# **International Trade Theories within a Unified Framework**

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### ABSTRACT

A framework is developed which reduces to the three most popular models of international trade under different sets of assumptions. The key intuition is to focus on differences in per unit costs of production as determinants of trade patterns. This focus on per-unit costs clearly defines the links between between the Ricardian, the Hecksher-Ohlin, and the Economies of Scale Trade Theories. Examining the assumptions that are sufficient (but not necessary) for each case to hold provides a foundation which Facilitates student understanding. Students of international trade can see how these theories are inter-related, instead of viewing them in isolation as in the standard textbook expositions.

#### Introduction

Most undergraduate International Economics textbooks present the major theories of trade in a somewhat disunited fashion. A typical textbook<sup>2</sup> exposition approaches International Trade Theory in the following manner. Starting with a brief description and critique of the Mercantilist doctrine of the seventeenth and eighteenth centuries, it discusses Smith's Theory of Absolute Advantage and concludes that this theory has limited applicability. This sets the stage for the introduction of Ricardo's improvement on it in the form of Comparative Advantage. The Trade Theory section of the text generally starts with the presentation of the Ricardian Theory, followed by the Hecksher-Ohlin Theory, and ending with the more modern Economies of Scale Approach. It employs tools such as production possibilities frontiers, consumption possibilities frontiers, and community indifference curves to gain an insight into these theories.

While such an exposition clearly has logic and value, it can leave the undergraduate student with the impression that each theory is a separate and distinct entity, existing independently of the others. A unified framework within which each trade theory can be derived as a special case would be a valuable complement to the conventional approach. Such an innovation would help students to more easily recognize the links between the various theories. This is particularly useful in an area as challenging as trade theory can be for undergraduate students.

In this note, we develop a framework which reduces to the three most popular models of international trade under different sets of assumptions, thus illustrating the links between these models. The key intuition behind our approach is our focus on differences in per unit costs of production as determinants of trade patterns. As we show in the following sections, this focus on per-unit costs allows us to clearly define the links between the Ricardian, the Hecksher-Ohlin and the Economies of Scale Trade Theories. By highlighting and evaluating the assumptions that are sufficient (albeit not necessary) for each case to hold, we provide a foundation which will

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<sup>&</sup>lt;sup>2</sup> We surveyed half a dozen popular undergraduate textbooks in International Economics. These are Husted and Melvin (2001), Jepma, C.J., H. Jager, E. Kamphius (1996), Kreinin (2001), Krugman and Obstfeld (2003), Pugel (2004), and Salvatore (2004).

make it easier for students of international trade theories to see how these theories are interrelated, instead of viewing them in isolation as in the standard textbook expositions.

The paper is organized as follows. In the first section, we develop the general theoretical framework upon which our analysis is based. In the following three sections, we delineate the assumptions needed to derive the predictions of each theory using our general framework. In the final section of our paper, we bring together the earlier sections by discussing our results.

### **The General Framework**

The starting point of our analysis is the assumption that production technology and market structure are such that the prices of traded goods are proportional to per unit costs.<sup>3</sup> Consequently, a country will export a good when the relative average cost of producing that good is lower than that of its trading partner. This focus on per unit costs, which is associated with labor productivity, factor prices and economies of scale (as we show below) facilitates comparisons between the three trade theories we consider.

Suppose lp is average labor productivity, Q is output, and L is employment. Then output per worker or average productivity is given by:

$$lp = \frac{Q}{L}$$
, or  $Q = lpL$ . (1)

Next, suppose that AC represents average cost and TC represents total cost. Average cost can then be written as:

$$AC = \frac{TC}{Q}.$$
Substituting for Q from Equation (1) yields:

Substituting for Q from Equation (1) yields:

$$AC = \frac{TC}{lpL} = \frac{TC}{L} \frac{1}{lp}.$$
(2)

Equation (2) states that the average cost of producing a good varies directly with the total cost per worker and inversely with labor productivity. Further, if we decompose total cost into its three component parts -- fixed cost (FC), labor cost, wL (where w is the wage rate), and capital cost, iK (where i is the interest rate, and K is the amount of capital), we get the following equation: TC = FC + wL + iK. (3)

Substituting (3) in (2) yields:

$$AC = \left(\frac{FC}{L} + w + \frac{iK}{L}\right)\frac{1}{lp}.$$
(4)

In other words, per unit cost depends upon the following variables:

(a) The fixed cost per worker, which is a traditional measure of economies of scale.

(b) The prices of the factors of production --- the wage rate, w, and the interest rate, i.

(c) The capital-labor ratio 
$$\frac{K}{L}$$
, and

(d) Labor productivity, *lp*.

<sup>&</sup>lt;sup>3</sup> Such an assumption is consistent with (a) a perfectly competitive market structure, where the proportionality constant is one; (b) a monopolistically competitive market structure with consumer preferences of the constant elasticity of demand variety. We are grateful to an anonymous referee for this observation.

