Mathematical extraction equation of horizontal shear transfer in interface between two concrete members

Dr. Hakim S. M. Al-Kurayshi

Civil Technical Department, Najaf Technical Institute

Al-Furat Al-Awsat Technical University, Iraq

Assistant Professor , E- mail: ntit.10006@gmail.com

Received 14 September 2015 Accepted 23 November 2015

Abstract

Shear is transferred through interface between two concrete members by two mechanisms either by aggregate interlock or by dowel action or by both in the same time. This media is very complex and there are so many variables affecting shear transfer like surface shape and final treatment of the concrete surfaces. Histories of the previous researchers are mentioned in this paper to find out the applicable results of their equations. It is noticed that most of them depend on only clamping stress due to dowel action while others used besides that the type of concrete used namely its compressive strength. A mathematical statistical trials are made in this paper to find out the most accurate and suitable equation which simulates the action happened in this media. An equation of six parameter polynomial is obtained containing clamping stress and compressive strength of concrete. Assumed values of the mentioned variables are used in this equation to find out the horizontal shear strength. The deviation recorded is about 4% which is an acceptable one.

Keywords: Shear transfer, polynomial, shear stress, slip, and interface.

Introduction

When two concrete members cast in different times, an interface will be formed between the contact surfaces of the two members as they are constructed one over the other. Shear is transferred across that interface between the two members which can slip relative to one another. An example of those members is a beam and deck sections and, in order of them to act compositely, horizontal shearing forces must be transferred through interface between them at contact surfaces Figure (1). The shear is transferred by two mechanisms namely aggregate interlock and dowel action due to the existence of shear connectors like steel bars or studs of steel sections. The aggregate interlock depends on the concrete of the two surfaces and as a result on the coefficient of friction between the two surfaces. It depends also on the finishing treatment of the concrete surface if it is smooth or rough or in between these two.

In most of the literature researchers, the researchers focus on the dowel action of steel bars pass through the two concrete members. Others were studying the effect of aggregate interlock besides dowel action. Unfortunately, the results of horizontal shear transfer equations suggested by them were not coincided to some extent. In this paper the work will be classified into two groups depending on the variables taken by them. First group that took the clamping stress as a main effective variable in shear evaluation. The second group which is near the truth was taking the compressive strength of concrete into account besides the clamping stress.

.

Shear transferred through dowel action only (first group work)

Shear transferred by this mechanism when shear connectors are used like bars or any other steel studs which are inserted in the two concrete members to join one to other. The ratio of steel used is taken to beρ_v and the clamping stress of these connectors is $\rho_v f_y$ where f_y is yield stress of steel used. There had been so many proposed equations to find out the horizontal shear strength through interface in composite sections. These equations can be summarized here according to the people who found it out as shown in table 1.

Shear transferred through dowel action and concrete strength (second group work)

These equations were taking into account the clamping stress and concrete compressive strength. They are as mentioned in table 2.

Present Work

In the literature it is seen that people who were working on equations without taking concrete compressive strength reached an incorrect results of horizontal shear strength while those who were taking that into considerations had far results. An equation is derived depending upon the results of the previous work. The equation governed depends on the following procedure:

1. The work of people who were working on shearing strength related to only clamping stress was drawn on one plot as shown on Figure (2). The values of clamping stress $\rho_v f_y$ were taken from 0 to 350 psi by an increment of 50 psi [from 0 to 2.415 MPa by an increment of 0.345 MPa] .The points of the relation between shear strength and clamping stress were represented by the most suitable and compatible curve. That curve was polynomial of six parameters. The equation obtained is very near to the truth and it represents the work of so many researchers. Each equation of the literatures work was represented through an average values taken as mentioned in Table (3).

2. The horizontal shear strength is also depending not only on clamping stress but on concrete strength also. The two parameters. The work of these peoples are assembled through using their governed equations and an assumed values of concrete compressive strength. The values of the compressive strengths of a normal weight concrete are taken from 10 MPa to 40 MPa by an increment of 5 MPa. Figures 3, 4, 5, 6, 7, 8 and 9 are representing the relation between shear strength and clamping stress. These curves are corresponding to 10 MPa concrete compressive strength, 15 MPa, 20 MPa, 25 MPa, 30 MPa, 35 MPa and 40 MPa respectively.

