

A WIDEBAND MODEL FOR ESTIMATING THE DIRECTION OF ARRIVAL IN THE PHASED ARRAY ANTENNA

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Abstract

In this paper, a new wideband direction-of-arrival (DOA) estimation method is proposed, related to the real time capability of DOA estimator the complexity estimation of the algorithms is used. This includes the calculation of different algorithms complexity and its applicability, taking into account frame and slot times, minimum number of samples and estimation bandwidth. The complexity of the algorithms as a function of key parameters is used. Based on these estimated functions, the execution time of the algorithms was calculated. Through the use of MATLAB package, which is used as reference for the calculation a DSP system with 400 MFLOPS processing power. Most algorithms can be executed within 400 μ s. It's provided also a method of two step estimation for further reduction of the execution time. Through this approach, a significant reduction was achieved for some combination of algorithms.

Key words: wideband, direction of arrival, array antenna, estimation, DOA.

(MATLAB)

.(DOA)

400 Mflops

(DSP)

.400 μ s

Introduction

Due to the LMDS (Local Multipoint Distribution System) system standardization process being held during the present year, the proposed system is based on the main standard documents: the European ETSI BRAN HIPERACCESS [Acampora et. al.] and the American IEEE 802.16.1 WirelessMAN [Deliverable D2.1]. As both systems have enough common characteristics, a single modified system was defined here. The coverage area is divided in rectangular cells of four 90° sectors, each of them covered by one or more sectorial antennas. The physical layer is a combination of Time Division Duplex (TDD), Time Division Multiplex (TDM) in the downlink and Time Division Multiple Access (TDMA) in the uplink. Therefore, the channel bandwidth is time-slotted and users are required to provide explicit or implicit information regarding their needs for bandwidth. The bandwidth management is based on granted access and access requests in contention or polling mode and the traffic is classified in a similar way as in Asynchronous Transfer Mode (ATM). The intended bit rate is up to 155 Mbps, employing multilevel modulations. As a TDD frame, the uplink and downlink transmissions share the same frequency, but are separated in time. The frame has a fixed length, but the length of each sub-period is a system parameter that may be dynamically modified. The frame time is 1ms. **Table (1)** reports the main network (cellular) level characteristics of the proposed system.

The available bandwidth in both directions is defined with a granularity of one time-slot, which is a multiple of 4 modulation symbols. The pulses are shaped employing a raised-cosine filter with a certain roll-off factor. The modulation rate is selected in order to obtain an integer number of time-slots within each frame.

The baud rate depends on the roll-off factor and the channel size. The channel bandwidth is 28MHz taking into account a 1 ms frame duration and a roll-off factor of 0.25. Therefore, for a symbol rate of 22.4 Mbaud (symbol period is 0.0446 μ s) the duration of each time slot will be equal to 0.179 μ s or a multiple of this value. The downlink sub-frame starts with a preamble for synchronization and equalization duties, based in one or more 16-symbols Constant Amplitude Zero Auto-Correlation (CAZAC) sequences. The preamble is followed by the protocol control data, which mainly maps the assignation of the different time intervals in the frame. Finally, the data is transmitted in physical strength order. The downlink transmission is governed by the base station (BS) and therefore operates in a broadcast mode, free of collisions. The data are transmitted in order of modulation robustness. The control information is always transmitted with the most robust physical mode.

1. Slot-Based Beam Switching Approach.

The main issue when the proposed beamforming capabilities of the beamformer are introduced in the BS, is how to cover a 90° sector employing a directive antenna. In this research, a solution that employs a space switched single-beam antenna covering the whole sector from the base station instead of the traditional omnidirectional or sectorial antenna is proposed, as depicted in **Figure (1)**. This approach differs from others in that the BS covers the service region with a narrow beam, which can be rapidly scanned to a number of different positions depending on the geographical user position [D3.1, D3.2]. This beam-switching approach matches perfectly to a time based system, as the BS antenna is illuminating only in the desired direction and just at the appropriate time.

As seen previously, the frame multiplex and access schemes are a combination of TDM in the downlink and a TDMA in the uplink (both in the same radio-channel as the TDD operation mode was chosen). In both downlink and uplink a protocol data control zone and a data transmission zone may be distinguished. The downlink control information is used to manage the system, to grant the required bandwidth by users and to map the different zones of the frame. The uplink signaling zones are used for unsolicited access in a contention mode, or unicast or multicast polling, for bandwidth request and configuration issues, etc. The control information zone must be broadcasted to all the active users in the sector. An also, all the users present in the sector should be able to access into the signaling zone. This is clearly infeasible with a beam-switched antenna as it

only points to a certain portion of the sector at a given instant of time. Two approximations to overcome this problem were identified [D5.1] and investigated [ETSI TS101-999 V1.1.1, Godara]: the inter-frame or frame-based beam-switching and the intra-frame or slot-based beam-switching.