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Following traditional analysis, we consider a two-good, two-country world; x and y being the two goods, 1 and 2 being the two countries. From equation (2), Country 1's relative cost per unit of Good x is:

$$\left(\frac{AC_x}{AC_y}\right)^1 = \frac{\left(\frac{TC}{L}\right)_x^1}{\left(\frac{TC}{L}\right)_y^1} \cdot \left(\frac{lp_y}{lp_x}\right)^1.$$
(5a)

Similarly, the Country 2's relative cost per unit of Good *x* is:

$$\left(\frac{AC_x}{AC_y}\right)^2 = \frac{\left(\frac{TC}{L}\right)_x^2}{\left(\frac{TC}{L}\right)_y^2} \left(\frac{lp_y}{lp_x}\right)^2.$$
(5b)

Trade flows are determined by relative costs per unit. If the relative cost per unit of Good x in Country 1 is less than the relative cost per unit of Good x in Country 2, i.e. if the expression in (5a) is smaller than that in (5b), then Country 1 will benefit from specializing in and exporting Good x. It follows that the inverse of (5b) is smaller than the inverse of (5a); consequently, Country 2 will benefit from specializing in and exporting Good y.

Substituting for total cost (TC) from Equation (3) into Equation (5a) and Equation (5b) gives us the following expressions for the relative cost per unit x in countries 1 and 2, respectively.

$$\left(\frac{AC_x}{AC_y}\right)^1 = \frac{\left(\frac{FC}{L}\right)_x^1 + w_1 + i_1 \left(\frac{K}{L}\right)_x^1}{\left(\frac{FC}{L}\right)_y^1 + w_1 + i_1 \left(\frac{K}{L}\right)_y^1} \left(\frac{lp_y}{lp_x}\right)^1$$
(6a)

and

$$\left(\frac{AC_{x}}{AC_{y}}\right)^{2} = \frac{\left(\frac{FC}{L}\right)_{x}^{2} + w_{2} + i_{2}\left(\frac{K}{L}\right)_{x}^{2}}{\left(\frac{FC}{L}\right)_{y}^{2} + w_{2} + i_{2}\left(\frac{K}{L}\right)_{y}^{2}} \left(\frac{lp_{y}}{lp_{x}}\right)^{2},$$
(6b)

where

$$\frac{\left(\frac{FC}{L}\right)_{x}^{1} + w_{1} + i_{1}\left(\frac{K}{L}\right)_{x}^{1}}{\left(\frac{FC}{L}\right)_{y}^{1} + w_{1} + i_{1}\left(\frac{K}{L}\right)_{y}^{1}} \text{ and } \frac{\left(\frac{FC}{L}\right)_{x}^{2} + w_{2} + i_{2}\left(\frac{K}{L}\right)_{x}^{2}}{\left(\frac{FC}{L}\right)_{y}^{2} + w_{2} + i_{2}\left(\frac{K}{L}\right)_{y}^{2}}$$

are relative cost per worker in countries 1 and 2, respectively.

Equations (6a) and (6b) give us a relationship between average costs, fixed costs, capitallabor ratios and the relative labor productivities that is fundamental to our analysis. In the following three sections, we will apply alternative assumptions to this relationship and demonstrate that it reduces to the Ricardian Theory, Hecksher-Ohlin, and Economies of Scale, respectively.

# The Ricardian Theory of Trade

Almost two centuries after David Ricardo's 1817 classic The Principles of Political Economy and Taxation (where his principle of comparative advantage made a debut) was first published, the Ricardian Theory of Trade is still regarded as an insightful and fundamental explanation of the patterns and gains from trade, and discussed at length in most International Economics texts. In his book, Ricardo attributed trade between countries to differences in relative labor productivities in the production of each good. In its simplest form, his theory states that if the relative labor productivity in producing Good x relative to producing Good y differs between countries 1 and 2, then both countries stand to gain from specializing in one of the goods and trading with each other, even if one country is more productive in absolute terms in both goods.

For instance, if labor in Country 1 is more efficient at producing Good x relative to Good y, then Country 1 should specialize in and export Good x. Harberler<sup>4</sup> interpreted comparative advantage in terms of opportunity costs -- a country with comparative advantage in Good xproduces Good x at a lower opportunity cost than the other country. In a two-good, two-country world, this necessarily means that Country 2 will have a comparative advantage in Good y.

If we assume that inputs are used in the same proportion in the production of each good, then labor productivity would be the only relevant determinant of comparative advantage. This is less restrictive than the classical assumption of the Labor Theory of Value where labor is the only determinant of the price of a product. We show below that a country should specialize in the product in which it has the highest relative labor productivity.

# **Proposition 1:**

Given the same relative cost per worker in both countries, each can gainfully trade with the other if it specializes in that good in which it has a higher relative labor productivity.

# **Proof:**

Assume:

The relative cost per worker in each country equals k. (Note that this condition, while (a) sufficient to prove Proposition 1, is not, however, necessary)

Relative labor productivity in Good *x* is higher in Country 1 than in Country 2. (b)

Assumption (b) implies that 
$$\left(\frac{lp_x}{lp_y}\right)^1 > \left(\frac{lp_x}{lp_y}\right)^2$$
 or that  $\left(\frac{lp_y}{lp_x}\right)^1 < \left(\frac{lp_y}{lp_x}\right)^2$ .

