## UNDERGRADUATE RESEARCH

# WORKING FOR THE WEEKEND: A TIME ALLOCATION MODEL FOR STUDENT WORKERS ${ }^{1}$ 

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#### Abstract

An important area of consumer choice is time allocation and its role in dictating behavior. For the typical student worker, the time allocation decision involves three primary activities: paid employment, academic pursuits, and leisure pursuits. Exogenous factors such as the wage rate, price of consumption, rate of effective studying, desired academic grade, and total time available influence an individual's choice of time to spend on work, study, and leisure. The effects of these exogenous factors reveal a bifurcation in the student worker's time allocation decision: a laborleisure tradeoff versus academics. The time allocation model developed here derives these effects for the hybrid case of the student worker.


Key Words: time allocation, student workers, employment, leisure, academics, utility maximization

JEL Classification: C30, D11

## Introduction

Utility maximization has traditionally been examined in relation to labor-leisure choice. ${ }^{4}$ If an individual is not satisfied with the amount of income she receives, she will likely choose to work more hours (assuming it is within her power to control the number of hours worked). On the other hand, if an individual feels overworked and is willing to forgo a portion of her income, she will likely choose to work fewer hours and devote more time to leisure. When the concept of utility maximization is presented in this manner, it becomes apparent that the individual's choice is dictated by another, arguably more important factor: time.

By its very nature, time is unlike any other resource. Resources such as capital and labor are subject to both marginal increases and decreases in quantity; time, on the other hand, is subject only to marginal decreases. People are only "losing" time, so to speak. In addition, other resources can be traded while time cannot, for "time cannot be borrowed, traded, sold, or stored; but only consumed at a constant rate" (Klein, 2007, p. 3). So, how can an individual be expected

[^0]to allocate efficiently a resource that acts unlike any other? How is an individual's utility influenced by the efficient (or inefficient) allocation of her own time?

A review of works applying utility maximization to time allocation, specifically, studentworker time allocation, forms the basis for the theoretical model presented below.

## Literature Review

Becker (1965) provided the first theoretical analysis of time allocation based on the idea that individuals in households divide time into alternating segments of production and consumption. As a result, time allocation is subject to one basic constraint because "time can be converted into goods by using less time at consumption and more at work" (Becker, 1965, p. 496). This theory, however, does not provide adequate explanation for individuals who do not spend all of their time either "producing" or "consuming." A notable case is that of the student.

Multiple studies have focused on student time allocation with regard to academic performance. Kelley (1975), Schmidt (1983), and Dolton, Marcenaro, and Navarro (2001) found positive correlations between good study habits and high performance in classes. Good study habits include taking notes, studying course material outside of class, and attending lectures. Similarly, negative correlations were found between excessive leisure time and high performance in classes.

Nevertheless, the modern student does not gain utility solely from academic performance. How do additional responsibilities and interests factor into a student's time allocation? What if students choose to allocate their time across leisure, school, and work?

The National Center for Education Statistics found that 40 percent of undergraduates ages 16 to 24 worked while enrolled in college full-time in 2010, while roughly 73 percent worked while enrolled in college part-time (National Center for Education Statistics, 2012). The effects of student employment on academic performance appear to be positive. "Quantitative studies consistently show that retention rates are higher for students who work a modest number of hours per week (ten to fifteen) than they are for students who do not work at all or those who work more than fifteen hours per week" (Perna, 2010, p. 30). Using data from the 2003-2008 American Time Use Surveys, Kalenkoski and Pabilonia (2011) found that employment decreases the amount of time high school students spend on both "productive" and "unproductive" activities. In fact, as students worked more hours, the amount of time spent on "unproductive" activities decreased more than the amount of time spent on "productive" activities.

Statistical analysis suggests that employment has a significant impact on the way students spend their time. Simply stated, time spent working may not be spent doing any other activity. Thus, a model of student time allocation should consider employment as seriously as leisure and schoolwork.

## The Time Allocation Model

## Constructing the Lagrangian Function

The traditional time allocation model examined labor-leisure tradeoffs and the production and consumption of households. A student worker's time allocation model should utilize the traditional labor-leisure tradeoff while incorporating academic activity. Suppose that the typical student worker gains utility from time spent in three activities: paid employment, academic pursuits, and leisure pursuits. The utility function of such a student worker can be represented as:

$$
\begin{equation*}
U=U\left(t_{1}, t_{2}, t_{3}\right) \tag{1}
\end{equation*}
$$

where $t_{1}$ indicates time spent in employment, $t_{2}$ indicates time spent on academics, and $t_{3}$
indicates time spent in leisure. Naturally, the generality of this utility function accommodates all forms of student workers, no one specific case.