In each of these figures same polynomial of six parameters is used to reach the equations which are representing the most acceptable relation between horizontal shear strength and clamping stress. To reach a unified equation that has clamping stress and compressive strength variables, it is assumed that the final equation can be represented by the following polynomial equation:

$$\boldsymbol{y = C_1X^5 - C_2X^4 + C_3X^3 - C_4X^2 + C_5X + C_6} \qquad (1)$$

Where y here represents the horizontal shear strength v_n (MPa) and X is the clamping stress ($\rho_v f_y$) also in MPa. C,s are functions of f_c'. Each equation governed corresponding to the compressive strength used has six parameters which are functions off_c'. These parameters can be seen in Table (4). So if C1 is taken to be drawn against the values of concrete compressive strength then seven values of C1 will be used against seven values of concrete compressive strength. This truth is translated to a plot shown in Figure (10) from which a correlated representative curve is obtained.

Figures 11,12,13,14 and 15 are plots against C2 ,C3 ,C4 ,C5 and C6 respectively. The following equations are taken from the plots:

$$\boldsymbol{C_1 = 0.0053f_c' + 0.2253,\ \ C_2 = -0.0386f_c' - 1.611,\ C_3 = 0.1068f_c' + 4.336,}$$
$$\boldsymbol{C_4 = -0.1431f_c' - 5.5252,\ \ C_5 = 0.1158f_c' + 3.8871,\ C_6 = 0.00005f_c' + 0.002}$$

The final equation is obtained by substituting these parameters in Eq.(1) to get:

$$\boldsymbol{v_n = [0.0053f_c' + 0.2253][\rho_v f_y]^5 - [0.0386f_c' + 1.611][\rho_v f_y]^4 + [0.1068f_c' + 4.336][\rho_v f_y]^3 - [0.1431f_c' + 5.5252][\rho_v f_y]^2 + [0.1158f_c' + 3.8871][\rho_v f_y] + [0.00005f_c' + 0.002]} \qquad (2)$$

Discussion of the results and conclusions

The obtained equation seems to be long and complicated due to its inclusion of so many numbers and parenthesis, so its validity must be checked by comparing the outputs with the results of first and second worker groups. To make the comparison logically and reasonable an example of the variables used in this equation must be assumed. The values of these variables are taken to be 20 Mpa for compressive strength of concrete and equal to 2 MPa for clamping stress. The horizontal shear strength obtained by Eq.(2) is 3.11 MPa.

1. Comparison with first group:

The shear strength equation of this group is that which is mentioned in Figure (2) as follows:

$$\boldsymbol{v_n = 0.6316[\rho_v f_y]^5 - 4.4973[\rho_v f_y]^4 + 12.012[\rho_v f_y]^3 - 14.961[\rho_v f_y]^2 + 9.6952[\rho_v f_y] + 0.0054}$$

After the substitution of the clamping stress value, the shear strength obtained is 3.9022 MPa. The deviation between the two results is 20%.

2. Comparison with second group:

The shear strength of the work of this group can be obtained from equation mentioned in Figure (5):

$$\boldsymbol{v_n = 0.339[\rho_v f_y]^5 - 2.4433[\rho_v f_y]^4 + 6.6586[\rho_v f_y]^3 - 8.6534[\rho_v f_y]^2 + 6.4146[\rho_v f_y] + 0.0027}$$

The result of the shear strength after substitution of the assumed clamping stress and concrete compressive strength is 3.2423 MPa, so the percentage deviation is 4%.Fig.(16) shows a comparison between the output of the present equation for so many values of clamping stress and compressive strength of concrete is 20 MPa with what was governed by using equations obtained by Walraven , Loov and Loov&Patnaik .It seems to be taking approximately an average style.

Conclutions

From the results of comparison, the following conclusion points can be drawn:

a. The values of horizontal shear strength governed by Eq.(2) always give lower bound values therefore the use of this equation in design is almost safe because the designer will be forced to increase the clamping stress through increasing number of dowel bars .

b. From the results it is seen that the deviation of the first group is very high (20%) compared to (4%) deviation of the second group work. This is noticed because aggregate interlock was not taken to account. It is advisable in design to follow this equation or second group work since they are near the acceptable results.

c. An average values are taken for horizontal shear strength in the work of all the researchers to stand on a moderate values and not on upper or lower bound values. For example the results of Loov & Patnaik were always high and gave upper bound values compared to others like Walraven for example.