We show in the appendix that under assumptions (a) and (b),  $\left(\frac{AC_x}{AC_y}\right)^1 < \left(\frac{AC_x}{AC_y}\right)^2$ . In other

words, given similar relative costs per worker, a higher relative labor productivity will lead to lower average costs. Therefore, Country 1 will benefit from specializing in Good x, while Country 2 should specialize in Good y.

This was first introduced in English in a chapter in the translation of his original German textbook published in 1936 as Theory of International Trade. (W. Hodge & Company, London).

Let us explore further our key assumption that relative cost per worker is the same in both countries. This assumption implies the following condition:

$$\frac{\left(\frac{FC}{L}\right)_{x}^{1} + w_{1} + i_{1}\left(\frac{K}{L}\right)_{x}^{1}}{\left(\frac{FC}{L}\right)_{y}^{1} + w_{1} + i_{1}\left(\frac{K}{L}\right)_{y}^{1}} = \frac{\left(\frac{FC}{L}\right)_{x}^{2} + w_{2} + i_{2}\left(\frac{K}{L}\right)_{x}^{2}}{\left(\frac{FC}{L}\right)_{y}^{2} + w_{2} + i_{2}\left(\frac{K}{L}\right)_{y}^{2}}.$$
(8)

Below, we discuss some situations under which equation (8) may hold.

### *Case (i): Labor is the only factor of production.*

An assumption standard in most textbook expositions of Ricardo's basic comparative advantage analysis, this implies that there are no fixed capital costs, reducing total cost to just labor costs. In other words,  $TC = wL^5$  Substituting for total cost in (8) reduces each side of equation (8) to '1' and thus the equality is satisfied.

Case (ii): Production takes place under constant returns to scale technology and within each country the capital-labor ratio is the same for each good, even though this ratio may be different between the two countries.

These two assumptions eliminate any fixed cost and imply that the capital-labor ratio in the production of both goods is the same. In other words, Case (ii) involves the following conditions: (i) FC=0

(ii) 
$$\left(\frac{K}{L}\right)_{x}^{1} = \left(\frac{K}{L}\right)_{y}^{1}$$
. Let this be equal to  $\left(\frac{K}{L}\right)^{1}$ .  
(iii)  $\left(\frac{K}{L}\right)_{x}^{2} = \left(\frac{K}{L}\right)_{y}^{2}$ . Let this be equal to  $\left(\frac{K}{L}\right)^{2}$ .  
Substituting conditions (i), (ii), and (iii) into (8) gives us  $\frac{w_{1} + i_{1}\left(\frac{K}{L}\right)^{1}}{w_{1} + i_{1}\left(\frac{K}{L}\right)^{1}} = \frac{w_{2} + i_{1}\left(\frac{K}{L}\right)^{2}}{w_{2} + i_{1}\left(\frac{K}{L}\right)^{2}} = 1;$ 

the equality is thus satisfied.

Case (iii): Production takes place under constant returns to scale; technology for each good is common across countries and the ratio of wages across countries equals the ratio of the costs of capital.

As in Case (ii), constant returns to scale implies that FC=0.

process. In this case, however, we can decompose the labor costs wL into  $w(L_0 + L_v)$ , where  $\frac{wL_0}{v}$  is the labor

<sup>&</sup>lt;sup>5</sup> Note that these labor costs may include a fixed number of labor hours required to start the production

involved in setup and  $\stackrel{WL}{v}$  is the variable labor cost. Such decomposition will not alter the argument that is laid out in this section.

Common technology across countries implies the following:

$$\left(\frac{K}{L}\right)_{x}^{1} = \left(\frac{K}{L}\right)_{x}^{2} = \left(\frac{K}{L}\right)_{x} \text{ and } \left(\frac{K}{L}\right)_{y}^{1} = \left(\frac{K}{L}\right)_{y}^{2} = \left(\frac{K}{L}\right)_{y}.$$

A further condition is the following:  $\frac{w_1}{w_2} = \frac{i_1}{i_2} \Rightarrow w_1 i_2 = w_2 i_1.$ 

Substituting th

Substituting the above three conditions in of (8) gives us: 
$$\frac{w_1 + i_1 \left(\frac{K}{L}\right)_x}{w_1 + i_1 \left(\frac{K}{L}\right)_y} = \frac{w_2 + i_2 \left(\frac{K}{L}\right)_x}{w_2 + i_2 \left(\frac{K}{L}\right)_y}.$$
which can be rewritten<sup>6</sup> as:  $(w_1 i_2 - w_2 i_1) \left(\frac{K}{L}\right)_y = (i_2 w_1 - i_1 w_2) \left(\frac{K}{L}\right)_x = 0,$ 

since  $w_1 i_2 = w_2 i_1$ . Thus, the equality (8) is satisfied.