In addition, an increase in any time-use variable, $t_{i}$, necessarily results in an increase in total utility.

$$
\begin{equation*}
\frac{\partial U}{\partial t_{i}}>0 \tag{2}
\end{equation*}
$$

for any time-use variable, $t_{i}$. Diminishing marginal utility ${ }^{5}$ implies

$$
\begin{equation*}
\frac{\partial^{2} U}{\partial t_{i}^{2}}<0 \tag{3}
\end{equation*}
$$

That is, each additional increase in time spent in a particular activity leads to a marginally smaller increase in total utility for the individual. ${ }^{6}$ Moreover, the time-use variables, $t_{i}$, are mutually exclusive; time spent working, $t_{1}$, does not directly affect performance in academics or leisure, time spent studying, $t_{2}$, does not directly affect performance in paid employment or leisure, and time spent relaxing, $t_{3}$, does not directly affect performance in paid employment or academics.

In this model, the student worker's utility function is maximized subject to three basic inequality constraints: two budget constraints and one time constraint. The budget constraints are based on interactions of the three activities at hand. Stemming from the traditional labor-leisure relationship,

$$
\begin{equation*}
w t_{1} \geq c t_{3} \tag{4}
\end{equation*}
$$

where $w$ represents the wage rate for paid employment and $c$ represents the price of consumption for leisure. An additional budget constraint relates study time to a desired grade in academics.
Here,

$$
\begin{equation*}
r t_{2} \geq \bar{R}, \tag{5}
\end{equation*}
$$

where $r$ denotes the rate of effective studying and $\bar{R}$ denotes the desired grade of the representative individual. The third constraint ensures the stability of the time-use variables of the model. For some total amount of activity time, $T$,

$$
\begin{equation*}
t_{1}+t_{2}+t_{3} \leq T \tag{6}
\end{equation*}
$$

Thus, in this model, the student worker maximizes the utility function,

$$
\begin{equation*}
U=U\left(t_{1}, t_{2}, t_{3}\right) \tag{1}
\end{equation*}
$$

subject to the constraints,

$$
\begin{gather*}
w t_{1} \geq c t_{3}  \tag{4}\\
r t_{2} \geq \bar{R}  \tag{5}\\
t_{1}+t_{2}+t_{3} \leq T . \tag{6}
\end{gather*}
$$

Obtaining a meaningful solution to the constrained maximization problem is a task most easily accomplished through utilization of the Lagrangian method. The aforementioned utility function and constraints can be expressed in the following Lagrangian form:
$L\left(t_{1}, t_{2}, t_{3}, \lambda_{1}, \lambda_{2}, \lambda_{3}\right)$
$=U\left(t_{1}, t_{2}, t_{3}\right)+\lambda_{1}\left(w t_{1}-c t_{3}\right)+\lambda_{2}\left(r t_{2}-\bar{R}\right)+\lambda_{3}\left(T-t_{1}-t_{2}-t_{3}\right)$,
5 William Stanley Jevons discussed diminishing marginal utility in his seminal work, The Theory of Political Economy, stating, "We may state as a general law, that the degree of utility varies with the quantity of commodity, and ultimately decreases as that quantity increases" (Jevons, 1888, p. 53).
$6 \quad$ For the remainder of this paper, all $\frac{\partial U}{\partial t_{i}}$ will be denoted as $U_{t_{i}}$, all $\frac{\partial^{2} U}{\partial t_{i}{ }^{2}}$ will be denoted as $U_{t_{i}{ }^{2}}$, and all $\frac{\partial^{2} U}{\partial t_{i} \partial t_{j}}$ will be denoted as $U_{t_{i} t_{j}}$. This practice greatly simplifies notation when representing second-order conditions in matrix form.
where each $\lambda_{i}$ represents the Lagrangian multiplier for the corresponding constraint.

## Deriving and Interpreting the First-Order Conditions

The presence of inequality constraints in the utility maximization model leads to the implementation of Kuhn-Tucker conditions as the first-order necessary conditions of the problem. Differentiation of the Lagrangian function produces the following set of Kuhn-Tucker conditions:

$$
\begin{gather*}
\text { foc }_{1}: U_{t_{1}}+\lambda_{1} w-\lambda_{3}=0  \tag{8.1}\\
\text { foc }_{2}: U_{t_{2}}+\lambda_{2} r-\lambda_{3}=0  \tag{8.2}\\
\text { foc }_{3}: U_{t_{3}}-\lambda_{1} c-\lambda_{3}=0  \tag{8.3}\\
\text { foc }_{4}: w t_{1}-c t_{3} \geq 0  \tag{8.4}\\
\text { foc }_{5}: \lambda_{1}\left(w t_{1}-c t_{3}\right)=0  \tag{8.5}\\
\text { foc }_{6}: \lambda_{1} \geq 0  \tag{8.6}\\
\text { foc }_{7}: r t_{2}-\bar{R} \geq 0  \tag{8.7}\\
\text { foc }_{8}: \lambda_{2}\left(r t_{2}-\bar{R}\right)=0  \tag{8.8}\\
\text { foc }_{9}: \lambda_{2} \geq 0  \tag{8.9}\\
\text { foc }_{10}: T-t_{1}-t_{2}-t_{3} \geq 0  \tag{8.10}\\
\text { foc }_{11}: \lambda_{3}\left(T-t_{1}-t_{2}-t_{3}\right)=0  \tag{8.11}\\
\text { foc }_{12}: \lambda_{3} \geq 0 \tag{8.12}
\end{gather*}
$$

Note that equations (8.5), (8.6), (8.8), (8.9), (8.11), and (8.12) represent the complementary slackness conditions ${ }^{7}$ of the problem. In this case, because the utility function is assumed to be concave and the constraints are linear, the problem can be modified into an equality-constrained problem and the constraints themselves can be assumed to be binding. Assuming binding constraints, complementary slackness conditions are no longer necessary, and the following set of first-order conditions is established in relation to the equality-constrained problem:

$$
\begin{gather*}
L_{t_{1}}: U_{t_{1}}+\lambda_{1} w-\lambda_{3}=0  \tag{9.1.a}\\
L_{t_{2}}: U_{t_{2}}+\lambda_{2} r-\lambda_{3}=0  \tag{9.2.a}\\
L_{t_{3}}: U_{t_{3}}-\lambda_{1} c-\lambda_{3}=0  \tag{9.3.a}\\
L_{\lambda_{1}}: w t_{1}-c t_{3}=0  \tag{9.4.a}\\
L_{\lambda_{2}}: r t_{2}-\bar{R}=0  \tag{9.5.a}\\
L_{\lambda_{3}}: T-t_{1}-t_{2}-t_{3}=0 \tag{9.6.a}
\end{gather*}
$$

The economic significance of the first-order conditions is dictated by the marginal effects of the Lagrangian multipliers and time-use variables on the student worker's utility. Equations (9.4.a), (9.5.a), and (9.6.a), representing the marginal effects of the Lagrangian multipliers on the objective function, lead to the following interpretations of the Lagrangian multipliers:

$$
\begin{gather*}
\lambda_{1}=\text { marginal utility of income (or lowering expenditures) }  \tag{10.1}\\
\lambda_{2}=\text { marginal utility of lowering academic standards }  \tag{10.2}\\
\lambda_{3}=\text { marginal utility of time } \tag{10.3}
\end{gather*}
$$

Rearranging equations (9.1.a), (9.2.a), and (9.3.a) allows one to observe the marginal effects of the time-use variables on an individual's utility:

$$
\begin{align*}
& U_{t_{1}}=\lambda_{3}-\lambda_{1} w,  \tag{9.1.b}\\
& U_{t_{2}}=\lambda_{3}-\lambda_{2} r, \tag{9.2.b}
\end{align*}
$$

[^1]\[

$$
\begin{equation*}
U_{t_{3}}=\lambda_{3}+\lambda_{1} c \tag{9.3.b}
\end{equation*}
$$

\]

Provided that $\lambda_{1} w$ is large enough, an individual is willing to have negative utility of work. Economic intuition supports this claim, dictating that an individual may be willing to obtain disutility, or "pain," from working if her expenditures are "low enough" and her wage is "high enough." Similar interpretation can be established regarding the marginal utility of studying. An individual who holds sufficiently "low" academic standards and a sufficiently "high" rate of effective studying may still be willing to obtain disutility from studying. On the other hand, no matter the size of $\lambda_{1} c$, an individual is not willing to have negative utility of relaxing. A rational individual will not willingly spend money and time to inflict pain or suffering on herself when she could instead work to earn money or study to obtain academic benefit.

An equivalency relationship regarding the time-use variables can be established:

$$
\begin{equation*}
U_{t_{1}}+\lambda_{1} w=U_{t_{2}}+\lambda_{2} r=U_{t_{3}}-\lambda_{1} c=\lambda_{3}=\text { marginal utility of time } \tag{11}
\end{equation*}
$$

Because the Lagrangian multipliers and exogenous rates are positive, the marginal utility of relaxing is necessarily larger than the marginal utility of working and the marginal utility of studying. This is expected because an individual is willing to obtain even disutility from working and studying if $\lambda_{1} w$ and $\lambda_{2} r$ are large enough while utility obtained from relaxing must be positive. In addition, from an economic standpoint, the negative correlation found between the wage effect and the price effect is expected, for production and consumption are inverse operations.

