_{3x}(\Phi-1-\Omega+p,\Phi-1-\Omega+p) \end{bmatrix} \qquad (10)$$

$$C_{wxx} = \begin{bmatrix} C_{wxx}(-\Phi+1,-\Phi+1) & C_{wxx}(-\Phi+2,-\Phi+2) & \cdots & C_{wxx}(-\Phi+1+q,-\Phi+1+q) \\ C_{wxx}(-\Phi+1,-\Phi+2) & C_{wxx}(-\Phi+2,-\Phi+3) & \cdots & C_{wxx}(-\Phi+1+q,-\Phi+2+q) \\ \vdots & \vdots & \cdots & \vdots \\ C_{wxx}(\Phi-1-\Omega,\Phi-1-\Omega) & C_{wxx}(\Phi-\Omega,\Phi-\Omega) & \cdots & C_{wxx}(\Phi-1-\Omega+q,\Phi-1-\Omega+q) \end{bmatrix} \qquad (11)$$

In these equations, $\Omega = \max(p,q)$. Equation (9) can be written as

$$\underline{c} = C\underline{\theta} + \underline{d} \qquad (12)$$

Where

$$C = [\, C_{3x} \quad C_{wxx}\,], \qquad (13)$$

$\underline{\theta}$ is a vector represents the parameters

$$\underline{\theta} = [1, a_1, \cdots, a_p, \ 1, b_1, \cdots, b_q]^{\mathrm{T}} \qquad (14)$$

The vector $\underline{d}$ represents the residual errors in fitting the model $C\underline{\theta}$ to the data $\underline{c}$. For a given estimate of $\underline{\theta}$, the squared error between $\underline{c}$ and the model $C\underline{\theta}$ is

$$e^2 = tr[(\underline{c}-C\underline{\theta})(\underline{c}-C\underline{\theta})^T]$$

$$= [(\underline{c}-C\underline{\theta})^T (\underline{c}-C\underline{\theta})] = \underline{d}^T \underline{d} \qquad (15)$$

Where tr denotes the trace. To obtain the least square estimate, we minimize Eq (15) by taking the gradient of e^2 with respect to $\underline{\theta}$

$$\frac{\partial}{\partial \underline{\theta}} e^2 = 2C^T(\underline{c}-C\underline{\theta}) \qquad (16)$$

The least squares estimate equates the gradient to zero. Then

$$C^T \underline{c} = C^T C \underline{\theta}$$

or

$$C^T C\underline{\theta} = C^T \underline{c} \qquad (17)$$

Now, we can write the matrix C as the singular value decomposition as follows.

$$C = U\Sigma V^T \qquad (18)$$

where U and V are orthogonal matrices, Σ is a matrix whose elements are zeros except possibly along the main diagonal (the singular value of C). That is, the singular values of C are the diagonal elements of Σ.

$$\Sigma = diag(\sigma_1, \sigma_2, \cdots, \sigma_r) \qquad (19)$$

such that

$$\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_r \geq 0 \qquad (20)$$

Notice that σ_i are called the singular values and are nonnegative numbers. The matrix U contains the left singular vectors of C; the matrix V contains the right singular vectors of C.

Then, the normal equation in (17) may be written as

$$V\Sigma^T \Sigma V^T \hat{\underline{\theta}} = V\Sigma^T U^T \underline{c} \qquad (21)$$

These equations can be solved as follows.

$$\hat{\underline{\theta}} = V\Sigma^{-1} U^T \underline{c} = \sum_{i=1}^{p} \underline{v}_i \sigma_i^{-1} \underline{u}_i^{-1} \underline{c} \qquad (22)$$

Where Σ^{-1} is the inverse and is found as

$$\Sigma^{-1} = diag[\sigma_1^{-1}, \cdots, \sigma_p^{-1}] \qquad (23)$$

To test the optimality of the proposed algorithm, the mean and the variance measures were considered. The signal-to-noise (SNR) was computed as follows.

$$(SNR)_{dB} = 10\log\left(\frac{\sigma_x^2}{\sigma_v^2}\right) \qquad (24)$$

Where σ_x^2 the signal is power and σ_v^2 is the measurement noise power. It should be noticed that the only available data in the ARMA modeling is the output sequence. However, the input sequence is necessary to compute the cross-cumulants, which is an intermediate step in the identification process. Hence, the observed output data was modeled by a high order AR model [15]. Thus, the system in (1) could be rewritten as

$$\hat{w}(n) = \sum_{i=0}^{M} \gamma_i x(n-i) \qquad (25)$$

Where γ_i are the parameters of the high AR model and are estimated as follows.