The frame-based beam-switching is based onto the sub-sectorization concept. The sub-sectorization consists on dividing a 90° sector in, e.g. four 22.5° degrees sectors. Each sub-sector will act as an independent sector. This way each beam position may be viewed as a kind of sub-sector, but only one of them can be active at each instant of time. However, with the given latency requirements of these systems (given by the frame duration of 1ms) the frame duration in each sub-sector, dependent of the number of beam positions, would lead to very inefficient bandwidth solutions as shown in [Godara].

The slot-based beam-switching solution has been chosen as the most optimal approximation for the convergence of the reference standards. This solution is based onto the control information replication idea. In this way, the control information would be retransmitted for all the beam positions in a kind of scanning. As may be seen in **Figure (2)** the use of the channel is shared between the equivalent sub-sectors but during a frame, as the beam is pointed to the desired terminal, which is equivalent to an "overall-sector" frame. However, this replication introduces protocol overhead and might make this approximation also unfeasible. The minimization of the control information in the slot-based beam-switching is required. An adapted Medium Access Control (MAC) protocol was developed in to minimize the control information replication [Godara].

It should also be remarked that the proposed slot-time based switching implies that a switching time in the ns range is required. For a given bandwidth and bit rate the efficiency depends on the ability of pointing the beam to different users as quick as possible during the guard time between slots. The guard time is 8 symbols [Acampora et. al.], so that, for the symbol rate of 22.4 Mbaud, the guard time will be 357 ns. Therefore switching times on the ns range are required.

2. DOA Estimation Requirements

The proposed system deals also with the following problem: in mobile systems, tracking becomes a must. The system must decide when a mobile should be pointed with an adjacent beam instead of the one being used, so it is a discrete tracking. DOA estimation must be done at uplink transmission. It can be based on a training sequence of constant power or on a pilot tone of lower frequency modulated on the carrier frequency (40 GHz), but in any case the duration of these signals should be much lower that the time assigned to each user inside each uplink sub-frame:

- * TRAINING SEQUENCE: the training sequence could be the burst preamble added in the uplink data burst. The short burst preamble consist of 16 or 32 symbols (0.71 and 1.43 μ s respectively), which are a repetition of 8 or 16 CAZAC sequences that are transmitted using the four corner points of the modulation constellation (maximum power).
- * PILOT TONE: the short-duration tone will be located after the burst preamble. The main drawback is that an additional hardware would be necessary in the user equipment in order to generate the pilot tone. One possible approach to overcome this drawback is to generate an additional training sequence (with a total length of 8, 16 symbols) after the burst preamble, so that, choosing an appropriated sequence (for instance 1, -1, 1, -1 ...) a pilot tone can be generated at lower integer multiples of the symbol rate, i.e. 11.2, 5.6, 3.733 and 2.8 MHz.

DOA Estimation at IF/BB Level

The conventional DOA estimation approach, which measures the maximum output power of different beams pointing to different angular positions, has fundamental resolution limitation. The digital signal processing (DSP) based algorithms such as subspace based methods or parametric methods have the high resolution required to achieve an accurate DOA estimation to resolve closely spaced signal sources. Thus, the estimation quality can be improved using powerful algorithms. An

accurate estimation is also a very useful task for the network management. The other advantage of the DSP based approach is the flexibility that different algorithms can be easily replaced without additional hardware, only loading new software code. In addition DOA estimation can be directly applied for the high bandwidth data signals. Consequently, no additional hardware for the transmission of extra pilot signal is required.

The estimation of the direction of arrival is an essential part for steering and forming the field distributions of array antennas in mobile communication systems. The spatial distribution of the mobile subscribers has to be determined with high accuracy in order to determine the desired look directions and (in case of beam forming) the nulls for suppressing interfering signals. In the previous deliverable [Horneffer and Plassmann], we proposed the set up and measurement concept for DOA estimation experiment using base band signal processing. Critical parameters are pointed out and we explore the solutions for proper adaptation to the system architecture. The implementation of the base band signal processor has been already finished.

Study on Adaptation Aspects to The Proposed System.

We can identify the critical points concerning adaptation to the system architecture as following:

- a) According to the system specifications, the real time capability of the system is an important issue to switch to its user simultaneously. Thus, the calculation of the processing power of the algorithms is a useful estimate for the assessment of the real time ability.
- b) The amount of the samples that are available within allocated time duration of each user must be estimated.
- c) The proposed system provides a high bandwidth data transmission up to 155 Mbit/s. The base band processor of the DOA estimator has a limited small bandwidth compared with that data rate. This fact seems to be a critical point for the practical implementation of the system.

