Using Volume of Fluid Approach to Simulate Nanofluid Flow

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ABSTRACT

A new implementation for the volume of fluid model with finite volume method is applied to simulate two-phase flow involving nano-particles. The model is used to investigate the laminar flow utilizing nanofluids in a lid-driven cavity. The fluid in the cavity is a water-based nanofluid containing Cu, CuO or Al_2O_3 nano-particles. The effects of adding extra phase (solid phase) and solid volume fraction for different nanofluids on hydrodynamic characteristics are investigated. The effective density and viscosity of nano-particles are calculated by Chon and Brinkman models, respectively. The CFD model is validated for laminar flow and the results showed good agreement with available numerical data. Then the model is tested for liquid and nano-particles where the results indicate that the effects of solid volume fraction approximately depend strongly on the effective density sequentially for Al_2O_3, CuO and Cu. In addition the effective viscosity has less effect on the flow field.

Keywords: Nanofluid, laminar, two-phase, finite volume method, cavity

الملخص

تطبيق جديد لنموذج حجم المائع مع طريقة الحجوم المحدودة لمحاكاة تدفق مشتمل على وجود طورين من المائع واحد هذي الاطوار حاوية على جزيئات نانوية. يستخدم هذا النموذج للتحقيق في تدفق الطباقي باستخدام مائع نانوي في تجويف ذات غطاء متحرك. المائع الموجود في التجويف هوعبارة عن سائل (ماء) وحاوي على جزيئات نانوية في حالات مختلفة والتي تحتوي على النحاس، CUO أو AL2O3 كجزيئات نانوية. تم دراسة تأثير إضافة مرحلة إضافية (المرحلة الصلبة) وكسر حجم جزء الصلب للجزيئات النانوية المختلفة على الخصائص الهيدروديناميكية. فيما يتعلق بالخصائص الهيدروديناميكية تحسب الكثافة واللزوجة فعالة من نانو جزيئات على اساس استخدام نماذج من قبل تشون وبرينكمان ، على التوالي. يتم التحقق من صحة نموذج CFD لتدفق الطباقي وأظهرت النتائج اتفاق جيد مع البيانات الرقمية المتاحة. ثم يتم اختبار نموذج لجزيئات النانو السائلة وحيث أظهرت النتائج أن آثاركسر حجم الجزء الصلب يعتمد بشدة على نحو متسلسل الكثافة فعال لـAL2O3، CUO والنحاس. بالإضافة لزوجة فعالة لها تأثير أقل على مجال التدفق.

1. Introduction

Over the last decade, liquids with unvarying suspended nanofluids (nano-particles) have been extensively investigated for improved thermal properties. There have been many researchers in literature that recorded in their work the enhancement of thermal conductivity for fluids with percentages of a solid volume fraction does not exceed 10\%. Many attempts in the modeling field have been made to formulate appropriate effective approach. The single phase or two phases approach could be used for simulating laminar, turbulent and convective heat transfer with nanofluids Maiga et al [1], Roy et al [2] and Khanafer et al [3]. The single phase approach is simpler and requires fewer computational time, which assumes that the fluid phase and particles are in thermal equilibrium and move with the similar velocity Akbarinia and Behzadmehr [4] and Akbarinia [5]. In order to close the system by using suitable expressions which calculate the properties of single phase nanofluid which are become important and notable. Moreover, the single phase approach has been implemented in different theoretical works of fluid flow and convective heat transfer with nanofluids, for example, Talebi et al. [6], Shahi et al [7] and Sundar et al [8]. Therefore the properties of nano-particles are not completely specified and there are not accurate formulas for predicting nanofluid mixture. The nanofluids were modeled by using single and two phase approach through a parametric study of thermal performance in a three dimensional transport equations have been used by Tokit[9] to investigate the generality of nano-particle velocity equation. The conservation equations (continuity, momentum and energy) were solved by using finite volume method. In heat transfer field, Oztop and Abu-Nada [10] discussed the natural convection in a rectangular cavity filled with a nanofluid containing Cu, Al_2O_3 and TiO_2 as nano-particles. They concluded that the greatest value of heat transfer is obtained by using Cu nano-particles. Generally speaking the predication of nanofluid using a single phase numerically is not coping the agreement with experimental results. So, this paper aims to simulate the nanofluid with a new method (volume of fluid) framework which is never used before as shown in the literatures. The method has been implemented in a cavity to study the nanofluid laminar behavior without heat transfer effect using. To do this, the Eulerian-Lagrangian approach of two-phase model is used. The Eulerian method is used for the mass and momentum conservation equations for liquid phase while the Lagrangian model is used for the solid phase. It should be pointed to that, this is the first paper implemented the two-phase nanofluid modeling in a cavity under laminar and ambient conditions. Moreover, from the previous work of many researchers, they used different nano-material but without optimized for the selection of the best one.

