NUMERICAL ANALYSIS OF FREE CONVECTION IN A SQUARE AND IN A RIGHT- ANGLE TRAPEZOIDAL ENCLOSURE FILLED WITH POROUS MEDIUM

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ABSTRACT:

A numerical study is conducted to investigate the natural convection heat transfer in a two cases of the enclosure ,the first case in a square enclosure of aspect ratio AR=1 and the second case in a right-angle trapezoidal enclosure with aspect ratios AR=0.45 and 0.25 .The enclosure was filled with a liquid saturated porous media .The bottom wall of the cavity was heated with a sinusoidal temperature distribution $\theta=0.5(1-\cos(2\pi x))$, the vertical wall was cooled at $\theta=0$ and the other walls were adiabatic. The governing equations were solved numerically using finite element software package (FLEXPDE). Flow and heat transfer characteristics are studied for the range of Rayleigh number ($100 \leq Ra \leq 1000$). Streamlines, isotherms and Nusselt numbers were presented. The obtained results show that the heat transfer coefficient increases with increasing of Rayleigh number and aspect ratio. A comparison of the flow field and isotherm field was made with that obtained by (Yasin et al., 2008), which revealed a good agreement.

KEYWORDS: Free convection, trapezoidal enclosures, porous medium

تحليل عددي للحمل الحر داخل غلاف مربع وداخل غلاف معيني دو زاويه قائمه مملوء بالمادة المتساميه

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الخلاصه :

اجريت دراسة عدديه للحمل الحر لحالتين ، الحاله الاولى داخل غلاف مربع الشكل ذو نسبة بعد AR=1 والحالة الثانية داخل غلاف معيني قائم الزاوية له نسبة إبعاد AR=0.45 و AR= 0.25 . الغلاف مملوء بمائع مشبع متسامي .الجدار السفلي للغلاف مسحن بدرجة حرارة غيرمنتظمة $\theta=0.5(1-\cos(2\pi x))$ والجدار العلوي عند درجة حرارة $\theta=0$ اما الجدران الاخرى فتكون معزولة . استخدمت الحقيبة البرمجية (FLEXPDE) لحل منظومة المعادلات الحاكمة لعملية انتقال الحرارة عدديا" بطريقة العناصر المحددة .تم دراسة الجريان ومعامل انتقال الحرارة لمدى محدد من رقم رايلي ($100 \leq Ra \leq 1000$). اظهرت النتائج التي تمثلت بخطوط الجريان و خطوط التحارر بان معدل انتقال الحرارة يزداد بازدياد

كل من AR وRa. قورنت النتائج المستحصلة مع ما منشور في المصدر (Yasin et al., 2008) واظهرت تقارب جيد.

NOMENCLATURE

AR Aspect ratio parameter H/L
g Acceleration due to gravity m/s^2
H Length of the top wall of the cavity m
K Permeability of the porous medium m^2
L Length of the bottom wall of the cavity m
Nu_L Local Nusselt number
Nu_m Mean Nusselt number
Ra Rayleigh number $Ra=g\beta k\,(T_h-T_c)/\upsilon\alpha$
T Temperature of the fluid porous medium
U, V Velocity components, uL/α , vL/α
X, Y Non-dimensional coordinates, X=x/L, Y=y/L

Greek Symboles

α Thermal diffusivity of porous media m^2/s
β Thermal expansion coefficient K^{-1}
θ Dimensionless temperature, $\theta=(T-T_c)/(T_h-T_c)$
υ Kinematics viscosity m^2/s
ψ Dimensionless Stream function

Subscripts

c Cold
h Hot
L Local
m Mean

INTRODUCTION:

Buoyancy induced convection and fluid flow in enclosures filled with fluid –saturated porous media can be seen in many applications in , solar power collectors , geothermal applications , double wall insulation , electric machinery , cooling system of electronic devices , natural circulation in the atmosphere , molten core of the earth , induces fibrous insulation , food processing and storage , thermal insulation of the buildings , geophysical systems , electro chemistry , metallurgy , the design of pebble bed nuclear reactors , underground disposal of nuclear or non-nuclear waste , etc. A great number of studies are related with the analysis of natural convection in square and rectangular enclosures. Some important numerical results can be found in the studies by (**Bejan and Tien, 1978**). (**Poalikakos and Bejan, 1983**) reported exact analytical solution of natural convection in an attic-shaped space filled with porous material. Natural convection heat transfer and fluid flow was studies for porous or non-porous medium trapezoidal enclosures mostly at differentially heated temperature boundary conditions see (**Kuypor and Hoogendoom, 1995**). (**Sadat and Salognac, 1995**) , (**Moukalled and Acharya, 2001**) , (**Bayates and Pop ,2001**) , (**Moukalled and Darwish , 2003**), (**Kumar and Kumar, 2004**) , (**Baytas et al.,2000**) presented a numerical study of natural convection in a two dimensional right angle triangular cavity filled with a fluid saturate porous medium using the Darcy–Bousseinesq equation media. Some studies also have been performed to investigate the effect of Boundary conditions and the effect of inclinations on natural convection in square/rectangular enclosures filled with porous media or viscous fluids (**Saeid, 2005**) and (**Oztop and varol, 2007**). The natural convection heat transfer in triangular enclosure has been studied by (**Varol et al., 2007**). Number of papers on natural convection flows in trapezoidal cavities filled with porous media (**Basak et al., 2009**) and (**Varol et al., 2009**). A numerical study of the steady buoyancy –induced flow and heat transfer in a trapezoidal cavity filled with a porous medium saturated with cold water has been performed by