1. Computational Complexity Estimation

The real time ability of a digital system is a topic related to the computational complexity of the algorithms. The applicability of the algorithms in real time may be assessed by estimating their execution time [IEEE 802 Committee]. As a result of this estimation, the complexity can be formulated as a function of some key parameters extracted from the algorithms. The choice of an algorithm and the amount of samples for the proper estimation are also related topics to the result of complexity estimation.

In the digital signal processing, the complexity of an algorithm can be simply estimated by counting all involved operations in the calculations. As a result of this estimation, the complexity is given as a function of key parameters, such as number of array elements, number of samples and amount of angular space, in which the maximum value is searched. For counting operations, the following rules are valid:

- * Additions and subtractions are one FLOP if the operands are real and two if complex. Multiplications and divisions count one FLOP each, if the result is real and six FLOPS, if it is not. Elementary functions count one if real and more if complex.
- * First, we verify some key calculations of the mostly used functions in the algorithms.
 - a) The calculation of sample covariance matrix requires $2M^2(4N-1)$ operations.
 - b) Singular Value Decomposition (SVD) needs $10(M^3+3.3M^2+13M-40)$ operations.
 - c) The process searching for maximum value requires $\log(N_\theta)$ operations.

The used parameters above are the number of antenna elements M , the number of samples N , the number of azimuth angles N_θ , the number of repeated loops N_{rep} , and the number of users $K=1$. The complexity estimation is performed for the following algorithms: BARTLETT, Multiple Signal Classification (MUSIC), Estimation of Signal Parameters via Rotational Invariance Techniques (ESPRIT) and Alternating Projection (AP) [Kortke and Schubert].

The estimation results are given as a function of above mentioned parameters and listed in the **Table (2)**. A calculation example about the required number of operations can be taken from **Table (3)**. In this calculation, following parameters are used:

- * number of array elements $M=4$.
- * number of samples $N=300$ bits.
- * number of azimuth angles $N_{\theta}=181$ (between -90 degrees and 90 degrees with 1 degree angular resolution).
- * number of loop repeats $N_{rep}=5$.
- * number of signal $K=1$.

For the calculation of the execution time, we also assume that a Digital Signal Processing (DSP) system with a processing power of 400 MFLOPS is used. Such DSP system is usually available in the market (as shown in **Table (4)**). The most algorithms may be executed for above parameters within $400 \mu s$. But the AP algorithm needs more processing power, while the ESPRIT algorithm requires the smallest calculation time because of no need of the search process.

2. Two Step Estimation For Reducing Complexity

In case of single user, the estimation process can be partitioned in two steps. First, we make a rough estimate with a simple algorithm, e.g. BARTLETT with a rough angular resolution (e.g. angular resolution 5 degrees). As a result of this estimation, we obtain an angular range to be estimated accurately. In the second step, we select a small angular range based on the angular range estimate of the first step. We apply a high resolution estimation algorithm or the same algorithm only for this small angular range. The main advantage of this approach is that we reduce the complexity for the same angular estimation accuracy (as shown in **Figure (3)**). Basically, the rough estimation can be achieved in the analogue domain as well as in the digital domain.

An estimation result can also be taken from the **Table (4)**. The complexity estimation is carried out for the previous parameters. The combined approach BARTLETT/AP shows a significant reduction of complexity compared with the estimation of AP alone, although the estimation accuracy remains the same.

3. The Amount Of Signal Samples Related To The Real Time Ability

Assuming that both uplink and downlink sub-frames together count as 1 ms frame and several users are transmitting on each uplink sub-frames. We can expect that each user is transmitting during tens of milliseconds on each frame. For instance, if we further assume that the duration of the each uplink and downlink sub-frames is 0.5 ms and 10 users are active within this sub-frame time duration, the allocation time of the each users T_{user} for obtaining the signal samples for DOA estimation may be $50 \mu s$. Note that the base band processor of the DOA experimental system has a constant sampling frequency $f_{sample}=16$ MHz. The amount of samples available within the allocation time T_{user} can be calculated as following:

$$N_{sample} = T_{user} \times f_{sample} = 800 \text{ complex valued samples} \quad (1)$$

Whether or not, the amount of samples is sufficient for the proper estimation.