Let us examine the implications of each of the above three cases more closely. While the assumption that labor is the only factor of production appears extremely restrictive, it is consistent with the classical view of the Labor Theory of Value. The assumption of constant returns to scale and similar technologies across goods is more general and less stringent than that of the Labor Theory of Value, while the third case is plausible, particularly if the trading partners have similar relative factor endowments.

Figure 1 graphically illustrates Proposition 1 by choosing sample values for the variables in Equations 6(a) and 6(b). In this figure, we choose values for the capital-labor ratio and the factors of production such that the relative costs per worker in both countries are constant. We also set the relative labor productivity for Good y in Country 2 to an arbitrarily chosen value of 3.5. The graph in Figure 1 then shows the effect of varying the relative labor productivity for Good y in Country 1 from a value of 0.5 to a value of 6.5 on the relationship between average

 $\operatorname{costs}\left(\frac{AC_{x}}{AC_{y}}\right)^{1} and \left(\frac{AC_{x}}{AC_{y}}\right)^{2} \text{As can be seen here, when the relative labor productivity in Good}$ 

$$\frac{w_{1}L_{x}+i_{1}\left(\frac{K}{L}\right)_{x}}{w_{1}L_{y}+i_{1}\left(\frac{K}{L}\right)_{y}} = \frac{w_{2}L_{x}+i_{2}\left(\frac{K}{L}\right)_{x}}{w_{2}L_{y}+i_{2}\left(\frac{K}{L}\right)_{y}} = \left[w_{1}L_{x}+i_{1}\left(\frac{K}{L}\right)_{x}\right]\left[w_{2}L_{y}+i_{2}\left(\frac{K}{L}\right)_{y}\right] = \left[w_{2}L_{y}+i_{2}\left(\frac{K}{L}\right)_{x}\right]\left[w_{1}L_{x}+i_{1}\left(\frac{K}{L}\right)_{y}\right] = \left[w_{1}L_{x}+i_{1}\left(\frac{K}{L}\right)_{x}\right]\left[w_{1}L_{x}+i_{1}\left(\frac{K}{L}\right)_{y}\right] = \left[w_{1}L_{x}+i_{2}\left(\frac{K}{L}\right)_{x}\right]\left[w_{1}L_{x}+i_{1}\left(\frac{K}{L}\right)_{y}\right] = \left[w_{1}L_{x}+i_{2}\left(\frac{K}{L}\right)_{x}\right]\left[w_{1}L_{x}+i_{2}\left(\frac{K}{L}\right)_{y}\right] = \left[w_{1}L_{x}+i_{2}\left(\frac{K}{L}\right)_{y}\right]$$

y is lower in Country 1 (i.e., is less than 3.5),  $\left(\frac{AC_x}{AC_y}\right)^1 < \left(\frac{AC_x}{AC_y}\right)^2$ , suggesting that Country 1

would export Good x, and this condition is reversed when the relative labor productivity in Good y is higher in Country 2 (i.e., greater than 3.5). (This graph obviously does not constitute a graphical proof of the proposition – an algebraic proof is supplied in the appendix. Instead this graph, as well as Figure 2 in the next section, is provided as a possible aid to textbook exposition of the proposition presented here.)



Figure 1 : Relative Labor Productivity and the Ratio of Average Costs

### THE HECKSHER-OHLIN MODEL

A typical textbook analysis of the Hechsher-Ohlin Theory of Trade considers a two countrytwo-good-two-factor model. The Hechsher-Ohlin Model rests on differences in relative factor endowments, which translate into different relative factor prices. The theory predicts that a relatively labor-abundant country can gainfully specialize in and export a labor-intensive product, while a relatively capital-abundant country should specialize in and export a capitalintensive product<sup>7</sup>. In contrast to Ricardo's Comparative Advantage Theory, the Hechsher-Ohlin Model assumes that technology of production for each good is the same in both countries.

<sup>&</sup>lt;sup>7</sup> As suggested by the Leontief Paradox, the role of factor endowments cannot be fully explained in a twofactor model of labor and capital, since it ignores the role of skilled vs. unskilled labor, and that of differing natural resource endowments. We are grateful to an anonymous referee for this observation.

# **Proposition 2:**

Under the standard Hecksher-Ohlin assumptions of constant returns to scale, similar product technologies for both countries, and no differences in labor productivity across countries, each country will gain from specializing in that product that uses it's relatively abundant factor more intensively.

# Proof:

Proposition 2 involves the following assumptions:

(i) Constant returns to scale, which implies that FC = 0.

(ii) Similar product technologies for both countries, which implies that:

$$\left(\frac{K}{L}\right)_{x}^{1} = \left(\frac{K}{L}\right)_{x}^{2}$$
 and  $\left(\frac{K}{L}\right)_{y}^{1} = \left(\frac{K}{L}\right)_{y}^{2}$ .

(iii) Similar relative labor productivities, which implies that:

$$\left(\frac{lp_x}{lp_y}\right)^1 = \left(\frac{lp_x}{lp_y}\right)^2$$
. Let us suppose that each of these equals  $\left(\frac{lp_x}{lp_y}\right)^2$ .