(**Yasin et al., 2010**). Thus the main objective of this paper is to examine the natural convection in a square cavity and in a right-angle trapezoidal filled with a porous medium. Different Rayleigh numbers and different aspect ratios were considered in these studies. The governing equations were solved numerically using finite element software package (FLEXPDE)

GOVERNING EQUATIONS:

A schematic of the two dimensional square enclosure and a right angle trapezoidal enclosure filled with saturated porous medium is shown in Fig.1 (a) and (b) respectively. The bottom wall of the cavity was heated at sinusoidal temperature distribution and the top wall was cooled, while the remaining walls were insulating.

The governing equations for the study, two dimensional, incompressible flows with Darcy-Boussinseq approximation and constant fluid properties can be written in non-dimensional form as follows:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \qquad (1)$$

$$\frac{\partial^2\psi}{\partial X^2}+\frac{\partial^2\psi}{\partial Y^2}=-Ra\frac{\partial\theta}{\partial X} \qquad (2)$$

$$\frac{\partial\psi}{\partial Y}\frac{\partial\theta}{\partial X}-\frac{\partial\psi}{\partial X}\frac{\partial\theta}{\partial Y}=\frac{\partial^2\theta}{\partial X^2}+\frac{\partial^2\theta}{\partial Y^2} \qquad (3)$$

Where the dimensionless variables are defined as:

$$X=\frac{x}{L},\ \ Y=\frac{y}{L},\ \ U=\frac{uL}{\alpha},\ \ V=\frac{vL}{\alpha},\ \theta=\frac{T-T_c}{T_h-T_c} \qquad (4)$$

Here X and Y are the Cartesian coordinates along the horizontal and vertical walls of the enclosure, respectively, U and V are the velocity components along the X- and Y- direction , θ is the non-dimensional fluid temperature , ψ is the non-dimensional stream function , which is defined as :

$$u=\frac{\partial\psi}{\partial y},\quad v=-\frac{\partial\psi}{\partial x} \qquad (5)$$

Where k is the permeability of the porous medium and α is the diffusivity of the fluid. The boundary conditions of Eqs (1-3) shown in Fig.1 are:

1-No-slip velocity boundary condition, U=0, V=0 and ψ=0 on all solid walls.

2- Sinusoidal temperature distribution on the bottom wall θ=0.5 (1-cos (2πx)), θ=0 on the top wall and $\frac{\partial\theta}{\partial n}=0$ on the other walls; where n denotes the direction normal to the adiabatic walls of the enclosure.

Besides the streamlines and isotherms, the physical quantities of interest in this problem are also the local Nusselt number Nu_L, and the mean Nusselt number Nu_m from the heated wall, which are given by:

$$Nu_L = -\left(\frac{\partial\theta}{\partial Y}\right)_{Y=0} \quad , \qquad Nu_m = \int_0^1 Nu_L\, dx \tag{6}$$

NUMERICAL SOLUTION:

In the present study, a finite element software package Flexpde (**Backstrom, 2005**) is relied in the solution of the nonlinear system of equations (2) and (3). Hence, the continuity equation (1) is used to check the error of the solution throughout the grids of domain.

SOFTWARE VALIDATION

To check the validation of software, three grid densities were examined 10^{-3}, 10^{-4}, and 10^{-5}. The grid dependency was checked with the continuity equation and the obtained results show the exactly validated of the velocity distribution for each above grid densities. i.e example, the griddled domain for the trapezoidal enclosure of AR=0.25 is shown in **Fig.2a** and the distribution of the values of ($\partial U/\partial X + \partial V/\partial Y$) over the domain, is presented in **Fig.2b**.

CODE VALIDATION

To check the validity of the numerical results, test calculations were performed for the triangular cavity filled with porous medium for Ra=1000. The boundary condition of the bottom wall is non-isothermally heated wall $\theta=0.5(1-\cos(2\pi x))$ and the vertical wall at $\theta=0$, while the inclined wall is kept adiabatic. The program was verified by a comparison with a theoretical work performed by (**Yasin et al., 2008**) as shown in Fig.3, who developed numerical results for the streamlines and isotherms in the triangular cavity filled with porous medium. The results show a very good agreement and from this comparison it can be decided that the current code can be used to predict the flow field for the present problem.