4. Bandwidth Considerations

The array receiver of the DOA estimation experimental system has a small bandwidth compared with the transmitted data signal provided in the proposed system. The transmitted channel bandwidth is 28 MHz and the proposed bit rates are between 44.8 Mbit/s and 134.4 Mbit/s corresponding to QPSK and 64-QAM using a roll-off factor of 0.25 (see **Tables (1)**). Assuming that no extra pilot signal or training sequence are available for DOA estimation, the transmitted high rate data signal or the synchronization signal must be used for the estimations, which provides much higher bandwidth than the DOA estimator. Suppose that single user signal is active, the received signal at the array antenna can be written as:

$$S_{Re}(t) = s(t)a(\theta) \tag{2}$$

where $a(\theta)=[1, \exp(j\mu_0), \dots, \exp(j(M-1)\mu_0)]T$ is the steering vector and $\mu_0=\pi \sin(\theta)$ is the spatial frequency. Because of the small bandwidth of the base band array receiver, only a small part of this transmitted spectrum (2.5 MHz) will be filtered. Thereby, the sampling theorem can not be obeyed. Consequently, we have to under-sample the signal, so that the transmitted signal can not be detected correctly. In the base band, the received signal will be sampled with a constant sampling frequency f_{sample} (16 MHz) and a sampling period T_{sample} (62.5 ns). The base band signal can now be written as:

$$S_{Re}(kT_{sample}) = S(kT_{sample})a(\theta) \tag{3}$$

However, as we can see from the equation (3), the DOA information remains unchanged after under-sampling. As a result, we can also perform the DOA estimation for high rated data signals directly without exploiting a pilot signal. Thus, the bandwidth of the DOA estimator is not a critical parameter.

Calibration of the Array

Before the DOA estimation is carried out in the real environment, the array must be calibrated, because the measured data may include errors such as [Robey et. al, Ziskind and Wax]:

- * Offset error.
- * Different amplification of individual antenna signals.
- * Different phase shift of individual antenna signals.

Since the calibration is accomplished in the digital domain, no additional hardware is needed.

a) Correction of offset error.

If signals incident on the array antenna with enough power and time duration, we can calculate the mean power of in phase- and quadrature-components for each signal path of the array receiver. The mean powers for each signal path can be written by:

$$\bar{m}_{i,k} = \frac{1}{N} \sum_{n=1}^N x(n)_{i,k} \tag{4}$$

$$\bar{m}_{q,k} = \frac{1}{N} \sum_{n=1}^N x(n)_{q,k} \tag{5}$$

where i and q the indexes for in phase- and quadrature-components, k for the array number, N the number of samples. Thus, the correction of offset error can be achieved by subtracting the mean power from the measurement vector \mathbf{x} .

$$\tilde{x}_{i,k} = x_{i,k} - \bar{m}_{i,k} \tag{6}$$

$$\tilde{x}_{q,k} = x_{q,k} - \bar{m}_{q,k} \tag{7}$$

b) Compensation of different amplifications of individual antenna signals.

Unfortunately, the subcomponents of DOA estimator have not the desired same characteristics. These differences affect unequally each of signal paths. Thus, the received signal powers of different paths must be compensated. First, the signal power for in phase- and quadrature-components must be calculated separately as following:

$$P_{i,k} = \frac{1}{N} \sum_{n=1}^N |\tilde{x}(n)_{i,k}|^2 \tag{8}$$

$$P_{q,k} = \frac{1}{N} \sum_{n=1}^N |\tilde{x}(n)_{q,k}|^2 \tag{9}$$

After that, we calculate the normalizing factors $d_{i,k}$ and $d_{q,k}$ for in phase- and quadrature-components.

$$d_{i,k} = \frac{1}{\sqrt{P_{i,k}}} \tag{10}$$

$$d_{q,k} = \frac{1}{\sqrt{P_{q,k}}} \tag{11}$$

The compensation of the amplifications can be achieved now by multiplying the signals $\tilde{x}_{i,k}$ and $\tilde{x}_{q,k}$ with the normalizing factors $d_{i,k}$ and $d_{q,k}$.

$$\tilde{x}_o^G = d_{i,k} \tilde{x}_{i,k} + j^* d_{q,k} \tilde{x}_{q,k} \tag{12}$$

c) Elimination of phase errors

The ideal signal output vector has the following form in the noiseless case:

$$\mathbf{x} = \mathbf{a}(\theta) s(t) \tag{13}$$

where $s(t)$ is the transmitted signal and \mathbf{a} is the steering vector with:

$$\mathbf{a}(\theta) = [1, e^{-j\pi \sin(\theta)}, \dots, e^{-j(K-1)\pi \sin(\theta)}]^T \tag{14}$$