## Deriving the Second-Order Conditions and Forming the Bordered Hessian ${ }^{8}$

Total differentiation of each of the first-order conditions yields the following set of second-order conditions:

$$
\begin{gather*}
\operatorname{soc}_{1}: U_{t_{1}{ }^{2}} d t_{1}+U_{t_{1} t_{2}} d t_{2}+U_{t_{1} t_{3}} d t_{3}+w d \lambda_{1}-d \lambda_{3}+\lambda_{1} d w=0,  \tag{12.1.a}\\
\operatorname{soc}_{2}: U_{t_{1} t_{2}} d t_{1}+U_{t_{2}{ }^{2}} d t_{2}+U_{t_{2} t_{3}} d t_{3}+r d \lambda_{2}-d \lambda_{3}+\lambda_{2} d r=0  \tag{12.2.a}\\
\operatorname{soc}_{3}: U_{t_{1} t_{3}} d t_{1}+U_{t_{2} t_{3}} d t_{2}+U_{t_{3}{ }^{2}} d t_{3}-c d \lambda_{1}-d \lambda_{3}-\lambda_{1} d c=0,  \tag{12.3.a}\\
\operatorname{soc}_{4}: w d t_{1}-c d t_{3}+t_{1} d w-t_{3} d c=0,  \tag{12.4.a}\\
\operatorname{soc}_{5}: r d t_{2}+t_{2} d r-d \bar{R}=0,  \tag{12.5.a}\\
\operatorname{soc}_{6}:-d t_{1}-d t_{2}-d t_{3}+d T=0 . \tag{12.6.a}
\end{gather*}
$$

Our goal is to explain how changes in the exogenous variables of the model affect the relative values of the endogenous variables. By separating the exogenous partial derivatives from the endogenous partial derivatives in the second-order conditions, one can easily obtain the desired comparisons. The set of second-order conditions may be rewritten in the following manner:

$$
\begin{gather*}
\operatorname{soc}_{1}: U_{t_{1}{ }^{2}} d t_{1}+U_{t_{1} t_{2}} d t_{2}+U_{t_{1} t_{3}} d t_{3}+w d \lambda_{1}-d \lambda_{3}=-\lambda_{1} d w  \tag{12.1.b}\\
\operatorname{soc}_{2}: U_{t_{1} t_{2}} d t_{1}+U_{t_{2}{ }^{2}} d t_{2}+U_{t_{2} t_{3}} d t_{3}+r d \lambda_{2}-d \lambda_{3}=-\lambda_{2} d r  \tag{12.2.b}\\
\operatorname{soc}_{3}: U_{t_{1} t_{3}} d t_{1}+U_{t_{2} t_{3}} d t_{2}+U_{t_{3}{ }^{2} d t_{3}-c d \lambda_{1}-d \lambda_{3}=\lambda_{1} d c}^{\operatorname{soc}_{4}: w d t_{1}-c d t_{3}=-t_{1} d w+t_{3} d c} \begin{array}{c}
\operatorname{soc}_{5}: r d t_{2}=-t_{2} d r+d \bar{R} \\
\operatorname{soc}_{6}:-d t_{1}-d t_{2}-d t_{3}=-d T
\end{array} . \tag{12.3.b}
\end{gather*}
$$

To determine comparative statics results, the most effective course of action is to solve for marginal effects by utilizing Cramer's Rule. With this process in mind, constructing the bordered Hessian, or matrix form, of second-order conditions is necessary:

$$
\left[\begin{array}{cccccc}
U_{t_{1}{ }^{2}} & U_{t_{1} t_{2}} & U_{t_{1} t_{3}} & w & 0 & -1  \tag{13}\\
U_{t_{1} t_{2}} & U_{t_{2}{ }^{2}} & U_{t_{2} t_{3}} & 0 & r & -1 \\
U_{t_{1} t_{3}} & U_{t_{2} t_{3}} & U_{t_{3}{ }^{2}} & -c & 0 & -1 \\
w & 0 & -c & 0 & 0 & 0 \\
0 & r & 0 & 0 & 0 & 0 \\
-1 & -1 & -1 & 0 & 0 & 0
\end{array}\right]\left[\begin{array}{c}
d t_{1} \\
d t_{2} \\
d t_{3} \\
d \lambda_{1} \\
d \lambda_{2} \\
d \lambda_{3}
\end{array}\right]=\left[\begin{array}{c}
-\lambda_{1} d w \\
-\lambda_{2} d r \\
\lambda_{1} d c \\
-t_{1} d w+t_{3} d c \\
-t_{2} d r+d \bar{R} \\
-d T
\end{array}\right] .
$$