RESULTS AND DISCUSSION:

The results of the flow fields and heat transfer for porous non-isothermally heated square and trapezoidal enclosures are studied in this paper. The temperature of the bottom wall is taken as sinusoidal temperature distribution $\theta=0.5(1-\cos(2\pi x))$ and the upper wall is $\theta=0$ while the other walls are taken as adiabatic . **Fig.4 (a)** shows the streamlines (on the left) and the isotherms (on the right) in a porous square enclosure for AR=1 and Ra=100. It can be noted from this Figure the double circulation cells were formed in vertical position, and the rotating direction is starting from middle of the bottom wall. For the trapezoidal enclosure AR=0.45 and 0.25 as given in **Fig.4b**(left) and **Fig.4c** (left) respectively , two cells were formed inside the enclosure with different rotation direction , the right cell is the dominated in the cavity . The left cell is located at the bottom corner due to effect of inclination of the left wall. The intersection of the isotherm lines near the corners with the bottom wall show that there is no heat transfer to the fluid , though most of the heat is transfer from the bottom wall toward the fluid. For small Rayleigh number the temperature distribution is the same as for pure conduction case, the isotherms show wavy variation and no vortices are observed. As shown from **Fig.4a**(right), for square cavity, the isotherms have a plum like distribution formed in middle of the bottom wall toward the upper wall , while for trapezoidal cavity AR=0.45 and 0.25 the plumes have skew ness toward the inclined adiabatic wall as shown in **Fig.4b**(right) and **Fig.4c**(right) respectively . **Fig.5** shows the effect of the aspect ratio on the flow and temperature fields for high Rayleigh number Ra=1000. As can be seen from the streamlines plot of **Fig.5a**(left) , that two circulation cells were formed and inclined at angle 68 ^{o} from horizontal , the flow strength is higher than that of **Fig,4a**(left) . Also it can be seen from **Fig.5 (b)** (left) and **Fig.5c** (left), the size of the cells is decreased with decreasing of the aspect ratio. Isotherms plot of **Fig.5a** (right) shows that, with high Rayleigh number the plum like distribution which generated from the bottom wall is occupied the whole space due to the increasing of the convection heat transfer. Also it can be seen from **Fig.5 b**(right) and **Fig.5 c**(right), the skew ness of the plum toward the left corner of the enclosure is more than that of **Fig.4 b**(right) and **Fig.4 c**(right) due to the dominant of the natural convection with increasing of Rayleigh number .

Variations of the local Nusselt number along the bottom wall for square and trapezoidal cavity at different values of aspect ratio and Rayleigh number are presented in **Figs.6 - 9** . As shown from Fig.6 , for the square enclosure AR=1 and with Ra=100, a symmetric variation of the local Nusselt number along the bottom wall is formed due to the symmetry in the temperature filed. The values of the local Nusselt number are negative near the corners of the bottom wall since the direction of the heat transfer is reversed in this rejoin. A positive value of the local Nusselt number is observed in the main part of the bottom wall depends on the recirculation intensity. It is clear from this figure , the value of Nu_L is small at the middle of the bottom wall due to the reducing of the recirculation intensity. Also it can be seen from **Fig.6**, for the trapezoidal enclosure AR=0.45 and AR=0.25, the maximum value of the local Nusselt number occurs at the right half part of the bottom wall. The values of the local Nusselt number are increases with increasing of the trapezoidal aspect ratio due to the effect of the inclined adiabatic wall. The heat losing from the left corner of the trapezoidal cavity is decreased with decreasing of the aspect ratio. **Fig.7** shows the variation of the local Nusselt number along the bottom wall for Ra=1000. The trend of the curves is the same as that in **Fig.6** , but the values of the local Nusselt number are higher. **Figs. 8-9** show the increases of the local Nusselt number with increasing of Rayleigh number along the heated wall of the square and trapezoidal enclosure respectively due to dominant of the natural convection. The negative values of the local Nusselt number near the corners of the bottom wall of the square cavity are increases with increasing of Rayleigh number as shown in **Fig.8** this is because of the increasing of the intensity of reversal stream lines. As can be seen from **Fig.9**, at the left corner of the trapezoidal cavity the losing of the heat is increased with increasing of Rayleigh number.

However, the local Nusselt numbers are negative at the corners of the bottom wall due to losing of the heat; positive values are observed in most part of the bottom wall which depends on the direction of the recirculation. Thus, the mean Nusselt numbers were found to be positive and presented in **Fig.10**. It can be seen from this **figure**, the mean Nussselt number for the square enclosure are higher than of trapezoidal enclosure. This is because of the heat transfer from the fluid losing through the bottom wall of trapezoidal cavity is higher then of the square cavity.