$$\underline{\gamma} = \left[\frac{1}{N+1}\sum_{k=0}^{N} \underline{\Psi}(k)\underline{\Psi}(k)^T\right]^{-1}\left[\frac{1}{N+1}\sum_{k=0}^{N}\underline{\Psi}(k)x(k)\right] \qquad (26)$$

With

$$\underline{\Psi}(k) = [-x(k-1) \quad -x(k-2) \quad - \ldots - x(k-M)]^T \qquad (27)$$

and M is the order of the high AR model. In this estimation γ_0 has been assumed to be one without loss of generality. Now, using $\hat{w}(n)$ in the place of $w(n)$, the identification procedures developed in this paper can be used.

III SIMULATION EXAMPLES

Simulation studies are presented to test the proposed ARMA model parameters estimation approach using SVD. Several examples were simulated at different levels of signal-to-noise ratio (SNR) on the output. The input sequence was generated as a zero-mean, i.i.d., and exponentially distributed random process. The length of the signal is N=1500. A comparison of the performance of the proposed algorithm with the GM and the QSS methods were made at different SNRs on the output signal. To guarantee statistical independence, all the results are a mean of 100 Monte Carlo runs, using different seed in each case. The computations were performed in MATLAB. In addition, *armaqs* and *armarts* commands were used from the higher-order spectral analysis toolbox [16] to estimate the ARMA parameters using QS and GM methods, respectively.

A Example 1
The time series to be considered is given by

$$x(t) + 0.7907x(t-1) + 0.042x(t-2) -0.5556x(t-3) - 0.0247x(t-4) + 0.3846x(t-5) + 0.3026x(t-6) = w(t) + 0.3452w(t-1) + 0.53w(t-2) + 0.3985w(t-3) + 0.8138w(t-4) \qquad (28)$$

This is an ARMA (6, 4). It has six poles and four zeros. The poles are located at $0.7102 \pm j0.41$, $-0.43 \pm j0.7448$, and $-0.6755 \pm j0.39$. The zeros are located at $0.485 \pm j0.84$ and $-0.6576 \pm j0.6576$. The signal $x(t)$ is observed in additive Gaussian noise $y(t) = x(t) + v(t)$. The input sequence was drawn from a zero-mean non-Gaussian distribution; namely, exponential distribution. Then the input signal was passed through the filter in Equation (28). After that, the output of the filter was corrupted with additive Gaussian noise at SNR of 20dB on the output sequence. It is assumed that the only available data is the output measurements. To estimate the ARMA model parameters, the cumulant matrix C in Equation (13) must be formulated. As it can be seen from (13), the matrix C consists of two matrices concatenated together. The matrix C_{3x} contains the cumulants of the observed output sequence, whereas C_{wxx} contains the cross-cumulants of the unobservable input and the observed output sequences. To estimate the input signal, Durbin [15] method was used. The ARMA parameters were estimated using the QS, the GM, and the proposed SVD methods. Table 1 shows the average estimate results of 100 Monte Carlo simulations at SNR of 20 dB on the output sequence. The performance measures considered for estimating the parameters are the arithmetic mean and the variance.

TABLE 1
True and estimated ARMA(6,4) parameters
(mean± variance).

	True	QS	GM	Proposed
a(1)	0.791	0.728 ±0.173	0.687±0.019	0.764±0.007
a(2)	0.042	-0.029± 0.102	-0.027±0.022	0.028 ± 0.005
a(3)	-0.551	-0.599± 0.030	-0.516±0.009	-0.554± 0.004
a(4)	-0.025	-0.022± 0.053	-0.035±0.007	-0.029 ± 0.003
a(5)	0.385	0.388 ± 0.017	0.366±0.011	0.391± 0.004
a(6)	0.306	0.291 ±0.023	0.293±0.011	0.308± 0.003
b(1)	0.3452	0.728± 0.297	0.282±0.066	0.382 ± 0.008
b(2)	0.53	-0.029± 0.208	0.630±0.309	0.474 ± 0.005
b(3)	0.3985	-0.599± 0.155	0.429±0.033	0.266 ±0.006
b(4)	0.8138	-0.022± 0.843	0.703±0.031	0.685 ± 0.005