Examining and Discussing the Marginal Effects of the Exogenous Variables ${ }^{9}$
The choice variables of the representative student worker are the three time-use variables; their marginal effects are the focus of the comparative statics analysis in this study. Because the time-use variables are affected by changes in the exogenous variables, the most logical approach to examining the student worker's time allocation decision is to determine the comparative statics results of the model and then interpret each of them in an economic setting to judge how the representative individual should behave. The following section is concerned with expanding the economic relevance of the time allocation model through comparative statics analysis.

## The Wage Rate

The first exogenous variable in the time allocation model is the wage rate, $w$. In order to observe the marginal effect of the wage rate on each of the time-use variables of the model, the bordered Hessian of second-order conditions must be modified utilizing the marginal wage effect. All other marginal exogenous effects can be momentarily disregarded. Applying this condition to the preceding matrix form of second-order conditions produces the following matrix form:

$$
\left[\begin{array}{cccccc}
U_{t_{1}{ }^{2}} & U_{t_{1} t_{2}} & U_{t_{1} t_{3}} & w & 0 & -1  \tag{14.1}\\
U_{t_{1} t_{2}} & U_{t_{2}{ }^{2}} & U_{t_{2} t_{3}} & 0 & r & -1 \\
U_{t_{1} t_{3}} & U_{t_{2} t_{3}} & U_{t_{3}{ }^{2}} & -c & 0 & -1 \\
w & 0 & -c & 0 & 0 & 0 \\
0 & r & 0 & 0 & 0 & 0 \\
-1 & -1 & -1 & 0 & 0 & 0
\end{array}\right]\left[\begin{array}{l}
d t_{1} \\
d t_{2} \\
d t_{3} \\
d \lambda_{1} \\
d \lambda_{2} \\
d \lambda_{3}
\end{array}\right]=\left[\begin{array}{c}
-\lambda_{1} d w \\
0 \\
0 \\
-t_{1} d w \\
0 \\
0
\end{array}\right] .
$$

Determining the marginal effect of the wage rate on time spent working is accomplished by solving for $\frac{d t_{1}}{d w}$. By implementing Cramer's Rule, $\frac{d t_{1}}{d w}$ is found in the following manner:

$$
\frac{d t_{1}}{d w}=\frac{\left|\begin{array}{cccccc}
-\lambda_{1} d w & U_{t_{1} t_{2}} & U_{t_{1} t_{3}} & w & 0 & -1  \tag{14.2.a}\\
0 & U_{t_{2}{ }^{2}} & U_{t_{2} t_{3}} & 0 & r & -1 \\
0 & U_{t_{2} t_{3}} & U_{t_{3}{ }^{2}} & -c & 0 & -1 \\
-t_{1} d w & 0 & -c & 0 & 0 & 0 \\
0 & r & 0 & 0 & 0 & 0 \\
0 & -1 & -1 & 0 & 0 & 0
\end{array}\right|}{\left|H^{b}\right|}=\frac{r^{2} t_{1} w+r^{2} t_{1} c}{\left|H^{b}\right|}
$$

where $H^{b}$ represents the bordered Hessian of second-order conditions. Comparative statics analysis is utilized to determine the economic significance of this effect. Because the bordered Hessian for the model contains three choice variables and three constraints, the sign of its

[^2]determinant must be negative to obtain a maximum. Therefore, applying signs to $\frac{d t_{1}}{d w}$ obtains the following result:
\[

$$
\begin{equation*}
\frac{d t_{1}}{d w}=\frac{r^{2} t_{1} w+r^{2} t_{1} c}{\left|H^{b}\right|}=\frac{(+)}{(-)}=(-)<0 \tag{14.2.b}
\end{equation*}
$$

\]

Thus, a wage rate increase causes the representative student worker to decrease the amount of time she spends working. This effect acts in accordance with the common microeconomic concept of the backward-bending labor supply curve. Though increases in the wage rate are typically met with increases in the number of hours worked at low levels of labor, the constraints placed on an individual in this model cause a redistribution of time to academics and leisure in response to a wage increase. The remaining comparative statics results of the time allocation model are displayed below in Table 1:

## Table 1: Comparative Statics Results

|  | Exogenous Variables |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Endogenous <br> Variables | $w$ | $r$ | $c$ | $\bar{R}$ | $T$ |  |
| $t_{1}$ | $(-)$ | $(+)$ | $(+)$ | $(-)$ | $(+)$ |  |
| $t_{2}$ | 0 | $(-)$ | 0 | $(+)$ | 0 |  |
| $t_{3}$ | $(+)$ | $(+)$ | $(-)$ | $(-)$ | $(+)$ |  |

Economic interpretation of the remaining results follows.
For the representative student worker, time spent studying is not affected by a change in the wage rate. At first glance, this result may appear to be slightly perplexing. As long as it can be shown that there exists an inverse relationship between time spent working and time spent relaxing with respect to a change in the wage rate, then the efficacy of the model is not jeopardized by this result.