The signal output vector of a real system with phase errors can be formulated as following:

$$\mathbf{x}_{real} = \mathbf{\Theta} \mathbf{a}(\theta) s(t) = \begin{pmatrix} e^{j\Delta\phi_1} & & & 0 \\ & e^{j\Delta\phi_2} & & \\ & & \ddots & \\ 0 & & & e^{j\Delta\phi_k} \end{pmatrix} \mathbf{a}(\theta) s(t) \tag{15}$$

If a single signal incidents on the array, the vector V_p of the covariance matrix R_{xx} must be the estimate of $\mathbf{\Theta} \mathbf{a}(\theta)$ for the real system. The vector V_p contains the eigen values of the matrix $\mathbf{\Theta} \mathbf{a}(\theta)$. If a single signal is transmitted from the azimuth angle at θ degrees, the diagonal elements of the matrix $\mathbf{\Theta}$ are corresponding to the elements of vector V_p because of $\mathbf{a}(\theta) = [1, 1, \dots, 1]$. From this, the phase errors are determined. The phase errors can be compensated by multiplying the matrix K with the array output vector \tilde{x}_o^G .

$$x = K \tilde{x}_o^G$$

.where

$$K = \begin{pmatrix} v_{p,1}^* & & & 0 \\ & v_{p,2}^* & & \\ & & \ddots & \\ 0 & & & v_{p,K}^* \end{pmatrix} \quad (16)$$

For the phase calibrations of the other angular directions the same formula (15) can be applied by the following modification:

$$\tilde{x}_{real} = x_{real} \otimes \mathbf{a}(\theta_k)^* = \mathbf{Q}\mathbf{a}(0)s(t) \quad (17)$$

where the operator \otimes denotes component wise multiplication and $\mathbf{a}(\theta_k)^*$ is the conjugate of the a priori known steering vector of the currently measuring directions.

Estimated Results

A calibration example and estimation errors are shown in **Figure (4)** as a function of azimuth angles. The measurements were performed in the angular range between -45 degrees and 45 degrees with 5 degree angular steps. The number of use samples N is 1000 . The used estimation algorithms are ESPRIT, MUSIC, Alternating Projection and Bartlett algorithms. Estimation errors are obtained by applying the calibration steps above. The calibration errors over whole angular ranges are below 0.2 degrees.

In **Figure (5)** the Root Mean Square Errors (RMSE) are shown as a function of the number of samples and the signal to noise ratio (SNR). The number of samples N is varied between 10 and 4500 . The signal noise ratio is changed between -10 dB and 10 dB in 5 dB steps. Single user signals in 5 degrees angular distance are measured. The estimation errors over all measured angular range are averaged. The used estimation algorithm is MUSIC. At high SNR values, the estimation performance is nearly not influenced by the number of samples, since there is a big performance difference at low SNR values. For all cases, 320 samples are a good choice for the reasonable performance.

Figure (6) shows the estimation result performing high bandwidth signal. The maximum estimation error is smaller than 0.4 degrees. The errors are derived from the imperfect calibration. However, there is no limitation because of the bandwidth as one have expected. Therefore, DOA estimation can be applied directly for high bandwidth received data signals as well as for pilot signals.

Conclusions

The adaptation aspects on the DOA estimation at IF/BB level have been studied in this point. The DOA estimation at IF/BB level has been investigated. The adaptation aspects of the DOA estimation using base band signal processing have been studied. We identified critical points concerning the implementation and adaptation to the system proposed here.

Related to the real time capability of DOA estimator the complexity estimation of the algorithms is performed. This includes the calculation of different algorithms complexity and its applicability, taking into account frame and slot times, minimum number of samples and estimation bandwidth. As a result, the complexity of the algorithms could be given as a function of key parameters. Based on these estimated functions, the execution time of the algorithms was calculated. As reference for the calculation a DSP system with 400 MFLOPS processing power was used. Most algorithms can be executed within 400 μ s. Also, a method of two step estimation for further reduction of the execution time is provided. Through this approach, significant reduction was achieved for some combination of algorithms.

Assuming that each user is continuously transmitting during 50 μ s of the up-link and taking the sampling frequency of the DOA experimental system of 16 MHz into account, the amount of the available samples N within this allocation time may be 800 complex valued samples. By means of the simulated results, the number of samples $N=320$ was identified as a good choice for proper estimation.

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Table (1). Report of the main proposed system parameters.

Number Of Sectors For Cells	Four 90° sectors	
	Cell Size	Mobile
	fixed	1 km
Frequency of Operation	40 GHz	
Duplexing Method	TDD	
Access Method	TDMA	
Frame Duration	1 ms	
Channel Size	28 MHz	
Bit Rate/Modulation Format	155 Mbps /Multi-level	

Table (2). Complexity estimation for the DOA algorithms as a function of key parameters [Kortke and Schubert, Li and Stoica, Milligan].