2. Governing Equations

The governing equations for fluid motion are the Navier-Stokes equations. The flow is assumed to be a steady, laminar and incompressible flow. When the density of a viscous fluid is constant, the equations are sufficient to model the flow in general form can be described in terms of the conservation of mass equation, or commonly known as the continuity equation. The continuity equation in divergence form can be written as:

$$\nabla\cdot(\rho_f \vec{V}_f) = 0 \qquad (1)$$

where ρ, V, and ∇ are the density, velocity vector and gradient operator respectively. The conservation of momentum equation is derived using the Newton’s second law applied to a fluid passing through an infinitesimal control volume. It is written in divergence form as:

$$\nabla\cdot(\rho_f\vec{V}_f\vec{V}_f)=\nabla\cdot(\mu_f\nabla\vec{V}_f)+\vec{\mathbf{S}_{\mathbf{f}}}-\nabla p \tag{2}$$

where μ, S and p are the fluid viscosity, the source term due to any external forces and the pressure gradient.

3. Volume of Fluid Approach

The Volume of fluid approach can handle the modeling of two or more phases by solving a single set of transport equations and tracking the volume fraction of each of the fluids throughout the domain. In this case for a fluid – solid system, the VOF method involves the construction of a new transport equation for the continuity and momentum equations for the indicator volume fraction, α to describe the evolution nano-fluid. Therefore the equations can be written as:

$$\nabla\cdot(\rho\alpha\vec{V})=0 \tag{3}$$

$$\nabla\cdot(\rho\alpha\vec{V}\vec{V})=\nabla\cdot(\mu\alpha\nabla\vec{V})+\alpha\vec{\mathbf{S}}-\alpha\nabla p+\nabla\cdot(\sum_{k=1}^{n}(1-\alpha)\rho_p\vec{V}_p\vec{V}_p) \tag{4}$$

Mathematically because the volume fraction α is unknown throughout the domain, an equation is needed for its computation. The continuity and momentum equations for the fluid can be used as a volume fraction to determine the fraction of the phase in the control volumes. The volume of fraction which is provided is not all used in the computations according to its definition below:

- $\alpha= 0$: The cell is empty (full with fluid).
- $\alpha = 1$: The cell is full (full with solid).
- $0 <\alpha < 1$: The cell contains the solid and fluid.

Therefore the volume of fraction transport equation can take the form of

$$\nabla\cdot(\rho_p\alpha\vec{V}_p)=\nabla\cdot(\rho_f(1-\alpha)\vec{V}_f) \tag{5}$$

where (p) and (f) are referring to the particle phase and liquid phase respectively. In the transport equation of the conservation of momentum, (4),V_p is defined as the drift velocity for secondary phase k (i.e., the nano-particles in this work), and can be written as

$$\vec{V}_p=\vec{V}_k-\vec{V} \tag{6}$$

The relative velocity (slip velocity) is denoted as the velocity of secondary phase (p) relative to the velocity of the primary phase (f)

$$\vec{V}_{pf} = \vec{V}_p - \vec{V}_f \tag{7}$$

The relation between the drift velocity and the relative velocity becomes

$$\vec{V}_{dp} = \vec{V}_f p - \sum_{k=1}^{n} \alpha \vec{V}_f \tag{8}$$

The drift velocity, calculated from equation (6) can be determined according to by Manninen et al. [11] while the relative velocity is taken from Schiller and Naumann [12] is used to calculate the drag coefficient

$$\vec{V}_{pf} = \frac{\rho_p d_p^2}{18\mu_f f_{drag}} \frac{(\rho_p - \rho)}{\rho_p} a \tag{9}$$

where (a) is the acceleration and can be determined from

$$a = g - (\vec{V} \cdot \nabla)\vec{V} \tag{10}$$

and (f_{drag}) is evaluated depending on the Reynolds number value from

$$f_{drag} = 1 + 0.15 Re_p^{0.687} \qquad Re_p \leqslant 1000 \tag{11}$$

or

$$f_{drag} = 0.0183 Re_p \qquad Re_p > 1000 \tag{12}$$

The density of the nanofluid containing a dilute suspension of small rigid spherical particles is given by