When the representative student worker experiences a wage increase, she chooses to spend more time relaxing, or "consuming." From an economic standpoint, neglecting inflation and taxes and holding all else constant, a wage increase provides an individual with more disposable income. Obtaining a higher level of purchasing power, a rational individual will choose to purchase more normal goods; therefore, it is reasonable to conclude that the individual will also devote more time to leisure.

## The Rate of Effective Studying

The second exogenous variable in the time allocation model is the rate of effective studying, $r$. There exists a positive correlation between the rate of effective studying and time spent working. Consider the rate of effective studying to be a productivity measure related to academics; then the representative student worker chooses to work more when her ability to study more efficiently improves. In most cases, a rate of studying would be measured subjectively, in terms of an individual's perception of her own productivity. Because there is no proper means of quantifying such a measure, the rate of effective studying is simply a conceptual construct devised to assist in the observation of behavior patterns in the time allocation model.

A negative correlation exists between the rate of effective studying and time spent studying. As the student worker's study rate increases, the individual chooses to devote less time to studying. To obtain some real-world perspective on this observation, it is helpful to imagine a
scenario in which two students, completely identical in all academic abilities, characteristics, and tendencies, prepare for the same exam under different studying conditions. One student studies in a quiet environment devoid of distractions and external influences; the other student studies in a noisy environment where friends are watching television and talking loudly. Based on the equivalent abilities of the two students, the first student necessarily has a higher rate of effective studying. For this reason, the first student will require less study time in order to achieve the same level of preparedness for the exam. Assuming that the individuals in this model act rationally, it is in the first student's best interest to achieve the desired level of preparedness and study for no additional time.

There exists a positive correlation between the rate of effective studying and time spent relaxing. It is important to note that an increase in $r$ is equivalent to an increase in the price of study time, where $t_{2}$ represents the quantity of study time "purchased." When $r$ increases, the relative prices of work time and relaxation time decrease. As a result, a substitution effect is experienced by the individual, and she chooses to substitute toward "purchasing" more of the relatively less expensive goods; in this case, the individual spends more time working and relaxing.

## The Price of Consumption

The third exogenous variable in the time allocation model is the price of consumption, $c$. As expected, the marginal effect of the price of consumption on time spent working is inversely related to the marginal effect of the wage rate on time spent working. Because the first budget constraint is linear, proportionality is observed with regard to the marginal effects of the exogenous rates. An increase in $c$ is equivalent to an increase in the price of leisure time, where $t_{3}$ represents the quantity of leisure time "purchased." When observing an increase in $c$, the equality constraint, $w t_{1}=c t_{3}$, necessitates an increase in either $w$ or $t_{1}$ to maintain the balance of the relationship. Because the wage rate remains fixed when examining the marginal effect of the price of consumption, the amount of time spent working must increase. Intuitively speaking, this is a sensible result; when an individual experiences a price increase with regard to consumption, there is necessarily a greater strain on the individual's disposable income. To maintain a comparable level of disposable income with the higher price, the individual chooses to work more hours.

An increase in the price of consumption has no effect on time spent studying. This conclusion comes as no surprise due to the observed marginal effect of the wage rate on time spent studying. Because production and consumption are inverse operations and their rates are proportional in the first budget constraint, their marginal effects should be inverses, as well. If an inverse relationship can be observed between time spent working and time spent relaxing with respect to a change in the price of consumption, then the efficacy of the model will not be negatively impacted by the result of $\frac{d t_{2}}{d c}$. The economic explanations of the results, $\frac{d t_{2}}{d w}$ and $\frac{d t_{2}}{d c}$, are less than satisfactory at this moment; speculation regarding the student worker's utility function will provide greater insight into these results at a later stage of this study.

There exists a negative correlation between the price of consumption and time spent relaxing. An increase in $c$ effectively represents an increase in the price of leisure time; as a result, because spending time in leisure activities has become relatively more expensive, a rational individual will substitute away from relaxing. When relaxing becomes relatively more expensive, working obviously becomes relatively less expensive because production and consumption are inverse operations. When examining the first budget constraint, one can also
hypothesize that an individual who consumes at a higher price is not able to sustain consuming for a long period of time; thus, time spent in leisure activities must lessen in order to accommodate the heightened price of consumption.

