

NUMERICAL SOLUTIONS FOR POTENTIAL FLOW PASTS 2 – D LIFTING AIRFOILS

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Abstract

The ability of the panel methods in solving the potential flow problem around 2-D lifting airfoils is tested in this study. The approximate solutions for potential flow pasts thick, thin, symmetrical, and non-symmetrical airfoils have been calculated by using panel methods and then compared with either the exact analytical solution or the numerical solution obtained by using a perturbation method. As a results, the linear-varying strength vortex method is seemed to be the better one in precision in solving the all four problems.

حلول عددية لجريان جهدي حول سطوح رفع هوائية ثنائية الأبعاد

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تدريسي في كلية الهندسة- جامعة الكوفة

الخلاصة

هذا البحث يفحص قابلية طرق الصفيحة في حل مشكلة الجريان الجهدي حول أسطح رفع هوائية ثنائية الأبعاد. الحلول التقريبية للجريان الجهدي حول أسطح هوائية سميكة و نحيفة و متناظرة و غير متناظرة حسب استخدام طرق الصفيحة و من ثم قورنت إما مع الحل التحليلي الحقيقي او مع حل عددي باستخدام طريقة الاضطراب. كنتيجة نهائية طريقة الدوامة ذات الشدة الخطية بدت الأحسن بالدقة في حل كل المسائل الأربعة.

Nomenclature

C_p	pressure coefficient
L.E	leading edge
T.E	trailing edge
c	airfoil chord
x	horizontal coordinate
α	angle of attack
ϵ	thickness ratio

Subscripts

i	collocation point
j	singularity element
n	number of panels
∞	free stream

Introduction

Due to the applicability of the results of the potential flow problem solution in airfoil design, this problem has received a good deal of attention. Also, due to the phenomenal growth in the use of computers, a method that is valid for completely arbitrary geometry is needed. Thus, panel methods have come to stay. In the panel methods the airfoil contour has been approximated by inscribed polygon. That is, the contour has been approximated by a number of straight-line segments. Each segment is distributed by singularities of certain kind that have an undetermined strength. These singularities deflect the oncoming stream so that it will flow around the airfoil. The requirement the oncoming flow be tangent to every segment at a particular location (collocation point) gives a set of equations which is used to compute the singularities strength. Thus the flow consisting of a uniform flow and the flow induced by the singularities on a finite number of segments becomes determined and the velocity and the pressure at any point in the flow field can be calculated. The requirement of the flow must be tangent to the segments at the collocation points leads to the normal component of the flow must be zero at these collocation points. This is the physical boundary condition (that is, the normal component of the fluid velocity must be zero on the solid boundaries). Therefore, two boundary conditions can be used to solve the Laplace's equation and they are; the Neumann B. C. which utilizes the physical B. C. directly, and the Dirichlet B. C. which specifies the velocity potential on the boundaries so that indirectly zero normal flow will be met.

Hess and Smith [1] is a review of such methods. Both source and vortex distributions have been used. The two-dimensional surface source method [1] approximates the body contour by an inscribed polygon. The number and distribution of the surface elements determine the accuracy of the numerical solution. The surface source strength is assumed to be constant on each element, and the zero normal velocity boundary condition is applied at the midpoint of each element (collocation point) to produce a set of linear algebraic equations for the values of the source strength on the elements. For lift problem an auxiliary constant-strength vortex singularity element has been distributed. The total strength of this distribution is adjusted to specify the Kutta condition, which requires the equality of the surface pressures at the collocation points adjacent to the trailing edge. Some difficulties have been encountered for airfoils that have very thin trailing edges. Moreover, the difficulties have been made more severe by the use of the relatively small element numbers that are appropriate for the three-dimensional cases. Even in the two-dimensional cases reduced element numbers are becoming more desirable to conserve computing time in multielement airfoil cases. One technique for increasing the computational accuracy is the so-called higher-order formulation, which is described in [2] for non-lifting case. While higher-order surface singularity distributions for lifting cases is described in [3].