$$\rho = (1-\alpha)\rho_f + \alpha\rho_p \tag{13}$$

and the effect of viscosity at reference temperature is given by

$$\mu = \frac{\mu_f}{(1-\alpha)^{2.5}} \tag{14}$$

4. The Complete Algorithm

The Navier - Stokes equations are solving using (SIMPLE) algorithm of Patankar and Spalding as cited in Versteeg [13]. To discretize the governing equations for fluid flow, the cell-centered finite volume method is selected. In this algorithm the overall solution procedure is iterative and is based on a pressure-correction equation is derived from the discretized equations for continuity and momentum. The discretized momentum equation for node (P_0) can be written as

$$\vec{u}'_{P_0} = \frac{\Delta V_{P_0}}{a_0} (\vec{\nabla} P')_{P_0} \tag{15}$$

where (∇) is the discrete gradient operator. The required mass flux correction is obtained from the velocity corrections, and substituting in the discretized continuity equation gives the pressure correction equation for cell (P_0) as:

$$\sum_{j}^{N_j} \rho \left[\overline{\frac{\Delta V_{P_0}}{a_0}}\right]_j (\vec{\nabla} P')_j \cdot \vec{S}_j = \sum_{j}^{N_j} (m_j) \tag{16}$$

where $(\vec{\nabla} P')_j \cdot \vec{S}_j$ is obtained using equation (11) and

$$(\vec{\nabla} P')_j \cdot \vec{S}_j = (\nabla P')_j S_j \tag{17}$$

Finally, the linear system of equations can be cast in the form of:

$$c_p P'_P + \sum_{j}^{N_j} c_{pj} P'_{Pj} = d_P \tag{18}$$

5. Problem Definition

The considered geometry of this work is depicted in Figure 1. This geometry includes a square cavity with nano-particles of different materials as listed in Table 1. The temperature of both liquid and nano-particles are kept at ambient temperature in order to investigate the effect driven velocity. The problem deals with a confined flow in a square enclosure. The flow is driven by the constant displacement of the upper wall where the tangential velocity is enforced. The description of the problem, configuration and the computational domain of the cavity is shown in Figure 2.For the selected test case, the computational domain with three different grid resolutions are used in this study as shown below. All test cases are run with value of Reynolds number 1000 to show the objective from this research. These are simulated and compared with benchmark results from Ghia et al. [14]. The details of the grid and code validation will be discussed briefly in the next section.

6. Code Validation

The validation of the proposed numerical solution is discussed in this section. The simulation results are compared with the numerical available results from the literature. In addition, because of the lack of experimental data for nanofluid flow in a steady laminar flow without heat transfer effect. Therefore for a single-phase fluid flow, the standard test case of a flow field within a two dimensional, steady -state, incompressible laminar flow is used to validate the solver. The lid which moves at a constant axial velocity drives the flow inside the cavity due to the transport of shear stress by the molecular viscosity. This problem has clearly defined in geometry and boundary conditions as shown in Figure 3. In addition, the balance of convection and diffusion in the momentum equations connected with recirculating zones introduce a good material to makes this problem an excellent case for testing the multi-grid techniques. The simulation of the tested case starts from the motion of the lid (the upper wall), the velocity at the top boundary of the cavity is set at 1 m/sec. The enclosure cavity is a square in shape with unit dimensions in width and height. The four boundaries are viscous walls where a no-slip condition is implemented. The flow is identified by the Reynolds number, which is defined by:

$$Re = \frac{\rho U_{lid} L}{\mu} \tag{19}$$

where (ρ) is the density of the fluid, (U_{lid}) is the velocity of the moving lid, (L) is the length and height of the cavity, and (μ) is the viscosity of the fluid. Due the lack of experimental data for the lid driven cavity with the presence of nano-particles, the present numerical solution is also validated by the available numerical results published by Ghia et al. [14]. Figures 4 a and b show comparisons between the present study and the available numerical data. Both axial and vertical velocity components expressed good agreement with referenced data.

7. Results and Discussions

Fluid driven cavity flow is solved for of nine computational cases are used to simulate the nanofluid flow configuration. First three cases of different volume of fraction values of [0.03, 0.05 and 0.1] and with three different Reynolds number values of [100, 400 and 1000]. After extracting the results, one case will be adopted in this work where the liquid volume fraction is (0.1) and the Reynolds number is (1000). Physical properties of nano-particles and fluid at the base temperature 20 °C are listed in Table 2.