Below we show using our general framework that it follows from (i), (ii) and (iii) that if Good x is more labor intensive than Good y, and Country 1 is relatively better endowed with labor as demonstrated by lower relative wages, then Country 1 should specialize in Good x (and Country 2 in Good y).

Substituting (i), (ii) and (iii) in equations (6a) and (6b) and omitting several in-between steps, which are detailed in the appendix, gives us the following condition:

If 
$$(w_1 i_2 - w_2 i_1) \left[ \left( \frac{K}{L} \right)_y - \left( \frac{K}{L} \right)_x \right]$$
 is negative, then  $\left( \frac{AC_x}{AC_y} \right)^1 < \left( \frac{AC_x}{AC_y} \right)^2$  and country 1

specializes in and exports Good x.

This condition holds if and only if:  $\frac{w_1}{i_1} < \frac{w_2}{i_2}$  and  $\left(\frac{K}{L}\right)_y > \left(\frac{K}{L}\right)_x$ . If we assume, as in most

textbook treatments<sup>8</sup>, that wages and capital rents are endogenously determined by the capitallabor ratio, this implies that country 1 specializes in and exports Good x if

$$\left(\frac{K}{L}\right)_y > \left(\frac{K}{L}\right)_x$$
 or  $\frac{w_1}{i_1} < \frac{w_2}{i_2}$  which is exactly what the Hecksher-Ohlin Model predicts.

In other words, if Country 1 is relatively labor-abundant and thus has lower relative wages, it will gain from specializing and exporting that good which uses relatively more labor and less capital, namely, Good x.

<sup>&</sup>lt;sup>8</sup> As noted in the standard textbook by Salvatore, given similar tastes and demand preferences in both countries, there is no difference between defining relative factor abundance in physical terms as well as in terms of factor prices. (Salvatore, 2004. Page 121)



Figure 2. Wage-Interest Rate ratio and the Ratio of Average Costs.

Figure 2 illustrates Proposition 2 by plotting the ratio of relative average costs of Good x against changes in the wage-interest rate ratio in Country 1. The figure assumes that the wage-interest rate ratio in Country 2 is fixed at 0.75, and other variables are assigned appropriate fixed values in accordance with our assumptions. As the graph shows, when the wage-interest ratio in Country 1 is smaller than in Country 2 (i.e., less than 0.75) and Good x is labor intensive, the relative average cost of Good x in Country 1 is lower than Country 2. Conversely, when Good x is capital-intensive, the reverse relationship holds.

### **Economies of Scale Theory**

In this section, we look at Economies of Scale with a focus on the reduction of fixed costs per worker as production is scaled up. This presentation of the Economies of Scale Theory is only applicable to a discussion of internal economies of scale achieved in monopolistically competitive markets, and does not cover external economies of scale that result in inter-industry trade. We present it here, in addition to models of inter-industry trade such as the Hecksher-Olin and the Ricardian Theory, since the fact that our framework can model certain intra-industry trade patterns (such as trade between developed countries in differentiated versions of a product) should aid textbook expositions of international trade.<sup>9</sup>

We define the variable cost per worker as:

$$VCW = w + i \left(\frac{K}{L}\right).$$

<sup>&</sup>lt;sup>9</sup> We thank an anonymous referee for pointing out this distinction as applied to our model.

The magnitude of the fixed cost per worker,  $\frac{FC}{L}$ , represents economies of scale.<sup>10</sup> The lower the

fixed cost per worker, the larger is the cost advantage for a producer stemming from the exploitation of the economies of scale.

### **Proposition 3:**

Under assumptions of similar relative labor productivities across countries and identical variable costs per worker in both countries for the production of the same good, each country can gainfully trade with the other by exploiting economies of scale under monopolistically competitive conditions.

# Proof:

(i) The assumption of similar relative labor productivities implies the following condition:

$$\left(\frac{lp_y}{lp_x}\right)^1 = \left(\frac{lp_y}{lp_x}\right)^2$$

(ii) The assumption of identical variable costs per worker across countries for the production of the same good implies the following:

 $VCW_x^1 = VCW_x^2$ ;  $VCW_y^1 = VCW_y^2$ . We denote the former by  $VCW_x$  and the latter by  $VCW_y$ .

Variable costs are given by production technologies and factor endowments across countries. If Good x is a differentiated product in which Country 1 is monopolistically competitive, then Country 1 can exploit increasing returns of scale in Good x, i.e., :

 $\left(\frac{FC}{L}\right)_{x}^{1} < \left(\frac{FC}{L}\right)_{x}^{2}$ . Similarly, Country 2 can exploit returns of scale in Good y i.e., when  $\left(\frac{FC}{L}\right)_{y}^{2} < \left(\frac{FC}{L}\right)_{y}^{1}$ .