CONCLUSIONS:

A numerical study was performed to investigate the flow and temperature fields for the square and trapezoidal porous enclosure. Based on the obtained results in the present study, findings are:

1-The flow and temperature fields are strongly depending on the aspect ratio and Rayleigh number. For the square cavity asymmetry distribution of the streamlines and isotherms, while the distribution of the flow for the trapezoidal cavity is occur near the left part of the bottom wall.

2-Aspect ratio affects the amount of the circulation mass inside the enclosure and the heat transfer is increases with increasing of the aspect ratio ,this mean that the heat transfer from the bottom wall of the square enclosure is higher then that of the trapezoidal enclosure .

3-The overall heat transfer coefficient is an increasing function of Rayleigh number. Conduction becomes dominants at small Rayleigh number.

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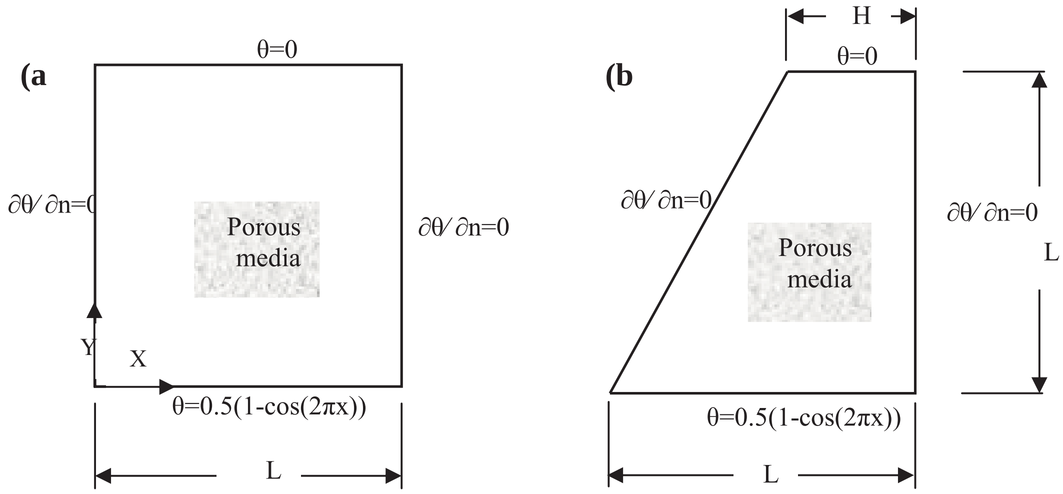

Fig.1 Schematic diagram of the physical domain a) square cavity b) trapezoidal cavity

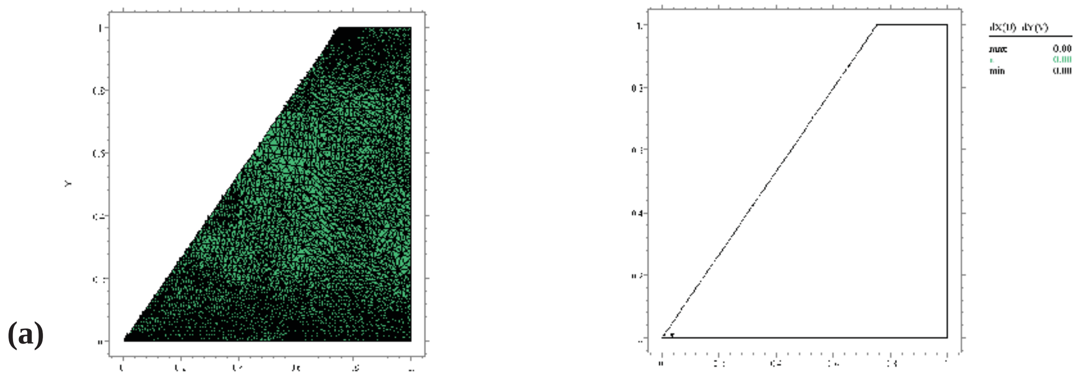

Fig. 2 For 10^{-5} accuracy (a) grid distribution over the domain (b) validation of

(Yasin et al. , 2008)