Figure 5a shows the mid plane axial velocity component (u) is plotted for various nanofluid. The most nanofluid identical to the pure fluid is (Al_2O_3), because of the density of the mixture is the approximately the closest one to the pure fluid (liquid water). It should be noted in this plot that the region near the upper wall of the cavity the forward motion of fluid is dominating due to shear action and all nanofluids are showed same behavior. Beyond this region the nanofluids flow in rearward direction. Figure 5b shows the mid plane vertical velocity component (v) where the value of solid volume fraction is (0.1). It is obvious from this figure that distribution of velocity is not symmetric with respect to the center of the cavity as in case of using air. It can be observed that the higher values of vertical velocity component are found near the upper wall which is matching the standard air case and then decrease gradually towards the bottom wall. Again the (Al_2O_3) shows a good agreement with liquid trend.

Figure 6 shows the stream line contour plots of two dimensional case of the considered problem for Reynolds number equals to 1000. In general it is found that the recirculation zone due to shearing action of upper velocity is clearly noticeable. In all cases which are identical with conventional wall shear cavity flow, it is found that one primary recirculation eddies regime is formed at the upper right corner of the cavity. The strength of these eddies for (H_2O) and (Al_2O_3) are approximately the same in size and (Cu) and (CuO) are also. Figure 7 can summarized all the investigation which is discussed above. The contour plots expressed the velocity magnitude for the liquid water and the nano-particles of different materials. In this work will be gone farther, where in Figure 8a presents the term $\nabla \cdot (\alpha \vec{V} u)$ the divergence of velocity extracted along the middle of the centerline of the cavity.

Nevertheless, it can be seen that due to the mass of the nano-particles in the corner is nearly conserved. The somewhat non-zero value of the divergence of velocity is probably because of the particle accumulation effect in the corner. In Figure 8b presents the $\nabla \cdot (\alpha \vec{V} u)$, divergence of velocity extracted along the middle of the centerline of the cavity.

It can be seen that the same trend for the nano-particles but due to the heavier of the particles the divergence of the velocity is increased.

8. Conclusions

Laminar flow in a lid-driven cavity utilizing nanofluid is simulated by volume of fluid model (VOF). This paper presents a new numerical study for the nanofluid which is implemented in a CFD code to simulate the nano-particles in laminar flow under ambient condition without the effect of heat transfer in a cavity. It treats the nanofluid as a suspended phase inside the liquid. The conclusions from the numerical analysis performed in this study are summarized as follows:

1- The two-phase model is performed well with the advanced finite volume method.
2- It is interesting to study the laminar flow without heat transfer effect.
3- The work object to optimize the flow of nano-particle under the room conditions to select the lower density material.

9. References

[1] S.E.Maiga, C.T. Nguyen, N. Galanis, and G. Roy. Heat transfer behaviors of nanofluids in a uniformly heated tube. Super Lattices Microstructure, 35 (36), 543–557, 2004.

[2] G. Roy, C.T. Nguyen and P.-R. Lajoie. Numerical investigation of laminar flow and heat transfer in a radial flow cooling system with the use of nanofluids. Superlattices Microstruct.35(36), 497–511,2004.

[3] K. Khanafer,K. Vafai and M. Lightstone. Buoyancy driven heat transfer enhancement in a two dimensional enclosure utilizing nanofluids. International Journal of Heat and Mass Transfer 46, 3639–3653,2003.

[4] A. Akbarinia and A. Behzadmehr. Numerical study of laminar mixed convection of a nanofluid in horizontal curved tubes. Journal of Applied Thermal Engineering. 27, 1327–1337,2007.

[5] A. Akbarinia. Impacts of nanofluid flow on skin friction factor and Nusselt number in curved tubes with constant mass flow. International Journal of Heat and Fluid flow 29 (1),229–241, 2008.

[6] F. Talebi, A.H. Mahmoudi and M. Shahi. Numerical study of mixed convection flows in a square lid-driven cavity utilizing nanofluid. International Communication of Heat and Mass Transfer. 37, 79–90,2010.

[7] M. Shahi, A.H. Mahmoudi and F. Talebi. Numerical study of mixed convective cooling in a square cavity ventilated and partially heated from the below utilizing nanofluid. International Communication of Heat and Mass Transfer. 37, 201– 213, 2010.

[8] L.S. Sundar, K.V. Sharma and S.Parveen. Heat transfer and friction factor analysis in a circular tube with Al2O3 nanofluid by using computational fluid dynamics. International Journal of Nanoparticles. 2 (16), 191–199, 2009.

[9] E. Mat Tokit, M. Z. Yusoff and H. Mohammed. Investigation on Nanoparticle Velocity in Two Phase Approach, International Journal of Mechanical, Industrial Science and Engineering Vol:7 No:10, 136-142, 2013.