Under assumptions (i) and (ii), the relative cost per unit becomes:

$$\frac{\left(\frac{AC_x}{AC_y}\right)^1}{\left(\frac{AC_x}{AC_y}\right)^2} = \frac{\frac{\left(\frac{FC}{L}\right)_x^1 + VCW_x}{\left(\frac{FC}{L}\right)_y^1 + VCW_y}}{\frac{\left(\frac{FC}{L}\right)_x^2 + VCW_x}{\left(\frac{FC}{L}\right)_y^2 + VCW_x}}.$$

As shown in the appendix, this expression is less than 1 if and only if:

<sup>&</sup>lt;sup>10</sup> This does not take into consideration the impact of specialization as plant size increases. We thank an anonymous referee for pointing out this simplification.

$$\left(\frac{FC}{L}\right)_{x}^{1} < \left(\frac{FC}{L}\right)_{x}^{2}$$
 and  $\left(\frac{FC}{L}\right)_{y}^{2} < \left(\frac{FC}{L}\right)_{y}^{1}$ , i.e., if both countries exploit economies of scale in

their respective good.

It is easy to see from Proposition 3 that in the economies of scale framework, even if both countries face the same fixed costs in the production of differentiated products, each country can obtain a cost advantage by exploiting economies of scale and, as a result, both countries can gainfully trade in these products. Attempting to explain trade flows under monopolistically competitive conditions by focusing on economies of scale only makes the implicit assumptions (i) and (ii) above, i.e. that both trading partners have the same relative labor productivities and similar variable costs per worker for each good.

### Conclusion

This paper illustrates how each of the three most popular trade theories can be linked together for purposes of instruction by means of a framework that focuses on average costs. It highlights the set of assumptions that is sufficient in order to obtain the important results of each theory. Such an approach can aid authors of International Economics textbooks in organizing the material they present in ways that highlight the connections between the various theories. Additionally, it will help undergraduate students in better understanding how different sets of assumptions may lead to different theories. Further, by explicitly specifying the sufficient conditions, it helps students evaluate competing theories not only on the merits of their predictions, but also on the reasonableness of their assumptions.

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### APPENDIX

### A1. Detail Proof of Proposition 1 (Theory of Comparative Advantage)

From Section I, the assumption that relative cost per worker in both countries *k* implies the following:

$$\frac{\left(\frac{FC}{L}\right)_{x}^{1} + w_{1} + i_{1}\left(\frac{K}{L}\right)_{x}^{1}}{\left(\frac{FC}{L}\right)_{y}^{1} + w_{1} + i_{1}\left(\frac{K}{L}\right)_{y}^{1}} = \frac{\left(\frac{FC}{L}\right)_{x}^{2} + w_{2} + i_{2}\left(\frac{K}{L}\right)_{x}^{2}}{\left(\frac{FC}{L}\right)_{y}^{2} + w_{2} + i_{2}\left(\frac{K}{L}\right)_{y}^{2}} = k.$$
(7)

The assumption that relative labor productivity in Good x is higher in Country 1, while for Good

y relative labor productivity is higher in Country 2 implies that 
$$\left(\frac{lp_y}{lp_x}\right)^1 < \left(\frac{lp_y}{lp_x}\right)^2$$
. (i)

Then:

$$\begin{aligned} & \left(\frac{AC_x}{AC_y}\right)^1 \\ & \left(\frac{AC_x}{AC_y}\right)^2 = \left[\frac{\left(\frac{FC}{L}\right)_x^1 + w_1 + i_1\left(\frac{K}{L}\right)_x^1}{\left(\frac{FC}{L}\right)_y^1 + w_1 + i_1\left(\frac{K}{L}\right)_y^1} \left(\frac{lp_y}{lp_x}\right)^1\right] \left[\frac{\left(\frac{FC}{L}\right)_y^2 + w_2 + i_2\left(\frac{K}{L}\right)_y^2}{\left(\frac{FC}{L}\right)_x^2 + w_2 + i_2\left(\frac{K}{L}\right)_x^2} \left(\frac{lp_x}{lp_y}\right)^2\right] \\ & = k \left(\frac{lp_y}{lp_x}\right)^1 \cdot \frac{1}{k} \left(\frac{lp_x}{lp_y}\right)^2 = \frac{\left(\frac{lp_y}{lp_x}\right)^1}{\left(\frac{lp_y}{lp_x}\right)^2}.\end{aligned}$$

The above expression is less than 1 because of Condition (i). Thus,

$$\left(\frac{AC_x}{AC_y}\right)^1 < \left(\frac{AC_x}{AC_y}\right)^2$$

This proves Proposition 1.

### A2. Detailed Proof of Proposition 2 (The Hecksher-Ohlin Model).

Since FC = 0, and labor productivities are similar, the ratio of equations (6a) and (6b) reduces to the following:

$$\frac{\left(\frac{AC_{x}}{AC_{y}}\right)^{1}}{\left(\frac{AC_{x}}{AC_{y}}\right)^{2}} = \frac{\frac{w_{1} + i_{1}\left(\frac{K}{L}\right)_{x}^{1}}{w_{1} + i_{1}\left(\frac{K}{L}\right)_{y}^{1}}}{\frac{w_{2} + i_{2}\left(\frac{K}{L}\right)_{x}^{2}}{w_{2} + i_{2}\left(\frac{K}{L}\right)_{y}^{2}}} = \left[\frac{w_{1} + i_{1}\left(\frac{K}{L}\right)_{x}^{1}}{w_{1} + i_{1}\left(\frac{K}{L}\right)_{y}^{1}}\right] \frac{w_{2} + i_{2}\left(\frac{K}{L}\right)_{y}^{2}}{w_{2} + i_{2}\left(\frac{K}{L}\right)_{y}^{2}}$$