Present

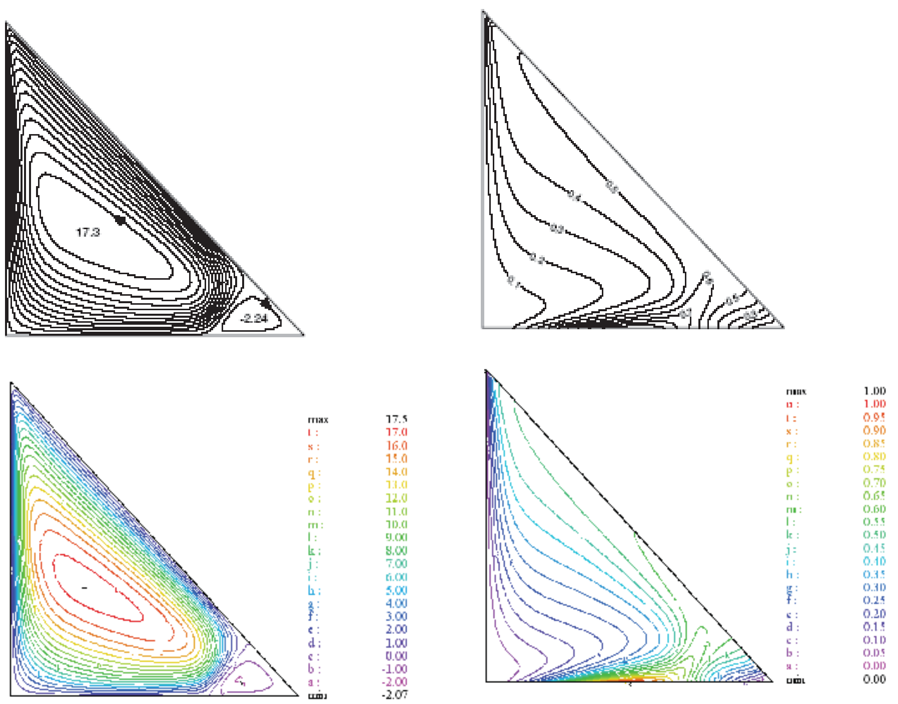

Fig. 3. Comparison of streamlines and isotherms at Ra =1000 with results of (Yasin et al., 2008)