Formulation of The Problem and its Numerical Solution

The general problem of calculating the incompressible, 2-D potential flow requires the solution of Laplace's equation ($\nabla^2\Phi = 0$) for the velocity potential Φ , with boundary conditions of zero normal velocity component on all solid surfaces ($V_n=0$), and zero velocity perturbation at large distances from the airfoil ($V_\infty=Constant$). Auxiliary conditions (Kutta conditions) are also imposed to fix the value of the circulation about the airfoils.

The numerical solution can be constructed by six ordinary steps, and they are:

1-Selection of the Singularity Element. The first and one of the most important decisions is the type of singularity element or elements that will be used. This includes the selection of source, doublet, or vortex representation and the method of discretizing these distributions (constant, linear, or quadratic). Six singularity distributions have been used in this study and they are; constant-strength doublet, constant-strength vortex, combined constant-strength source and doublet, linear-varying strength doublet, linear-varying strength vortex, and quadratic-varying strength doublet and for more details it is preferred to return to [4].

2-Descretization of Geometry. Once the basic solution element is selected, the geometry of the problem has to be subdivided such that it will consist of those basic solution elements. In this generation process, the elements' end points and the collocation points are defined. The collocation points are points where the boundary conditions, such as the zero normal flow to a solid surface, will be enforced.

3- Influence Coefficients. The influence coefficient a_{ij} is the normal component of the flow velocity induced by a unit strength element j at a collocation point i . In this phase, for each of the elements an algebraic equation (based on the boundary condition) is derived at the collocation point.

4- Establish the Right- Hand Side (RHS). The RHS of the matrix equation is the known portion of the free-stream velocity or the potential and requires mainly the computation of geometric quantities.

5- Solve Linear set of Equations. The influence coefficients and the RHS equations are obtained in the previous steps and now the equations are solved by a standard matrix technique (Gauss-elimination technique).

6- Secondary Computations. The solution of the matrix equation results in the singularity strengths and the velocity field and any secondary information can be computed. The pressure will be computed by the Bernoulli's equation, and the loads and the aerodynamic coefficients will be computed by adding up the contributions of the elements.

Actually the results of the first four steps mentioned above is the following system:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ \cdot & & & & \\ \cdot & & & & \\ a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \cdot \\ \cdot \\ \sigma_n \end{bmatrix} = \begin{bmatrix} \text{RHS}_1 \\ \text{RHS}_2 \\ \cdot \\ \cdot \\ \text{RHS}_n \end{bmatrix}$$

Where a_{ij} is the influence coefficient
 σ_j is the singularity unknown strength

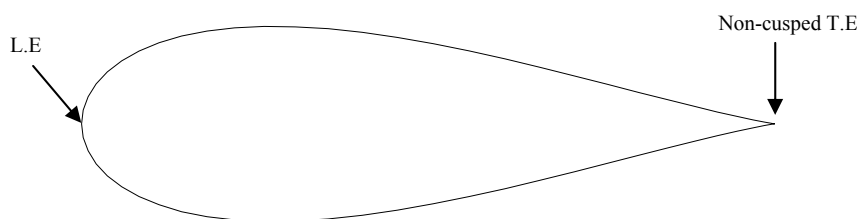
The above system of equation have been solved by using the Guassian elimination method for σ_j

Fortran 77 programming language have been used to solved the problem, just few second have been consumed to get upon the final results.