Expanding this expression gives us:

$$\frac{w_1w_2 + w_1i_2\left(\frac{K}{L}\right)_y^2 + w_2i_1\left(\frac{K}{L}\right)_x^1 + i_1i_2\left(\frac{K}{L}\right)_y^2\left(\frac{K}{L}\right)_x^1}{w_1w_2 + w_2i_1\left(\frac{K}{L}\right)_y^1 + w_1i_2\left(\frac{K}{L}\right)_x^2 + i_1i_2\left(\frac{K}{L}\right)_y^1\left(\frac{K}{L}\right)_x^2}$$

Adding and subtracting  $w_2 i_1 \left(\frac{K}{L}\right)_y + w_1 i_2 \left(\frac{K}{L}\right)_x$  from the numerator, the above expression

becomes:

$$\frac{w_1w_2 + w_1i_2\left(\frac{K}{L}\right)_y + w_2i_1\left(\frac{K}{L}\right)_x + i_1i_2\left(\frac{K}{L}\right)_y\left(\frac{K}{L}\right)_x + w_2i_1\left(\frac{K}{L}\right)_y + w_1i_2\left(\frac{K}{L}\right)_x - w_2i_1\left(\frac{K}{L}\right)_y - w_1i_2\left(\frac{K}{L}\right)_x}{w_1w_2 + w_2i_1\left(\frac{K}{L}\right)_y + w_1i_2\left(\frac{K}{L}\right)_x + i_1i_2\left(\frac{K}{L}\right)_y\left(\frac{K}{L}\right)_x}$$

Rearranging terms yields:

$$\frac{w_1w_2 + i_1i_2\left(\frac{K}{L}\right)_y\left(\frac{K}{L}\right)_x + w_2i_1\left(\frac{K}{L}\right)_y + w_1i_2\left(\frac{K}{L}\right)_x + w_1i_2\left(\frac{K}{L}\right)_y + w_2i_1\left(\frac{K}{L}\right)_x - w_2i_1\left(\frac{K}{L}\right)_y - w_1i_2\left(\frac{K}{L}\right)_x}{w_1w_2 + w_2i_1\left(\frac{K}{L}\right)_y + w_1i_2\left(\frac{K}{L}\right)_x + i_1i_2\left(\frac{K}{L}\right)_y\left(\frac{K}{L}\right)_x},$$

which can be rewritten:

$$1 + \frac{w_1 i_2 \left(\frac{K}{L}\right)_y + w_2 i_1 \left(\frac{K}{L}\right)_x - w_2 i_1 \left(\frac{K}{L}\right)_y - w_1 i_2 \left(\frac{K}{L}\right)_x}{w_1 w_2 + w_2 i_1 \left(\frac{K}{L}\right)_y + w_1 i_2 \left(\frac{K}{L}\right)_x + i_1 i_2 \left(\frac{K}{L}\right)_y \left(\frac{K}{L}\right)_x}$$

$$= 1 + \frac{\left( \frac{K}{L} \right)_{y} - \left( \frac{K}{L} \right)_{x}}{w_{1}w_{2} + w_{2}i_{1} \left( \frac{K}{L} \right)_{y} + w_{1}i_{2} \left( \frac{K}{L} \right)_{x} + i_{1}i_{2} \left( \frac{K}{L} \right)_{y} \left( \frac{K}{L} \right)_{x}}.$$

This implies that

$$\frac{\left(\frac{AC_x}{AC_y}\right)^1}{\left(\frac{AC_x}{AC_y}\right)^2} - 1 = \frac{\Psi_1 i_2 - w_2 i_1 \left[\left(\frac{K}{L}\right)_y - \left(\frac{K}{L}\right)_x\right]}{w_1 w_2 + w_2 i_1 \left(\frac{K}{L}\right)_y + w_1 i_2 \left(\frac{K}{L}\right)_x + i_1 i_2 \left(\frac{K}{L}\right)_y \left(\frac{K}{L}\right)_x}.$$

The denominator of the right hand side of the above expression is positive. Hence, the expression on the right hand side is negative if and only if:

$$\Psi_{1}i_{2} - w_{2}i_{1} \text{ and } \left[ \left( \frac{K}{L} \right)_{y} - \left( \frac{K}{L} \right)_{x} \right] \text{ are of opposite signs, i.e., if:} \\ \left[ \left( \frac{K}{L} \right)_{y} > \left( \frac{K}{L} \right)_{x} \right], \quad \Psi_{1}i_{2} < w_{2}i_{1} \text{ or } \frac{w_{1}}{i_{1}} < \frac{w_{2}}{i_{2}}.$$

This is just what the Hecksher-Ohlin Model predicts, thus completing our proof.