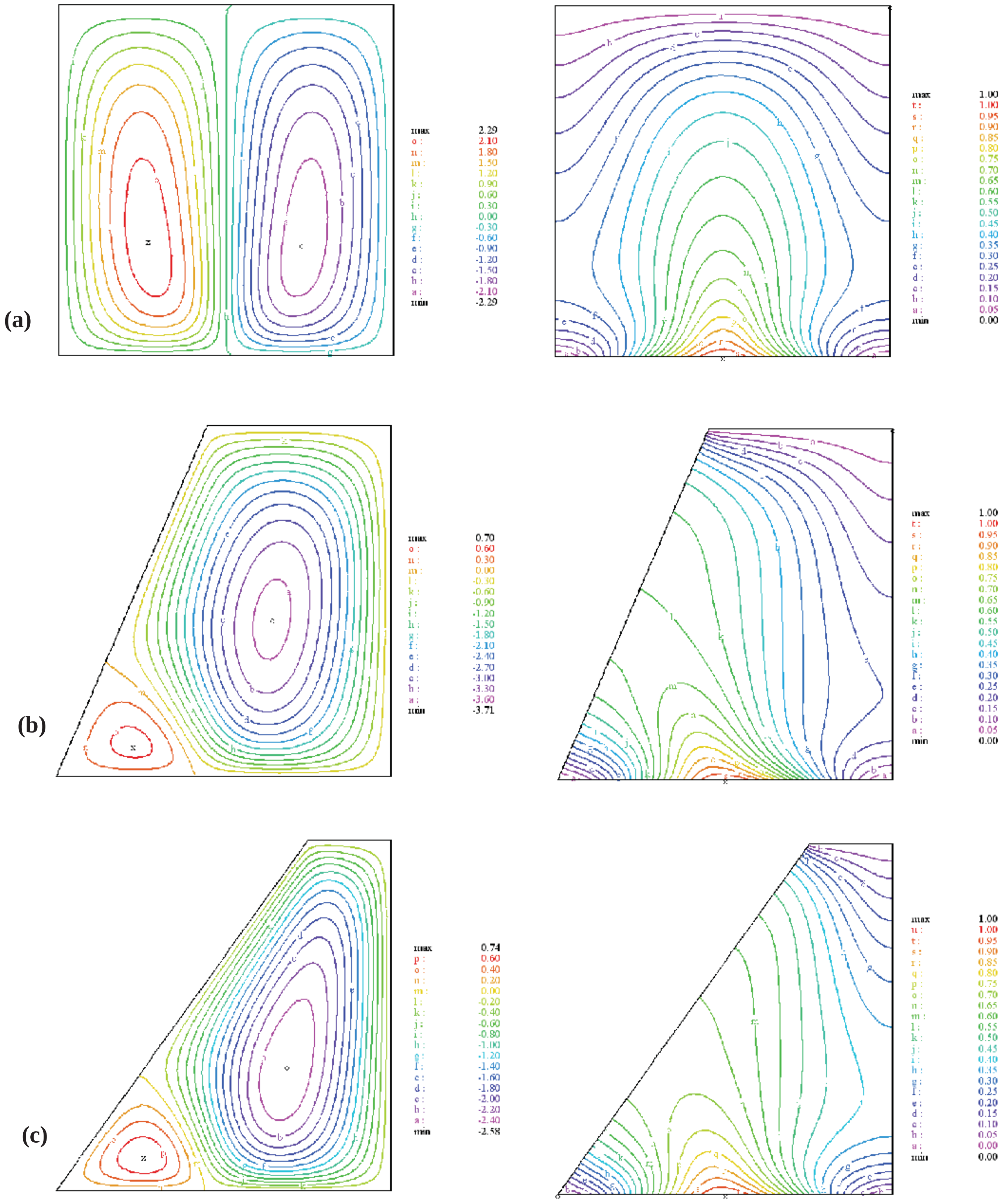

Fig.4 Streamlines (left) isotherms (right) at Ra=100 a) AR=1 b) AR=0.45 c) AR=0.25

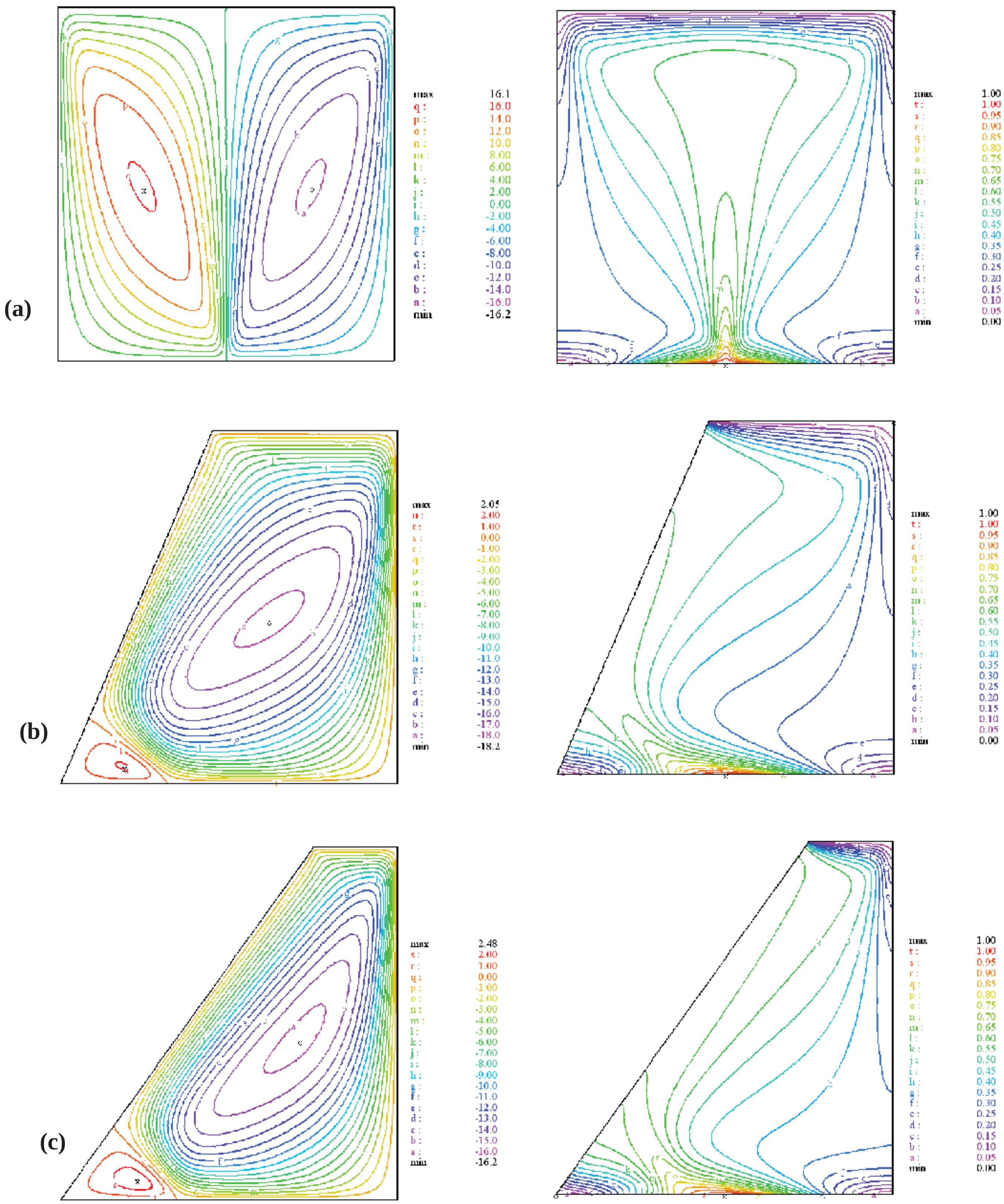

Fig.5 Streamlines (left) isotherms (right) at Ra=1000 a) AR=1 b) AR=0.45 c) AR=0.25