References

[1] Birkeland , P.W. ,and Birkeland, H.W.(1966), “Connections in Precast Concrete Construction”, ACI

[2] Building Code Requirements for Structural Concrete, ACI318-02(2002).American Concrete Institute, Farmington Hills, Michigan.

[3] Hsu, T.T.C., Mau, S.T., and Chen, B. (1987), “Theory of Shear Transfer Strength of Reinforced Concrete”, ACI Journal, Vol.84, No.2, pp.149-160.

[4] Loov, R.E.(1978),”Design of Precast Connections”, Paper Presented at a Seminar Organized by Compa International Pte , Ltd.

[5] Loov ,R.E. ,Patnaik ,A.K.(1994),”Horizontal Shear Strength of Composite Concrete Beams with a Rough Interface”, PCI Journal, Vol.39, No. 1, pp.48-66.

[6]] Mast, R.F.(1968),"Auxiliary Reinforcement in Concrete Connections", ASCE Journal, Vol.94,No.ST6.,PP.1485-1504.

[7] Mattock, A.H.(1974),"Shear Transfer in Concrete Having Reinforcement at an Angle to the Shear Plane", Shear in Reinforced Concrete, ACI Special Publication SP-42,Vol.1, American Concrete Institute,Detroit, MI,pp.17-42.

[8] Mattock, A.H. , Johal ,L., and Chow, H.C.(1975),"Shear Transfer in Reinforced Concrete with Moment or Tension Across the Shear Plane", PCI Journal, Vol.20, No. 4, pp.76-93.

[9] PCI Design Handbook-Precast and Prestressed Concrete (1992), Fourth Edition, Precast/Prestressed Concrete Institute. Chicago, IL.

[10] Saemann , J.C. ,and Wash ,G.W.(1994) ," Horizontal Shear Connections Between Precast Beams and Cast-in-Place Slabs" , ACI Journal,Vol.61,No.11, pp.1383-1408.

[11] Shaikh, A.F. (1978) ," Proposed Revisions to Shear-Friction Provisions", PCI Journal, Vol.23, No.2, pp.12-21.

[12] Walraven, J. ,Frenay , J., and Pruijssers ,a.(1987) ," Influence of Concrete Strength and Load History on the Shear Friction Capacity of Concrete Members ",PCI Journal, Vol.32, No.1, pp.66-84.

Table (1): shear transferred through dowel action only (fist group work)

Equation number	Researchers	Proposed equation	Surface treatment and concrete compressive strength
1	Saemann and Washa (1964)	$v_n = \frac{2700}{d+5} + 300\rho_v\left(\frac{33-d}{d^2+6d+5}\right)$ Where v_n= ultimate shear strength in psi. d = effective depth. ρ_v= percent steel crossing the interface.	Not taken into consideration
2	Birkeland and Birkeland (1966)	$v_n = 33.5\sqrt{\rho_v f_y}$	Not taken into consideration
3	Mast (1968)	$v_n = \mu\rho_v f_y$ Where μ is the coefficient of friction at the interface.	Not taken into consideration
4	Shaikh (1978)	$v_n = \emptyset\mu\rho_v f_y$ Where $\emptyset$ = 0.85 for shear $\mu = \frac{1000\lambda^2}{v_n}$ (psi) λ=1.0 for normal weight concrete. This equation can be simplified into (for normal weight concrete);	Taken into consideration

		$v_n = 29.15\sqrt{\rho_v f_y} \leq 0.25 f_c'$	
5	ACI Code 318 / 318 R -2002	$V_u \leq \emptyset V_{nh}$ $V_{nh} = A_{vf} f_{y\mu} \leq \min(0.2 f_c' A_c or 800 A_c)\ lbs\ \ and$ $if\ \ V_u > \emptyset(500 b_v d)\ \ \ \ \ lbs$…..…. (a) $V_{nh} = 80bd\ , lbs(when\ there\ is\ no\ shear\ reinf.)\ \ and$ $if\ V_u \leq \emptyset(500 b_v d)\ \ \ \ \ lbs$ ……….b) $V_{nh} = 80 b_v d\ , lbs(when\ there\ is\ min.\ shear\ reinf.)\ and$ $if\ V_u \leq \emptyset(500 b_v d)\ \ \ \ lbs$.……..... c) $A_{vmin} = 0.75\sqrt{f_c'}\frac{b_v s}{f_y} \geq \frac{50 b_v s}{f_y}$ $V_{nh} = (260 + 0.6\rho_v f_y)\lambda b_v d \leq 500 b_v d\ , lbs$ (When there is min. shear reinforcement and contact surfaces are roughened to a full amplitude of approximately ¼ in) and if $V_u \leq \emptyset(500 b_v d)\ \ \ \ \ \ \ \ \ lbs$…..….d)	Taken into consideration
		Vu =factored shear force, ø=0.75, bv=the width of the interface, d=distance from extreme compression fiber to centroid of tension reinforcement for entire composite section. Avf =area of reinforcement. fy =yield stress of shear reinforcement. Ac= the area of concrete section resisting shear transfer. fc'=concrete strength. µ=1.0 for normal weight concrete placed against hardened concrete with surface intentionally roughened. s= spacing of shear reinforcement.	