[10] H.F. Oztop and E.Abu-Nada. Numerical study of natural convection in partially heated rectangular enclosures filled with nanofluids, International Journal of Heat and Fluid Flow, 29,1326-1336,2008.

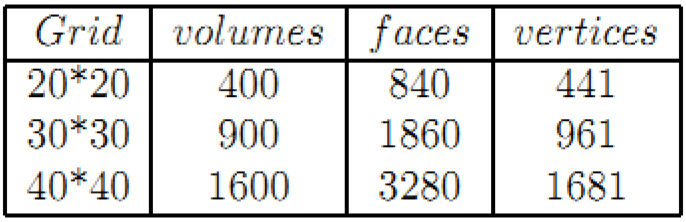
[11] M. Manninen, V. Taivassalo, and S. Kallio. On the mixture model for multiphase flow. Techical Report. 288, VTT Technical Research Centre of Finland, 1996.

[12] L. Schiller and A. Naumann. A drag coefficient correlation. Zeitschrift des Vereins Deutscher Ingenieure.77, 318–320, 1935.

[13] H. K. Versteeg and W. Malalasekera. An Introduction to Computational Fluid Dynamics. Pearson, second edition, 2006.

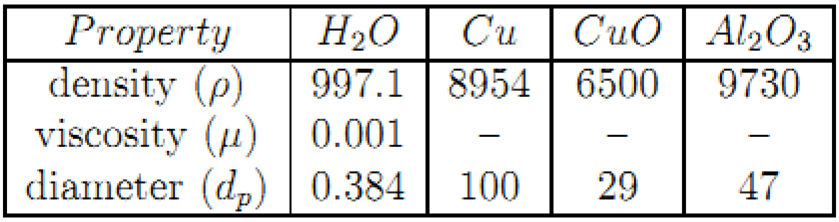
[14] U. Ghia, K.N. Ghia and C.T. Shin. High-Resolutions for incompressible flow using the Navier Stokes equations and a multigrid method. Journal of Computational Physics. 48, 387–411,1982.

Table 1: Structured grid information

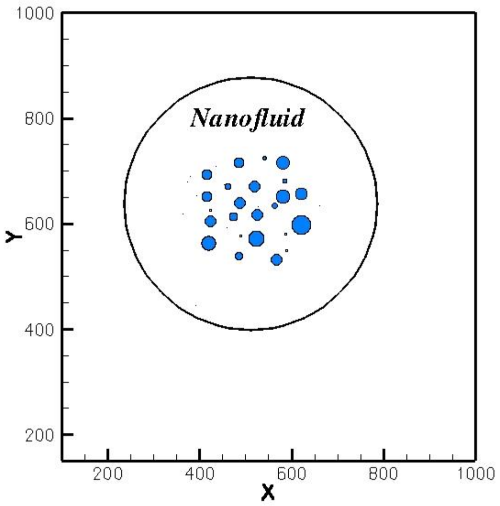


Grid	*volumes*	*faces*	*vertices*
20*20	400	840	441
30*30	900	1860	961
40*40	1600	3280	1681

Table 2: Physical properties of different phases.



Property	H_2O	Cu	CuO	Al_2O_3
density (ρ)	997.1	8954	6500	9730
viscosity (μ)	0.001	–	–	–
diameter (d_p)	0.384	100	29	47



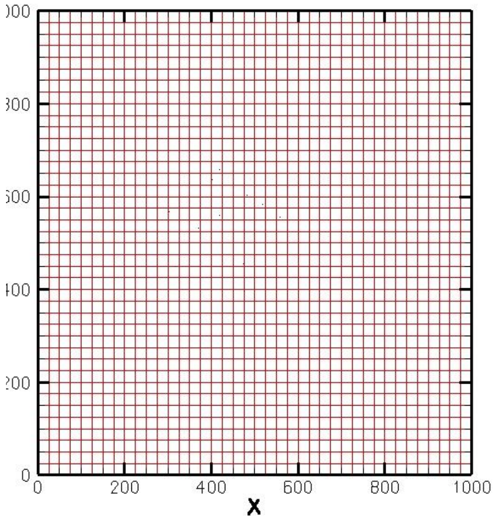


Figure 1: Description of the problem.

Figure 2: Computational domain.