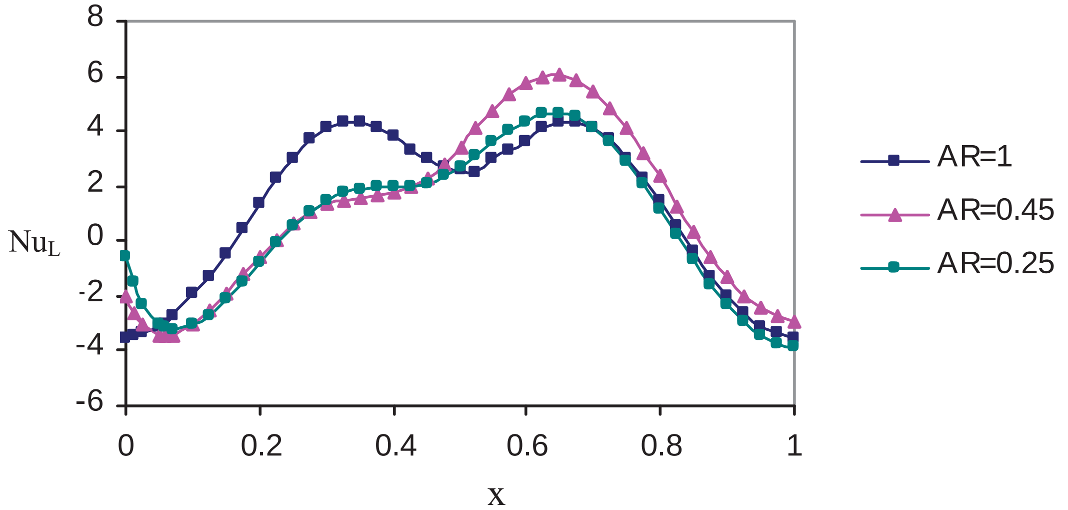

Fig.6 Variation of local Nusselt number along the bottom wall , Ra=100

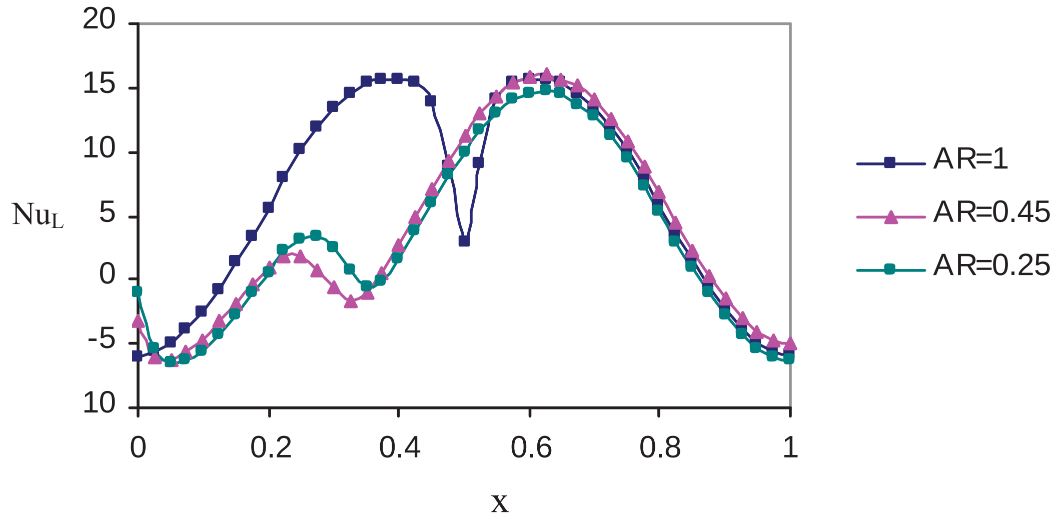

Fig.7 Variation of local Nusselt number along the bottom wall , Ra=1000

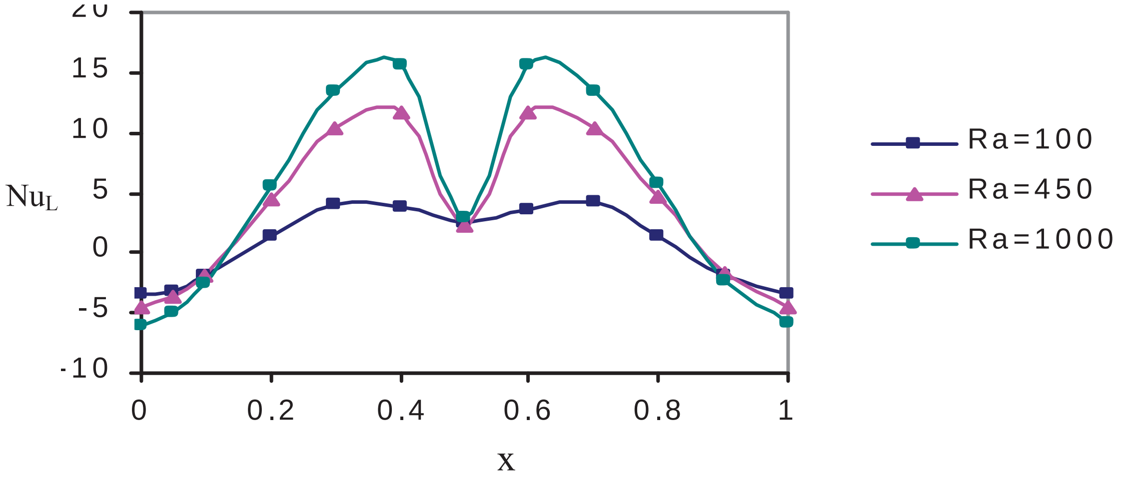

Fig.8 Variation of the local Nusselt number along the bottom wall of the square enclosure , AR=1