Table (2): shear transferred through dowel action and concrete strength (second group work)

Equation number	Researchers	Proposed equation	Surface treatment and concrete compressive strength
6	Walraven et al (1987)	$v_n = 16.8 {f_c'}^{0.406}[0.00007\rho_v f_y]^{0.0371 {f_c'}^{0.303}}$ Where f_c' here is equal to 0.85 times the compressive strength of 150 mm cubes.	Taken into consideration to some extent
7	Mattock (1974)	$v_n = 4.5 {f_c'}^{0.0.545} + 0.8[\rho_v f_y + \sigma_n]$ (Psi) $v_n \leq 0.3 f_c'$	Taken into consideration to some extent
8	Mattock (1975)	$v_n = 400 + 0.8\rho_v f_y$ (Psi) $v_n \leq 0.3 f_c'$	Taken into consideration to some extent
9	Loov (1978)	$v_n = k\ .\sqrt{\rho_v f_y f_c'}$ k is constant and equal to 0.5 for initially uncracked surface. Similar equation was	Taken into consideration to some extent

		proposed by Hsu et al (1987) using k equal to 0.66 for both cracked and uncracked surfaces.	
10	Loov and Patnaik (1994)	$v_n = k\,.\lambda\sqrt{(15+\rho_v f_y)\ f_c'} \le 0.25 f_c'$ k=0.6 and λ =1.0 for normal weight concrete.	Taken into consideration to some extent

Table (3): Horizontal shear strength and clamping stress for different concrete strengths

v_n Mpa	Clamping stress Mpa								f_c' Mpa
	0	0.345	0.690	1.035	1.380	1.725	2.070	2.415	
Brikeland	0	1.635	2.312	2.831	3.269	3.655	4.004	4.324	0
Mattock	2.76	3.040	3.310	3.590	3.860	4.140	4.420	4.690	0
Shaikh	0	1.420	2.010	2.460	2.840	3.180	3.480	3.760	0
Average	0.92	2.030	2.544	2.960	3.323	3.659	3.968	4.258	0
Walraven	0	0.720	0.910	1.040	1.150	1.237	1.316	1.368	10
Loov	0	1.220	1.730	2.120	2.450	2.740	3.000	3.240	10
Loov &patnaik	0	1.270	1.690	2.020	2.310	2.550	2.800	3.010	10
Average	0	1.070	1.440	1.730	1.790	2.180	2.370	2.550	10
Walraven	0	0.734	0.955	1.115	1.243	1.353	1.450	1.540	15
Loov	0	1.500	2.100	2.600	3.000	3.400	3.700	3.970	15
Loov &patnaik	0	1.560	2.070	2.480	2.830	3.140	3.420	3.690	15
Average	0	1.260	1.710	2.070	2.360	2.630	2.860	3.070	15
Walraven	0	0.732	0.980	1.160	1.310	1.440	1.550	1.660	20
Loov	0	1.750	2.490	3.050	3.530	3.940	4.320	4.670	20
Loov &patnaik	0	1.830	2.430	2.910	3.320	3.690	4.020	4.330	20
Average	0	1.440	1.970	2.370	2.720	3.020	3.300	3.550	20
Walraven	0	0.730	0.990	1.180	1.350	1.490	1.620	1.730	25
Loov	0	1.940	2.740	3.360	3.880	4.330	4.750	5.130	25
Loov &patnaik	0	2.010	2.670	3.200	3.650	4.060	4.420	4.760	25
Average	0	1.560	2.130	2.580	2.960	3.290	3.600	3.870	25
Walraven	0	0.720	1.000	1.210	1.380	1.530	1.670	1.800	30
Loov	0	2.120	3.000	3.680	4.250	4.750	5.200	5.620	30
Loov &patnaik	0	2.200	2.930	3.510	4.000	4.440	4.840	5.120	30
Average	0	1.680	2.310	2.800	3.210	3.570	3.900	4.180	30
Walraven	0	0.711	1.000	1.220	1.410	1.570	1.720	1.850	35
Loov	0	2.290	3.240	3.970	4.590	5.130	5.620	6.070	35
Loov &patnaik	0	2.380	3.160	3.790	4.320	4.800	5.230	5.630	35
Average	0	1.790	2.470	3.000	3.440	3.830	4.190	4.520	35
Walraven	0	0.707	1.010	1.240	1.430	1.610	1.760	1.910	40
Loov	0	2.450	3.470	4.250	4.900	5.480	6.000	6.490	40
Loov &patnaik	0	2,54	3.380	4.050	4.620	5.130	5.590	6.020	40
Average	0	1.052	2.620	3.180	3.650	4.073	4.450	4.807	40