Algorithm	Estimate Computational Operations
BARTLETT	$2M^2(4N - 1) + N_\theta(M + 1)[6M + 2(M - 1)] + \log(N_\theta)$
MUSIC	$2M^2(4N - 1) + 10(M^3 + 3.3M^2 + 13M - 40) + (M - 1)M[6(M - 1) + 2(M - 2)] + N_\theta(2M + 1)[6M + 2(M - 1)] + \log(N_\theta)$
ESPRIT	$2M^2(4N - 1) + 10(M^3 + 3.3M^2 + 13M - 40) + 125(M^3 + 2M^2 - 20M - 36)$
Alternating Projection	$2M^2(4N - 1) + N_{rep} \{ N_\theta [2M + 32(M + 0.55) + 6M^2 + M^2 [6M + 2(M - 1)] + 2M] + \log_{10}(N_\theta) \}$

Table (3). Complexity estimation example: $N_\theta=181$, $N=300$, $M=4$, $N_{rep}=5$ and $K=1$.

Algorithm	Number of Operations [FLOPS]	Calculation Time by [400 MFLOPS/s]
BARTLETT	65520	163 μs
MUSIC	88790	221 μs
ESPRIT	37160	92 μs
Alternating Projection	705910	1764 μs
BARTLETT+AP: Rough estimate (resolution 5°) and Accurate estimation (resolution 1°)	156050	399 μs
BARTLETT+ BARTLETT: Rough estimate (resolution 5°) and Accurate estimation (resolution 1°)	47070	117 μs

Table (4). An example of DSP system (source: Analog Device).

	ADSP-21065L	ADSP-21160N	ADSP-21161L
Clock Cycle	66 MHz	95 MHz	100 MHz
Instruction Cycle Time	15 ns	10.5 ns	10 ns
MFLOPS Sustained	132 MFLOPS	380 MFLOPS	400 MFLOPS
MFLOPS Peak	198 MFLOPS	570 MFLOPS	600 MFLOPS
1024 Point complex FFT (Radix with bit reversal)	279 μ s	97 μ s	92 μ s
FIR Filter (per tap)	15 ns	5.2 ns	5 ns
IIR Filter (per biquad)	61 ns	21 ns	20 ns
Matrix Multiply (pipelined)			
$[3 \times 3] * [3 \times 1]$	136 ns	47 ns	45 ns
$[4 \times 4] * [4 \times 1]$	242 ns	83 ns	80 ns
Divide (y/x)	91 ns	31 ns	30 ns
Inverse Square Root	136 ns	47 ns	45 ns

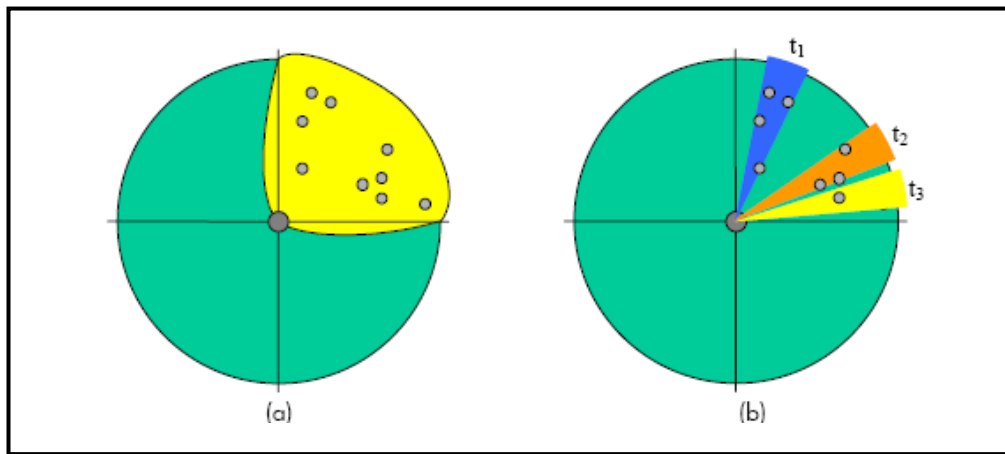


Figure (1). (a) Traditional sector with a 90° antenna. (b) Our modification employing a beam-switching antenna.