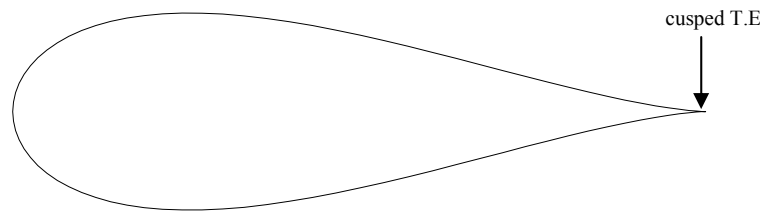
The Airfoils Used in the Study

Four types of airfoils were used in this study in solving the thickness, thinness, symmetry, and non-symmetry problems and they are:

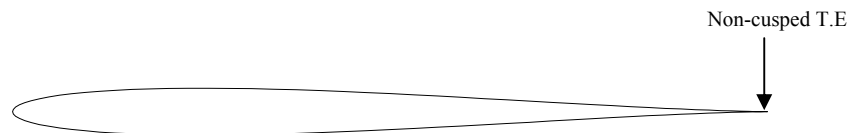
A-Van de Vooren airfoil with non-cusped trailing edge: To illustrate the effectiveness of the methods used in solving the problem for ordinary shapes of airfoils with finite trailing edge angles.



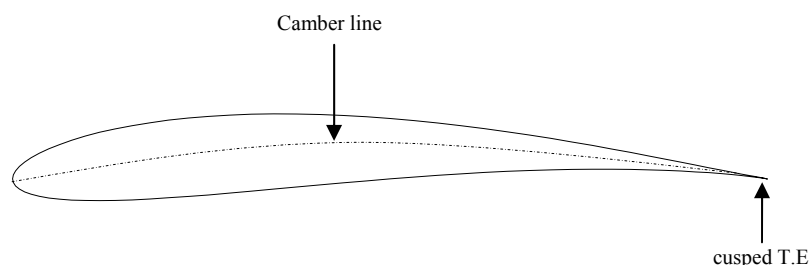
B- Van de Vooren airfoil with cusped trailing edge: To illustrate the effectiveness of the methods used in solving the problem for ordinary shapes of airfoils with zero trailing edge angles.



C- Symmetric Joukowski airfoil: To illustrate the effectiveness of the methods used in solving the problem for thin airfoils with cusped trailing edge.



D- Cambered Joukowski airfoil: To illustrate the effectiveness of the methods used in solving the problem of camber for thin airfoils with cusped trailing edge.



Numerical Results and Discussion

The pressure distribution for the seven surface singularity methods on a thick Van de Voorn airfoil with non-cusped trailing edge and with cusped trailing edge are shown in **Fig. (1)** and **Fig. (2)** respectively. The agreement between the analytical solution [5] and the numerical solution of the panel methods for the first case is excellent as shown in **Fig. (1)** especially for ($n=60$). That is due to the collocation points position be near the surface area of the airfoil where the analytical solution have been calculated (i.e, in the panal method, the collocation points are some where inside the airfoil counter and they will be nearest to the airfoil counter as the number of the used panel have been increased). Some deviation of the numerical solutions from the analytical solution appears near the trailing edge as shown in **Fig. (2)**, and that is due to the very thin region there (cusped) and a very high interaction of adjacent singularities will appear.

Depending on the earlier results, the pressure distributions for two surface singularity methods on thin symmetrical and non-symmetrical (cambered) airfoils are shown in **Fig. (3)** and **Fig. n(4)** respectively. Generally, very good agreement between the solutions obtained by these methods and the solutions obtained by method of [6].

CONCLUSIONS

1-The methods depending upon the Dirichlet B.C. are better than the methods depending upon the Neumann B.C. in the solution of the potential flow around thick airfoils without cusped trailing edges, especially the constant-strength doublet method because of the ease construction, low computational effort, and good results even for low density of panels.

2-The linear-varying strength vortex method with the Neumann B.C. and the constant-strength doublet, combined constant-strength source and doublet, and quadratic-varying strength doublet methods with Dirichlet B.C. can be used to solve the potential flow problem around symmetric thick and thin airfoils with cusped trailing edges. But only the linear-varying strength vortex method with Neumann B.C. and the combined constant-strength source and doublet method with Dirichlet B.C. can be used to solve the potential flow problem around asymmetric and thin airfoils with cusped trailing edges.