Table (4): Values of polynomial parameters corresponding to concrete compressive strengths

f_c' MPa	C1	C2	C3	C4	C5	C6
10	0.2708	-1.9313	5.2039	-6.6769	4.8195	0.0023
15	0.3069	-2.2018	5.9617	-7.6844	5.6319	0.0027
20	0.3390	-2.4433	6.6586	-8.6534	6.4146	0.0027
25	0.3666	-2.6403	7.1863	-9.3234	6.9258	0.0034
30	0.3784	-2.7410	7.5077	-9.8271	7.4124	0.0034
35	0.4040	-2.9094	7.9426	-10.389	7.8772	0.0036
40	0.4407	-3.1606	8.5854	-11.5854	8.3942	0.0036

.

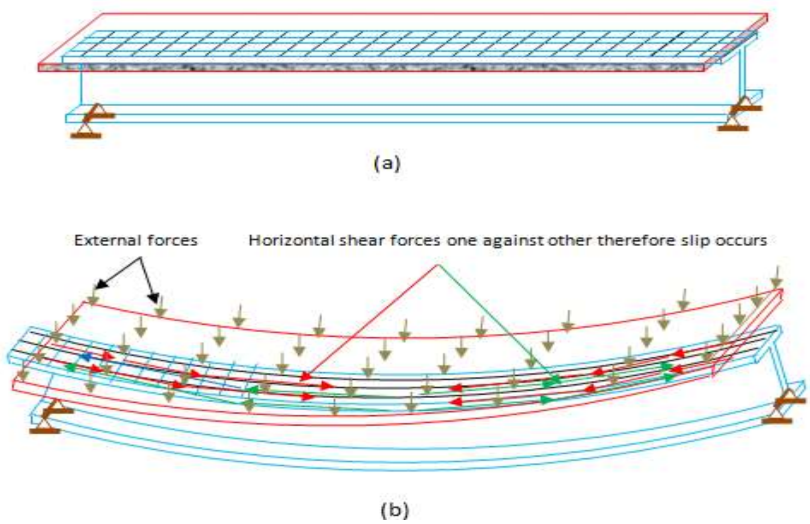
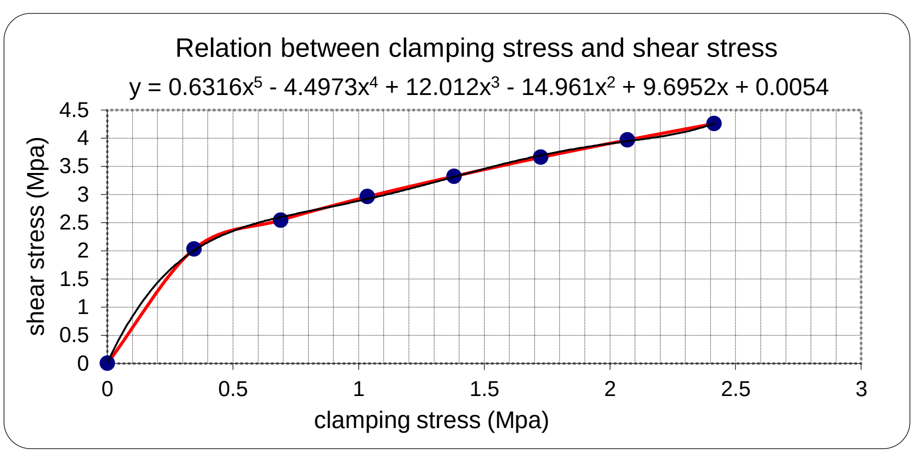


Figure (1): Horizontal shear forces (a) before slip. (b)after slip.