B Example 2
The time series to be considered is given by

$$x(t) - 1.5x(t-1) + 1.2x(t-2) -0.455x(t-3) = w(t) + 0.2w(t-1) + 0.9w(t-2) \qquad (29)$$

This is an ARMA (3,2). It has three poles and two zeros. The poles are located at $0.3939 \pm j0.6955$ and 0.7121. The zeros are located at $-0.1 \pm j0.9434$. The data was generated and analyzed as in Example 1. The ARMA parameters were estimated using the QS, the GM, and the proposed SVD methods. Table 2 shows the average estimate results of 100 Monte Carlo simulations at SNR of 20 dB on the output sequence.

TABLE 2
True and estimated ARMA(3,2) parameters
(mean± variance).

	True	QS	GM	Proposed
a(1)	-1.5	-1.489 ± 0.015	-1.483±0.015	-1.495±0.011
a(2)	1.2	1.197 ± 0.027	1.191 ± 0.028	1.195 ±0.015
a(3)	-.455	-0.451±0.009	-0.449 ±0.009	-0.449± 0.003
b(1)	0.2	0.201± 0.084	0.195 ± 0.028	0.186 ± 0.011
b(2)	0.9	0.901± 0.111	0.866 ± 0.003	0.857 ± 0.043

C Example 3
The time series to be considered is given by

$$x(t) + 0.39x(t-1) + 0.3x(t-2) + 0.2x(t-3) = w(t) - 1.85 w(t-1) + 0.825w(t-2) \qquad (30)$$

This is an ARMA (3,2). It has three poles and two zeros. The poles are located at $0.0711 \pm j0.6088$ and -0.5323. The zeros are located at 1.1 and 0.75. Notice that this is a non-minimum phase model since it has one of its zeros outside the unit circle. The data was generated as in the previous examples. The ARMA parameters were estimated using the QS, the GM, and the proposed SVD methods. Table 3 shows

the average estimate results of 100 Monte Carlo simulations at SNR of 20 dB on the output sequence.

TABLE 3
True and estimated ARMA(3,2) parameters (mean± variance).

	True	QS	GM	Proposed
a(1)	0.39	0.408±0.015	0.395± 0.014	0.340±0.006
a(2)	0.3	0.290±0.009	0.284±0.009	0.270±0.004
a(3)	0.2	0.197±0.005	0.193±0.005	0.179 ±0.002
b(1)	-1.85	-2.124±7.542	-3.956±42.061	-1.4794±0.005
b(2)	0.825	0.943±1.521	0.768±0.149	0.600±0.006

IV DISCUSSION

In the above examples, ARMA parameters were estimated using the proposed SVD method and compared with two well known methods; namely, GM and the QS methods. The computations were performed in MATLAB. In addition, *armaqs* and *armarts* commands were used from the higher-order spectral analysis toolbox [16] to estimate the ARMA parameters for the QS and GM methods, respectively. Tables 1, 2, and 3 show the average estimate results of 100 Monte Carlo simulations at SNR of 20 dB on the output sequence. The performance measures considered for estimating the parameters are the arithmetic mean and the variance. It can be seen that the proposed ARMA parameters estimation technique performs better than the GM and the QS methods. The proposed SVD method in this paper was more computationally efficient than the other two methods since the AR and MA parameters are estimated in one step only. That is Equation (22) has both AR and MA coefficients. However, in the other two methods the AR coefficients must first be estimated and then the MA coefficients can be estimated. It should be emphasized that the main reason for using HOS for ARMA parameter estimation is the fact that additive Gaussian noise of unknown variance does not affect the theoretical cumulant statistics. However, when dealing with finite-length data observations, the computed cumulants do not vanish but will be very small. From the author's experience with higher-order cumulants, these cumulants have little effects on these results at moderate SNR. However, the performance deteriorates at low SNR.

V CONCLUSION

This paper presents an approach to estimate ARMA parameter of a given system using the singular value decomposition of a covariance matrix of a third order cumulants of the observed output sequence only. A comparison of the performance of the proposed algorithm with the QS and the GM methods was made at 20 dB SNR on the output signal. The presented simulation results demonstrate the effectiveness of the proposed method.

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