#### **Detailed Proof of Proposition 3 (Economies of Scale Theory of Trade).** A3.

$$\frac{\left(\frac{AC_x}{AC_y}\right)^1}{\left(\frac{AC_x}{AC_y}\right)^2} = \frac{\frac{\left(\frac{FC}{L}\right)_x^1 + VCW_x^1}{\left(\frac{FC}{L}\right)_y^1 + VCW_y^1}}{\frac{\left(\frac{FC}{L}\right)_x^2 + VCW_x^2}{\left(\frac{FC}{L}\right)_y^2 + VCW_x^2}} = \frac{\frac{\left(\frac{FC}{L}\right)_x^1 + VCW_y}{\left(\frac{FC}{L}\right)_y^2 + VCW_y}}{\frac{\left(\frac{FC}{L}\right)_y^2 + VCW_y^2}{\left(\frac{FC}{L}\right)_y^2 + VCW_y}},$$

since  $VCW_x^1 = VCW_x^2 = VCW_x$ , and  $VCW_y^1 = VCW_y^2 = VCW_y$ .

The right hand side of above expression can be written as:

$$\begin{bmatrix} \left(\frac{FC}{L}\right)_{x}^{1} + VCW_{x} \\ \left(\frac{FC}{L}\right)_{y}^{1} + VCW_{y} \end{bmatrix} \begin{bmatrix} \left(\frac{FC}{L}\right)_{y}^{2} + VCW_{y} \\ \left(\frac{FC}{L}\right)_{x}^{2} + VCW_{x} \end{bmatrix}.$$
$$= \frac{\left(\frac{FC}{L}\right)_{x}^{1} \left(\frac{FC}{L}\right)_{y}^{2} + \left(\frac{FC}{L}\right)_{x}^{1} VCW_{y} + \left(\frac{FC}{L}\right)_{y}^{2} VCW_{x} + VCW_{x}VCW_{y} \\ \left(\frac{FC}{L}\right)_{y}^{1} \left(\frac{FC}{L}\right)_{x}^{2} + \left(\frac{FC}{L}\right)_{y}^{1} VCW_{x} + \left(\frac{FC}{L}\right)_{x}^{2} VCW_{y} + VCW_{x}VCW_{y} \end{bmatrix}.$$

Adding and Subtracting

$$\left(\frac{FC}{L}\right)_{y}^{1}\left(\frac{FC}{L}\right)_{x}^{2} + \left(\frac{FC}{L}\right)_{y}^{1}VCW_{x} + \left(\frac{FC}{L}\right)_{x}^{2}VCW_{y}$$
 from the numerator of the above expression,

and rearranging yields:

$$1 + \frac{\left[\left(\frac{FC}{L}\right)_{x}^{1}\left(\frac{FC}{L}\right)_{y}^{2} - \left(\frac{FC}{L}\right)_{y}^{1}\left(\frac{FC}{L}\right)_{x}^{2}\right] + \left[\left(\frac{FC}{L}\right)_{x}^{1} - \left(\frac{FC}{L}\right)_{x}^{2}\right]VCW_{y} + \left[\left(\frac{FC}{L}\right)_{y}^{2} - \left(\frac{FC}{L}\right)_{y}^{1}\right]VCW_{x}}{\left(\frac{FC}{L}\right)_{y}^{1}\left(\frac{FC}{L}\right)_{x}^{2} + \left(\frac{FC}{L}\right)_{y}^{1}VCW_{x} + \left(\frac{FC}{L}\right)_{x}^{2}VCW_{y} + VCW_{x}VCW_{y}}\right]}.$$

Thus,

$$\frac{\left(\frac{AC_x}{AC_y}\right)^1}{\left(\frac{AC_x}{AC_y}\right)^2} - 1 = \frac{\left[\left(\frac{FC}{L}\right)_x^1 \left(\frac{FC}{L}\right)_y^2 - \left(\frac{FC}{L}\right)_y^1 \left(\frac{FC}{L}\right)_x^2\right] + \left[\left(\frac{FC}{L}\right)_x^1 - \left(\frac{FC}{L}\right)_x^2\right] VCW_y + \left[\left(\frac{FC}{L}\right)_y^2 - \left(\frac{FC}{L}\right)_y^1\right] VCW_x}{\left(\frac{FC}{L}\right)_y^1 \left(\frac{FC}{L}\right)_x^2 + \left(\frac{FC}{L}\right)_y^1 VCW_x + \left(\frac{FC}{L}\right)_x^2 VCW_y + VCW_x VCW_y}\right]}.$$

The denominator of the right hand side of the above expression is positive. Hence, the expression on the right hand side is negative if:

$$\left(\frac{FC}{L}\right)_{x}^{1} < \left(\frac{FC}{L}\right)_{x}^{2} \text{ and } \left(\frac{FC}{L}\right)_{y}^{2} < \left(\frac{FC}{L}\right)_{y}^{1}.$$

This implies that Country 1 has economies of scale in Good x and Country 2 has economies of scale in Good y. This proves Proposition 3.