3-From the above conclusions, the decision of the type of the elementary solution distribution and the boundary condition selection depends upon the case under testing.

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6. T. C. Wong, C. H. Liu, and J. Geer, "Comparison of Uniform Perturbation and Numerical Solutions for Some Potential Flows Past Slender Bodies," Computers & Fluids, Vol. 13, No. 3, pp. 271-283, 1985.

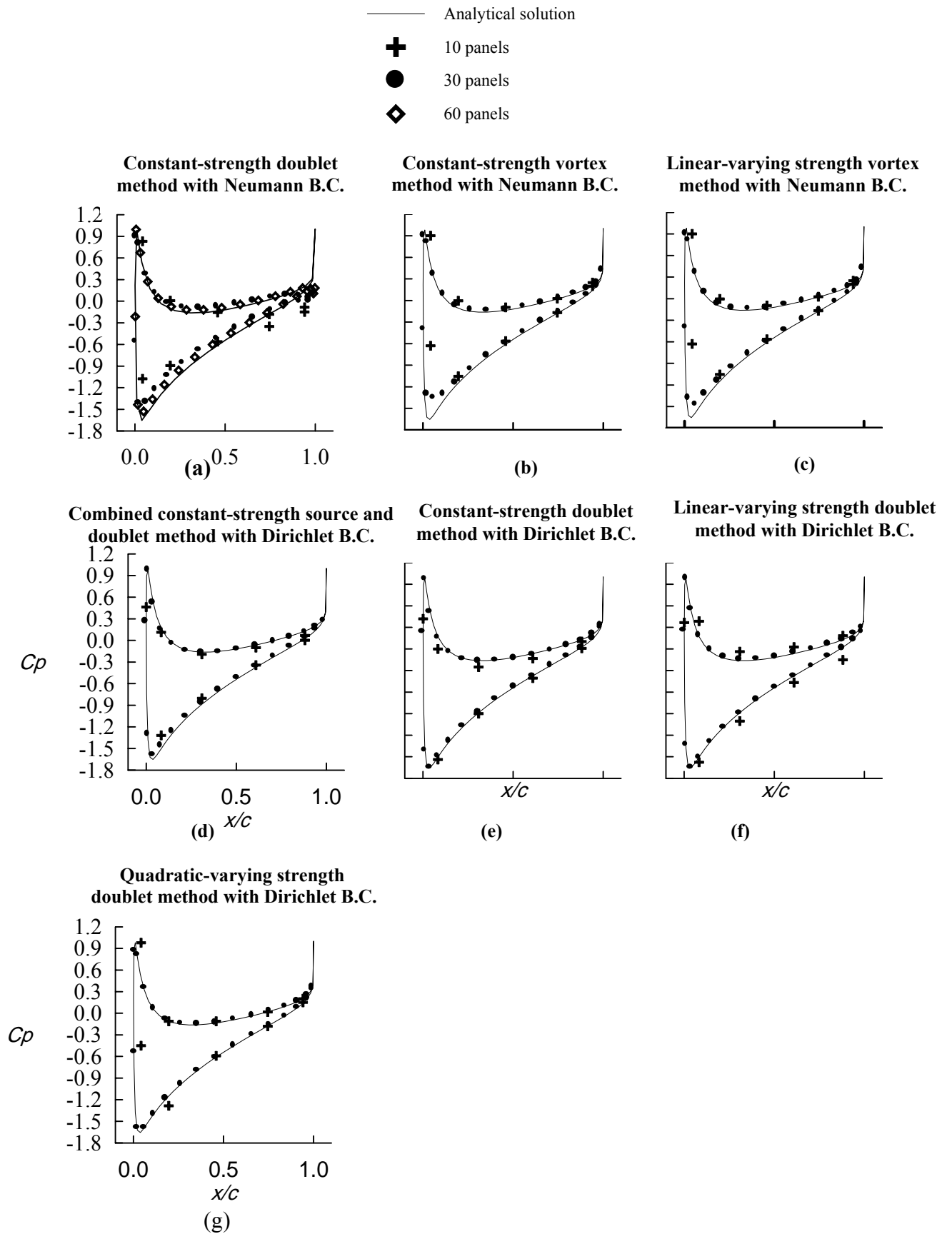


Fig. 1 Pressure coefficient distribution for a 15% thick Van de Vooren airfoil with non-cusped trailing edge at an angle of attack (α) of 5° .