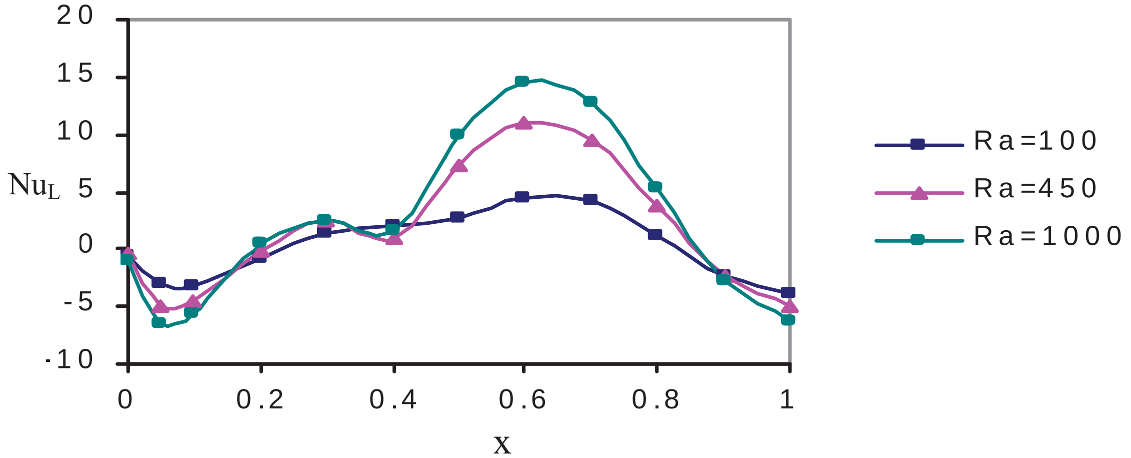

Fig.9 Variation of the local Nusselt number along the bottom wall of the trapezoidal enclosure , AR=0.25

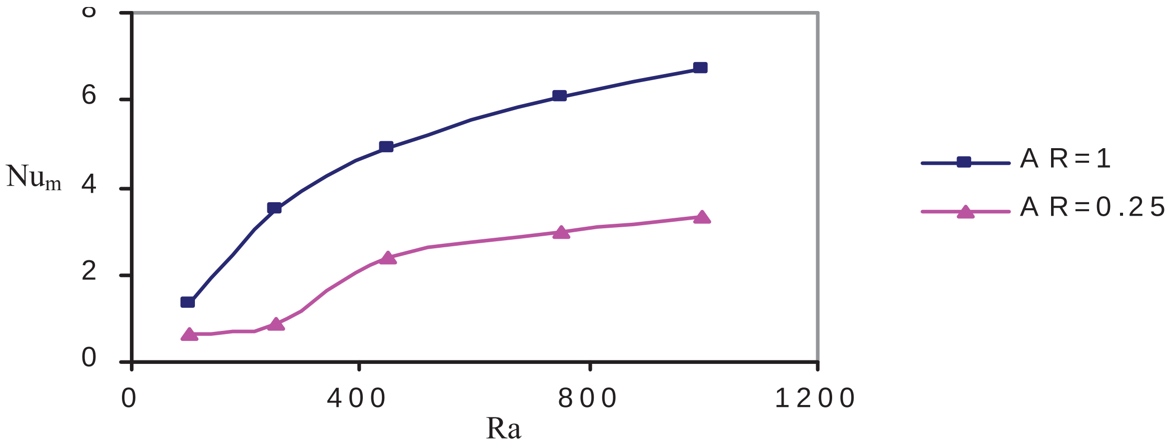

Fig.10 Variation of the mean Nusselt number with Rayleigh number for the square enclosure AR=1 and the trapezoidal enclosure AR=0.25 .