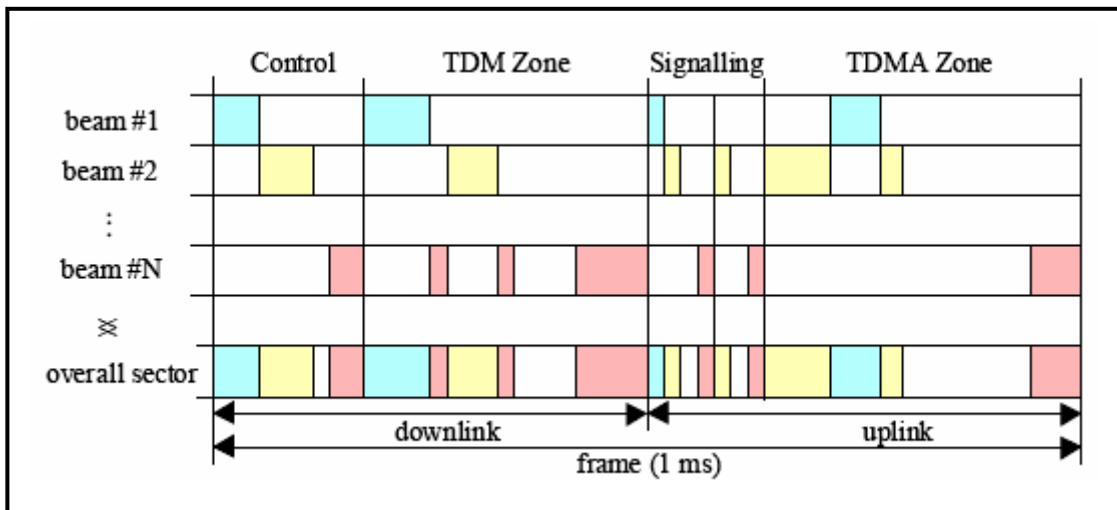


Figure (2). Slot-based beam-switching.

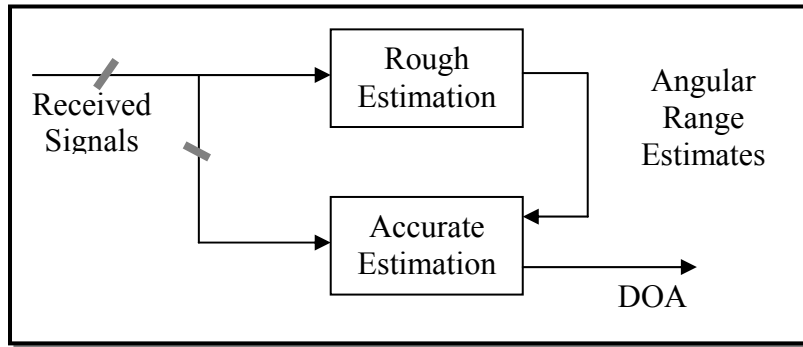
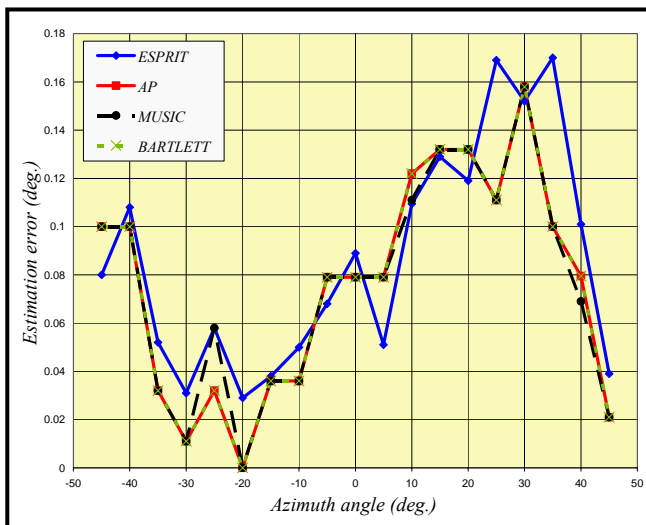


Figure (3). Reduction of execution time of DOA estimation using two steps estimation.

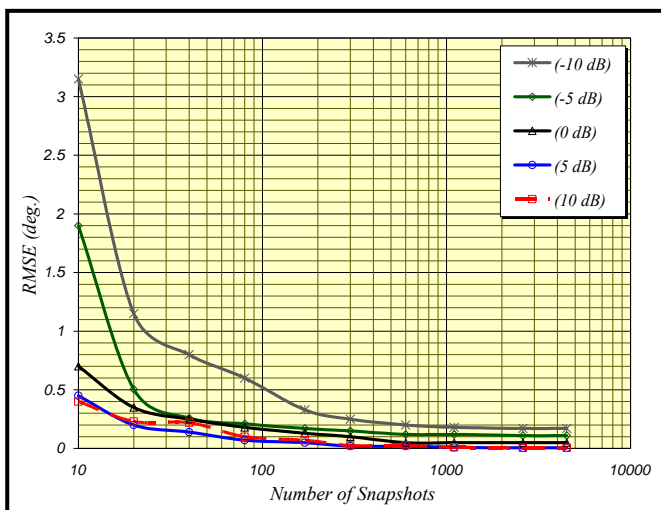


(a)