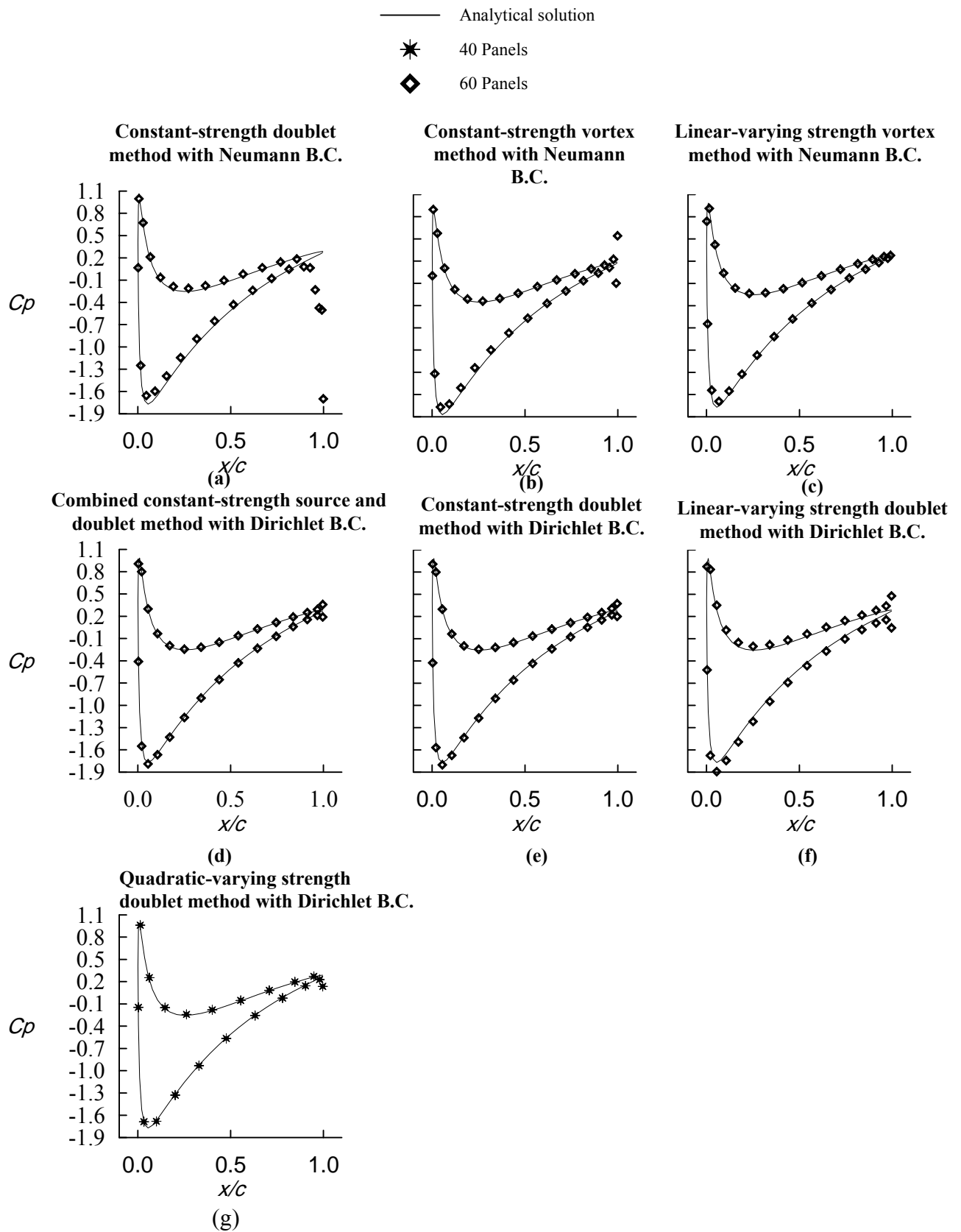


Fig. 2 Pressure coefficient distribution for a 15 % thick Van de Vooren airfoil with cusped trailing edge at an angle of attack (α) of 5° .