Figure(2): Relation between clamping stress $\rho_v f_y$ and shear strength v_n.

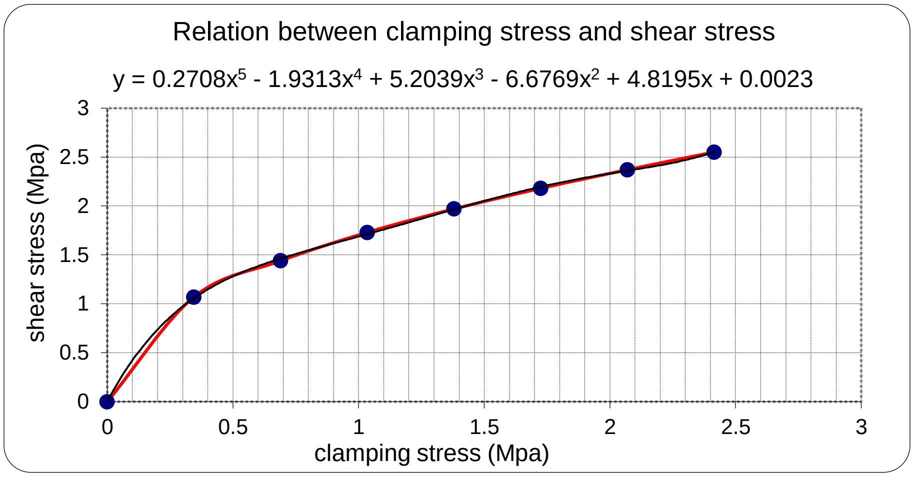


Figure (3): Relation between clamping stress $\rho_v f_y$ and shear strength v_n for f_c'=10 MPa.

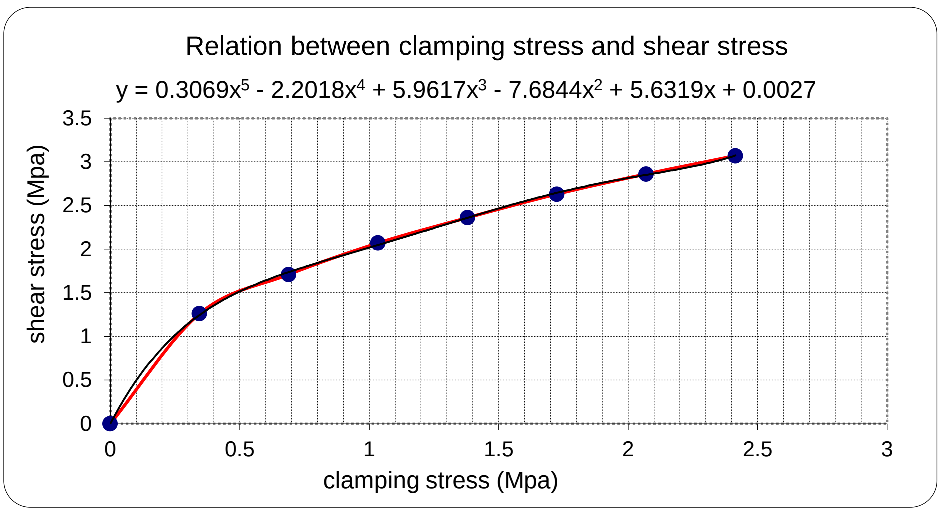


Figure (4): Relation between clamping stress $\rho_v f_y$ and shear strength v_n for f_c'=15 MPa.