## The Student's Desired Grade

The fourth exogenous variable in the time allocation model is the student's desired grade, $\bar{R}$. Because a student completes a number of assignments, $\bar{R}$ denotes a preferred grade average across all assignments, as determined by the respective student's set of preferences. There exists a negative correlation between academic standards and paid employment. This result corresponds with real-world expectations, for it provides insight into the varying work and academic preferences of student workers and workers. Student workers, on average, do not work as many hours as workers; by pursuing higher education, the student worker forgoes obtaining some monetary gains in the present in order to obtain potentially greater gains in the future. Observing the choice to work as a continuum, it is reasonable to assume that some student workers place higher stock in potential future gains than others. Therefore, it follows that student workers who place higher importance on achieving academic success in the present choose to work fewer hours because their labor-leisure motivations are subordinated by their academic motivations.

As a student's academic standards increase, the individual chooses to spend more time studying. Note that this result does not imply that students who study more necessarily earn better grades, and the result in no way displays a positive correlation between time spent studying and academic performance. An interpretation of this result was referenced while examining the marginal effect of the student's desired grade on time spent working; certain student workers favor academic motivations over labor-leisure motivations. Future research could observe the impact of this bifurcation on the life-cycle of the student worker.

A negative correlation exists between the student's desired grade and time spent relaxing. At first glance, it may seem puzzling that a marginal increase in the student's desired grade would negatively impact both time spent working and time spent relaxing. Because production and consumption are inverse operations, one may expect their time-use variables to shift in opposite directions regardless of the observed marginal effect. However, academic motivations of the individual are distinguished from labor-leisure motivations, so the combined weight of time spent working and time spent relaxing is measured against the weight of time spent studying. This trade-off, in essence, demonstrates the true nature of the student worker's time allocation decision, and it is the bifurcation of this decision that dictates that the student spend less time both working and relaxing when her academic aspirations increase.

## The Total Allotment of Activity Time

The fifth exogenous variable in the time allocation model is the total allotment of activity time, $T$. While $T$ can be assigned a specific value such as 24 hours for a day or 7 days for a week, it is best to maintain generality and assume that an individual performs miscellaneous activities not incorporated into the three choice activities of the model.

An increase in the total allotment of activity time causes an increase in the amount of time spent working for the representative student worker. Naturally, if an individual is granted more activity time, she will allocate it to those activities that are most beneficial from a rational standpoint. Thus, it is no surprise that an individual chooses to devote more time to working, for the marginal income of each additional hour worked is the wage rate. As a result, an increase in
the total allotment of activity time necessarily leads to an increase in the representative student worker's income, assuming that the individual responds rationally to the change.

Comparative statics analysis shows that a change in the total allotment of activity time has no direct effect on the relative amount of time spent studying for the representative student worker. One may wonder why an individual would not allocate more time toward all beneficial activities if given the opportunity. In this model, the time-use variable associated with academic pursuits is not measured against another time-use variable as those related to employment and leisure are. Because the product of the rate of effective studying and time spent studying must maintain an equivalency relationship with only the student's desired grade in the second budget constraint, an increase in total activity time does not necessitate an increase in study time. An individual, when reaching a level of academic preparedness that equals the desired grade objective, chooses to spend more time working and relaxing because those activities have tangible benefits and their relationship in the first budget constraint imposes balance on production and consumption.

An increase in the total allotment of activity time causes the representative student worker to spend more time relaxing. It is interesting that an individual faced with more activity time in this model chooses to work and relax more but study less. As previously noted, this is due to the separation of labor-leisure and academic motivations in this model. The preferred amount of study time is constrained by a constant, specifically the individual's desired grade; on the other hand, the preferred amounts of work and relaxation time are constrained by an equivalency relationship that is weighted by the wage rate and the price of consumption. Though the representative individual is a student, she still adheres to the consumer choice principle of nonsatiation. Therefore, as the student works more hours, she proportionally increases consumption under the assumption that "more is better," so long as no wage or price changes occur. Production fuels consumption and vice versa; thus, as the total allotment of activity time increases, a rational individual chooses to spend more time working and relaxing.