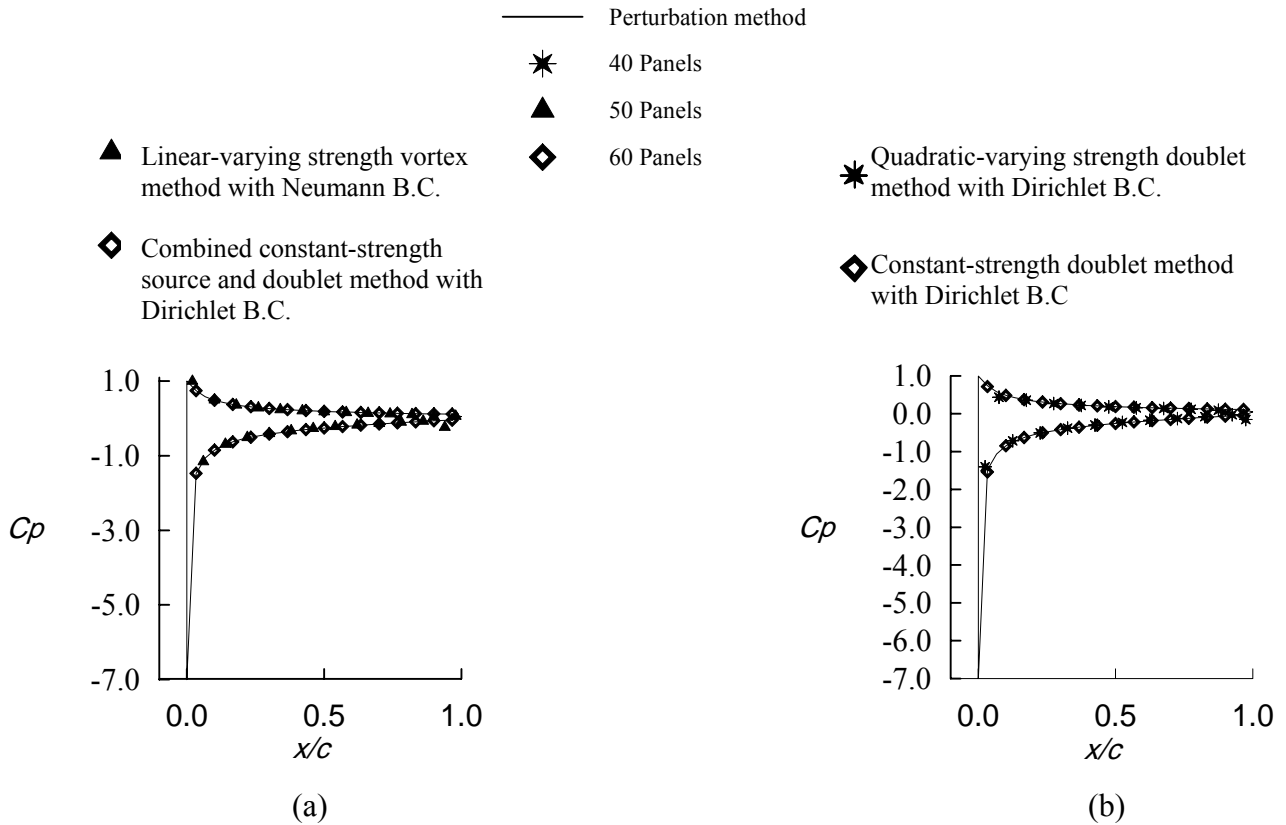


Fig. 3 Pressure coefficient distribution for a thin ($\epsilon = 0.01624$) and symmetric Joukowski airfoil with cusped trailing edge at $\alpha = 6^\circ$.