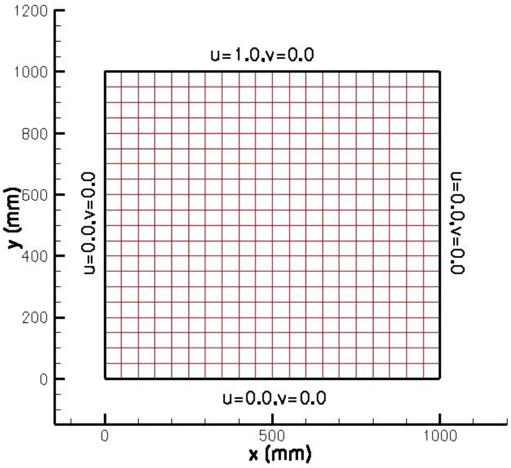


Figure 3: Description of boundary conditions of the problem.

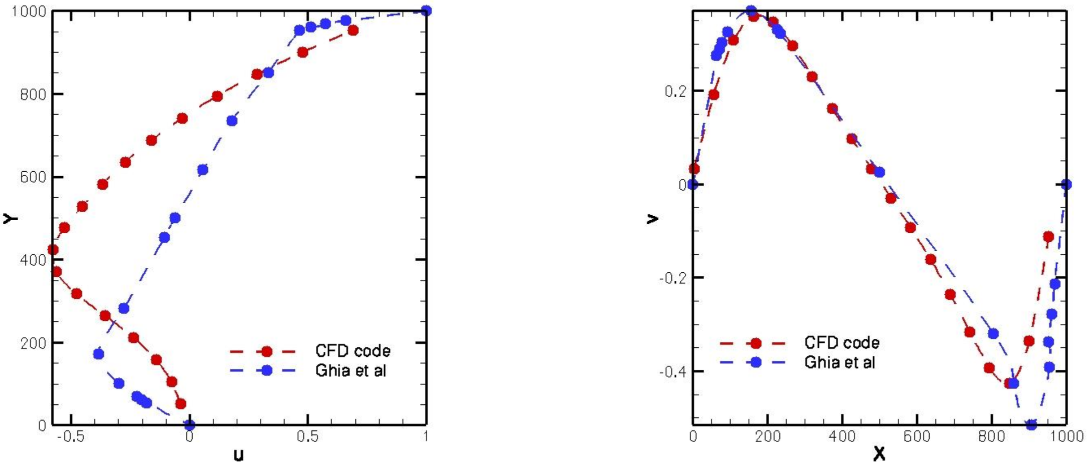
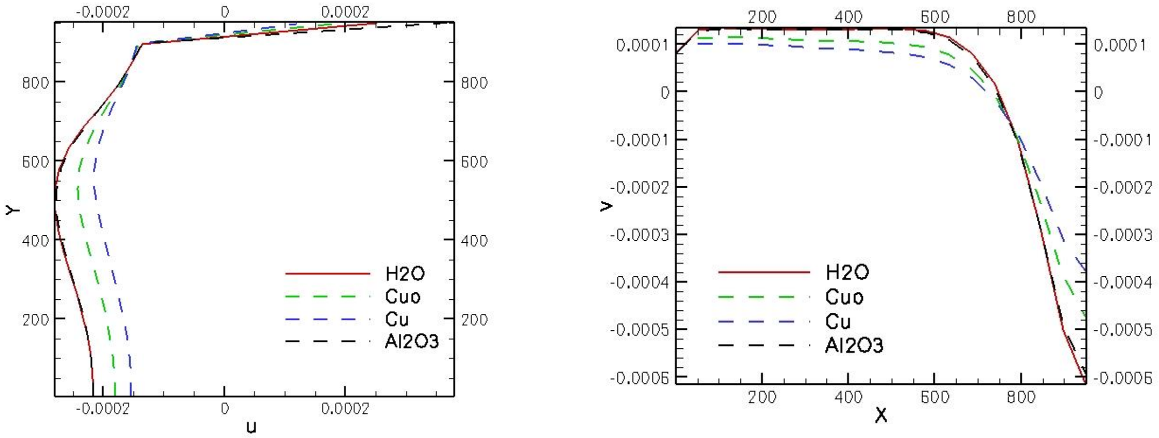


Figure 4: Comparisons with the referenced data.



(a) (b)

Figure 5: a-Variation of axial velocity component at x=0.5 m with vertical distance. b-Variation of vertical velocity component at y=0.5 m with axial distance.