## Discussing the Student Worker's Time Allocation Decision

The preceding comparative statics results provide tremendous insight into the time allocation decision of the representative student worker. The following marginal effects are observed to be unambiguously positive: $\frac{d t_{3}}{d w}, \frac{d t_{1}}{d r}, \frac{d t_{3}}{d r}, \frac{d t_{1}}{d c}, \frac{d t_{2}}{d \bar{R}}, \frac{d t_{1}}{d T}$, and $\frac{d t_{3}}{d T}$. Conversely, the following marginal effects are observed to be unambiguously negative: $\frac{d t_{1}}{d w}, \frac{d t_{2}}{d r}, \frac{d t_{3}}{d c}, \frac{d t_{1}}{d \bar{R}}$, and $\frac{d t_{3}}{d \bar{R}}$. In addition, three of the marginal effects equal zero: $\frac{d t_{2}}{d w}, \frac{d t_{2}}{d c}$, and $\frac{d t_{2}}{d T}$; these effects in no way directly influence the relative values of the time-use variables. Based on the results, one may surmise that academics exist independently of employment and leisure in this model; however, because the student obtains utility from all three activities, and the values of time spent in each activity are dependently related in the time constraint, it seems as though this conclusion is not accurate. Instead, though the student worker performs three general activities, a bifurcation exists; that is, a labor-leisure tradeoff is weighed against academics in the student's time allocation decision. A fitting explanation is that the typical student worker leads a "double life," so to speak.

While changes in the wage rate and price of consumption have no direct effect on time spent studying, changes in the rate of effective studying and desired grade of the individual influence the relative values of time spent working and relaxing. The constraints are constructed in such a way that the magnitudes of production and consumption are balanced, but the
magnitude of academic preparation is measured against the academic standards of the respective student. Thus, it follows that the decision regarding study time is strongly influenced by the academic motivations of the student worker and not the exogenous effects related to the wage rate, price of consumption, and total allotment of activity time.

Economic intuition supports such an observation, dictating that a rational individual who strongly favors labor motivations over academic motivations does not choose to pursue higher education in the first place. Those individuals who do pursue higher education naturally give much consideration to time spent on academics; as a result, their decisions regarding time spent in paid employment and leisure activities are strongly influenced by academic motivations.

## Conclusion

The human understanding of economics relies on individual decision-making. In terms of microeconomic analysis, rational, self-interested individuals are fueled by the desire to maximize utility. As a result, utility maximization forms the very foundation of consumer choice. While consumer choice is typically examined in relation to production and consumption, it shares a strong connection with time allocation. After all, how individuals choose to spend their time directly impacts their market behavior. Many economic agents occupy the role of worker, allocating time between alternating segments of production and consumption; however, some individuals also allocate time toward academic pursuits. Because academic pursuits necessarily exist outside of the scope of traditional labor-leisure choice, the time allocation decision of the student differs from that of the worker.

The model of this study intuitively explains the bifurcation found in the student worker's time allocation decision. Considering the nature of the time and budget constraints placed on the representative student worker, the marginal effects of the wage rate and price of consumption behave as expected with regard to the individual's labor-leisure decision; an increase in the wage rate causes a decrease in time spent working and an increase in time spent relaxing, while an increase in the price of consumption causes an increase in time spent working and a decrease in time spent relaxing. At the same time, the student worker's decision to spend more or less time studying is not affected by changes in the wage rate and price of consumption. The representative student worker only chooses to spend additional time studying if her academic standards increase or her ability to study effectively diminishes.

Observing labor-leisure choice and academic choice in this model allows one to form significant conclusions regarding the preferences of the typical student worker. The relative weight of a student worker's academic motivations dictates how she addresses her time allocation decision. Having higher academic standards draws the individual toward studying and away from working and relaxing, while having lower academic standards has the opposite effect. Here, an individual's choice to work and relax more characterizes her behavior as being "present-oriented," while an individual's choice to study more characterizes her behavior as being "future-oriented." It is this assessment of expected utility that reveals the true nature of the student's time allocation decision. Overall, this model provides valuable information about human behavior and individual preferences by shedding light on the motivations driving the typical student worker's time allocation decision.

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[^0]:    1 This research was undertaken in partial fulfillment of the requirements for graduation through the University Honors College with Distinction at Middle Tennessee State University during the Fall semester of 2013. Dr. Eff acted as Honors Faculty Advisor.
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    4 Labor-leisure choice refers to the decision made by an individual as to how many hours out of the day to work. The choice is modeled assuming that consumers desire leisure as well as the consumption of goods (Silberberg, 2001).

[^1]:    7 Complementary slackness conditions imply that strictly satisfying a constraint causes the corresponding Lagrangian multiplier, $\lambda_{i}$, to equal zero. On the other hand, if $\lambda_{i}>0$, then the constraint must equal zero.

[^2]:    9 When implementing Cramer's Rule to solve for marginal effects, determinants were solved using wxMaxima.