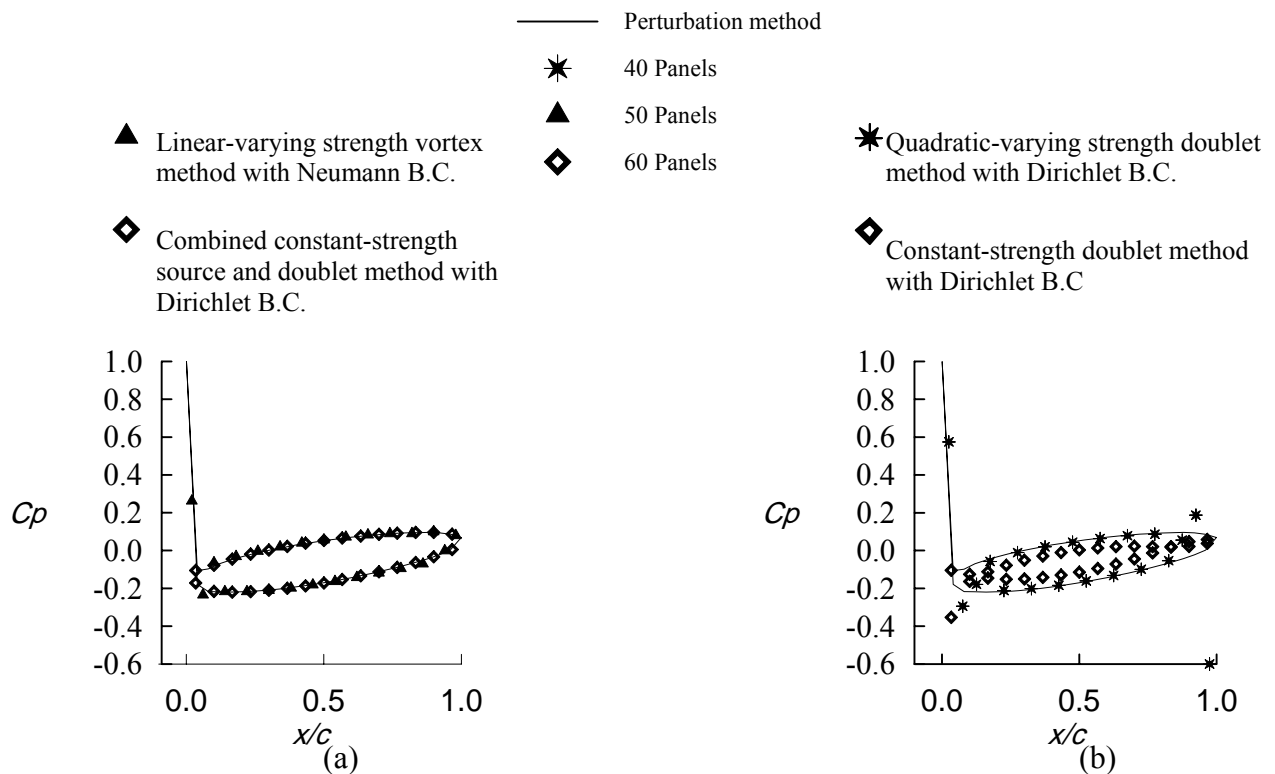


Fig. 4 Pressure coefficient distribution for a thin ($\epsilon = 0.02688$) and cambered Joukowski airfoil with cusped trailing edge at $\alpha = 0^\circ$.