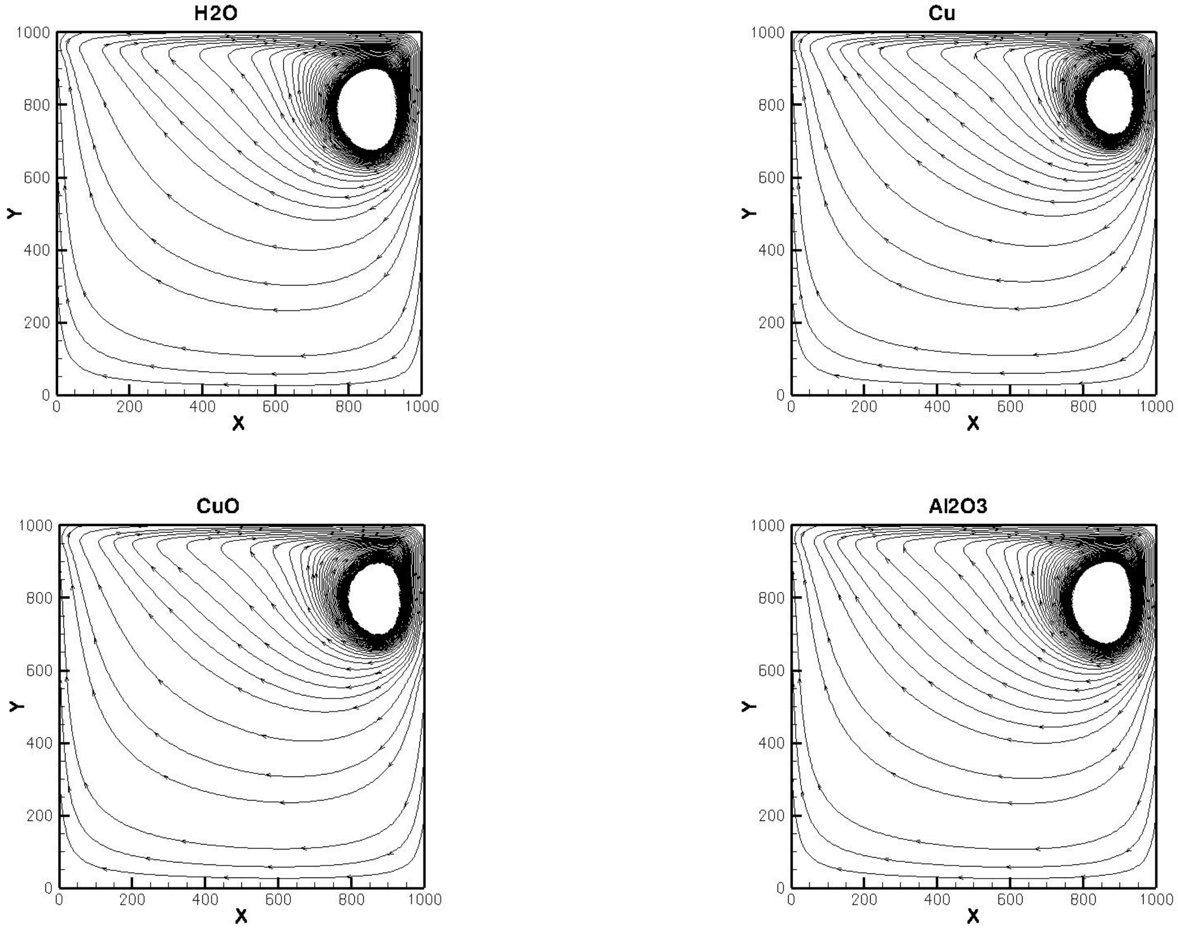


Figure 6: Streamlines for liquid water and different nano-particles.