Azimuth angle	Estimating error (deg.)			
	ESPRIT	AP	MUSIC	BARTLETT
-45	0.08	0.1	0.1	0.1
-40	0.108	0.1	0.1	0.1
-35	0.053	0.032	0.032	0.032
-30	0.032	0.012	0.012	0.012
-25	0.059	0.032	0.059	0.032
-20	0.03	0	0	0
-15	0.039	0.038	0.038	0.038
-10	0.05	0.038	0.038	0.038
-5	0.068	0.0797	0.0797	0.0797
0	0.089	0.0797	0.0797	0.0797
5	0.052	0.797	0.797	0.797
10	0.11	0.122	0.112	0.122
15	0.13	0.0.133	0.0.133	0.0.133
20	0.12	0.133	0.133	0.133
25	0.17	0.112	0.112	0.112
30	0.154	0.1595	0.1595	0.1595
35	0.17	0.1	0.1	0.1
40	0.101	0.08	0.069	0.08
45	0.0395	0.021	0.021	0.021

(b)

Figure (4). Calibration example, estimation errors vs. azimuth angles, SNR=25 dB ((a) Graph and (b) Table of results).

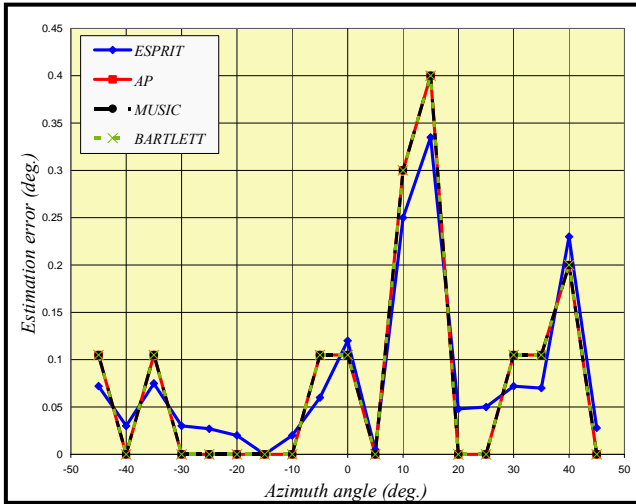


(a)

Number of Samples	RMSE (deg.)				
	-10 dB	-5 dB	0 dB	5 dB	10 dB
10	3.15	1.9	0.7	0.45	0.4
20	1.15	0.501	0.32	0.2	0.22
40	0.8	0.25	0.25	0.12	0.22
80	0.6	0.2	0.18	0.085	0.1
180	0.33	0.19	0.12	0.07	0.09
300	0.26	0.18	0.1	0.01	0.01
600	0.2	0.16	0.03	0	0
1100	0.195	0.16	0.03	0	0
2500	0.192	0.15	0.03	0	0
4500	0.192	0.15	0.03	0	0

(b)

Figure (5). Root Mean Square Errors (RMSE) as a function of the number of samples and SNR: number of samples $N = [10, 20, 40, 80, 160, 320, 640, 1280, 2560, 4500]$ and SNR = $[10, 5, 0, -5, -10]$ dB ((a) Graph and (b) Table of results).



(a)

Azimuth angle	Estimating error (deg.)			
	ESPRIT	AP	MUSIC	BARTLETT
-45	0.07	0.105	0.105	0.105
-40	0.035	0	0	0
-35	0.07	0.105	0.105	0.105
-30	0.035	0	0	0
-25	0.031	0	0	0
-20	0.025	0	0	0
-15	0	0	0	0
-10	0.025	0	0	0
-5	0.06	0.107	0.107	0.107
0	0.125	0.107	0.107	0.107
5	0	0	0	0
10	0.25	0.3	0.3	0.3
15	0.34	0.4	0.4	0.4
20	0.049	0	0	0
25	0.04	0	0	0
30	0.073	0.107	0.107	0.107
35	0.07	0.107	0.107	0.107
40	0.23	0.2	0.2	0.2
45	0.035	0	0	0

(b)

Figure (6). DOA estimation example performing high bandwidth signal, angular range between – 45 degrees and 45 degrees in 5 degrees steps, SNR=25 dB, maximum error= 0.4 degrees. ((a) Graph and (b) Table of results).