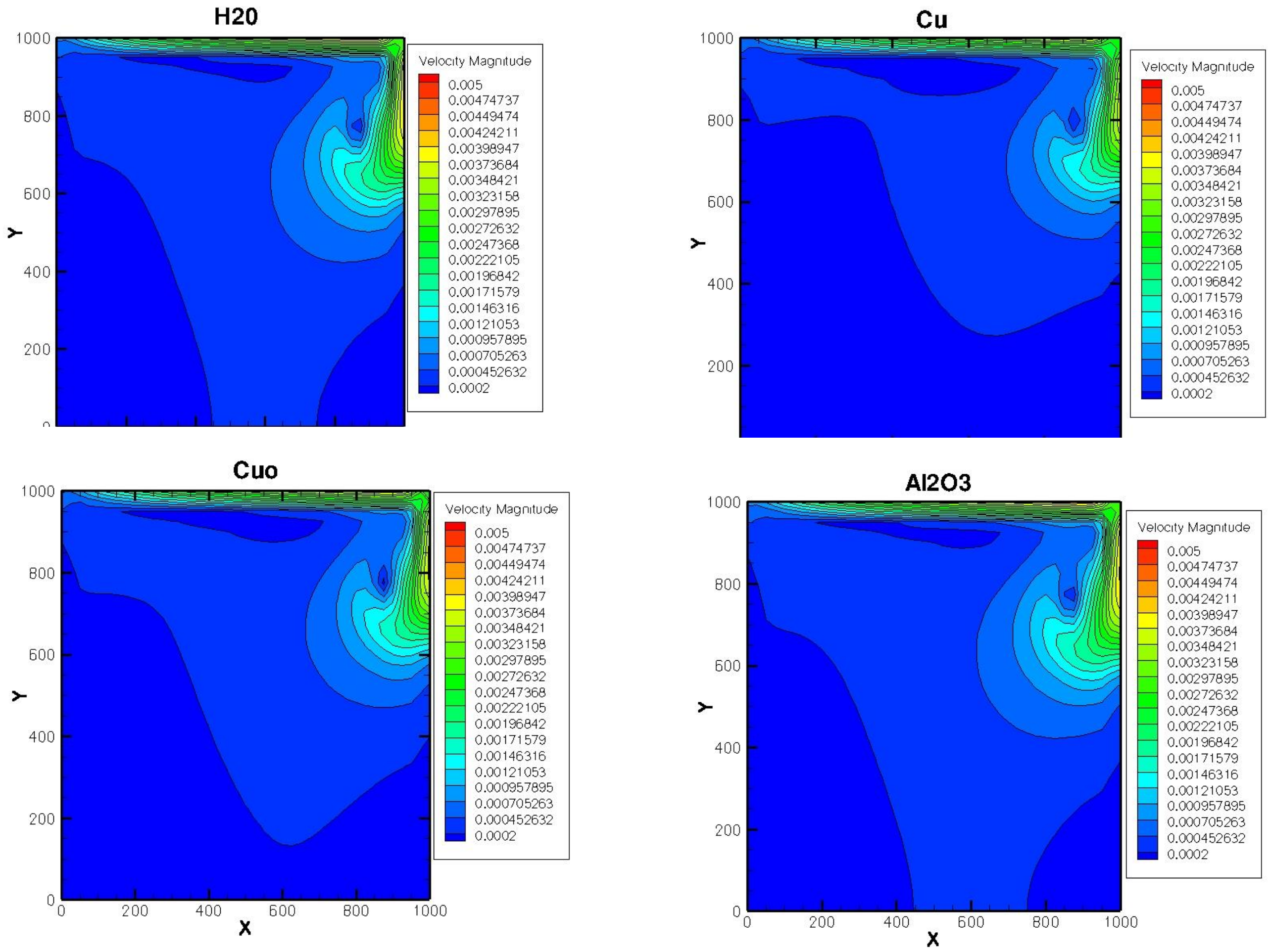


Figure 7: Velocity magnitude for liquid water and different nano-particles.

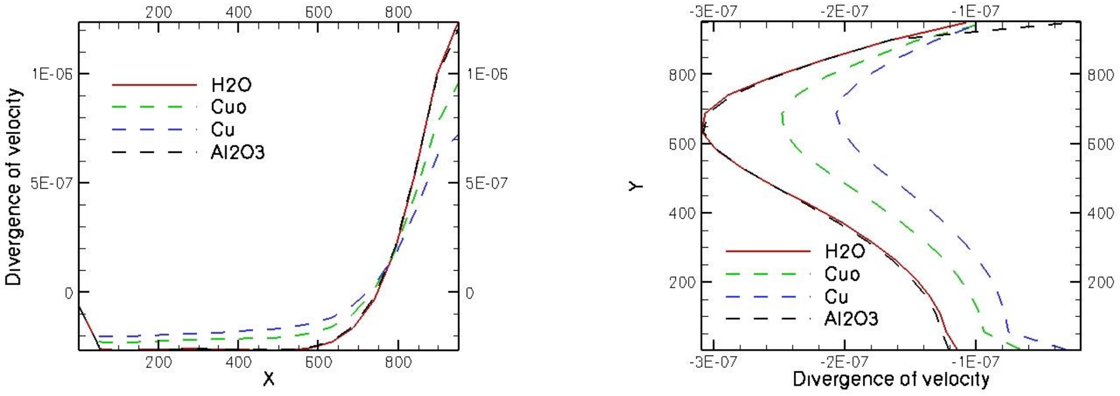


Figure 8: a-Variation of axial velocity divergence at x=0.5 m with vertical distance. b-Variation of vertical velocity divergence at y=0.5 m with axial distance.}