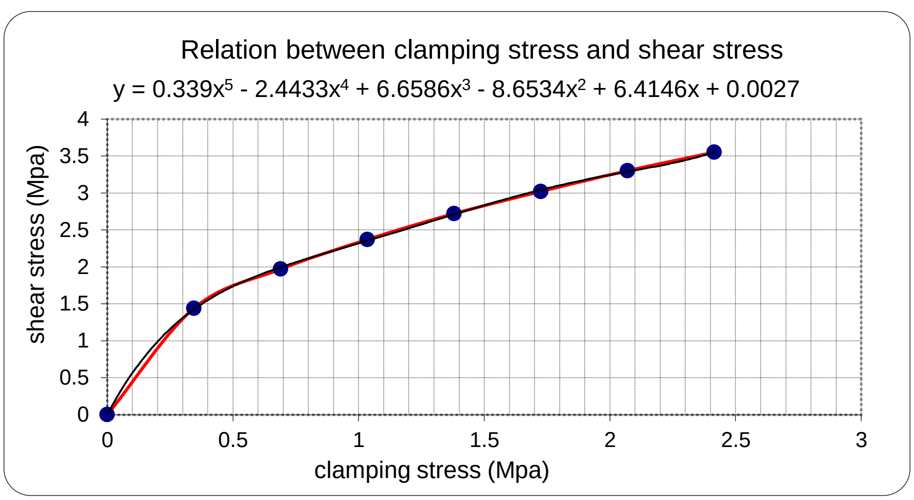


Figure (5): Relation between clamping stress $\rho_v f_y$ and shear strength v_n for f_c'=20 MPa.

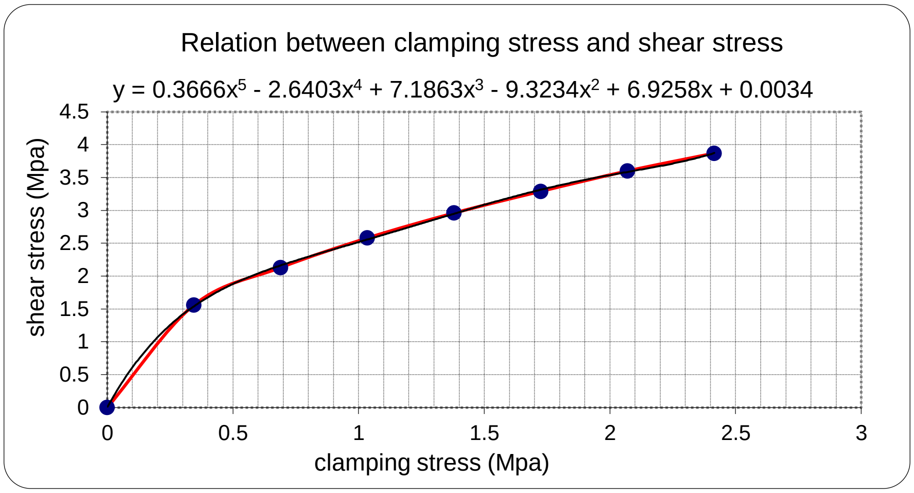


Figure (6): Relation between clamping stress $\rho_v f_y$ and shear strength v_n for f_c'=25 MPa.

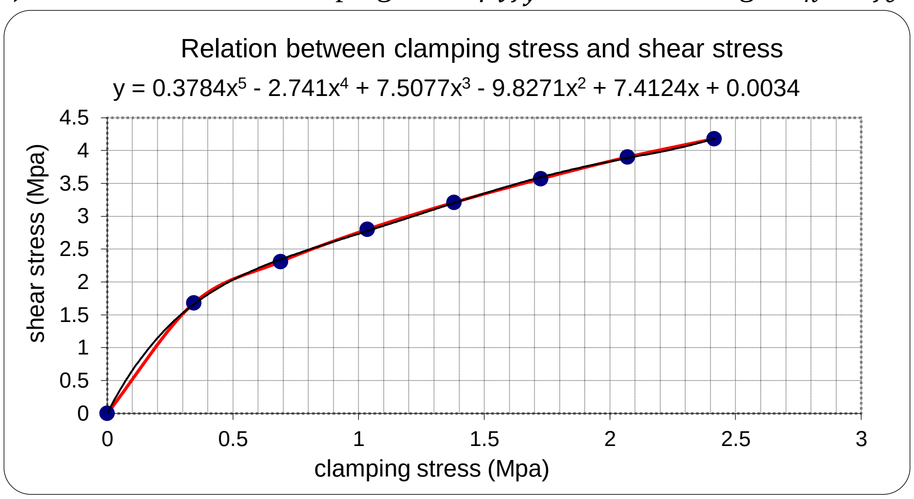


Figure (7): Relation between clamping stress $\rho_v f_y$ and shear strength v_n for f_c'=30 MPa.

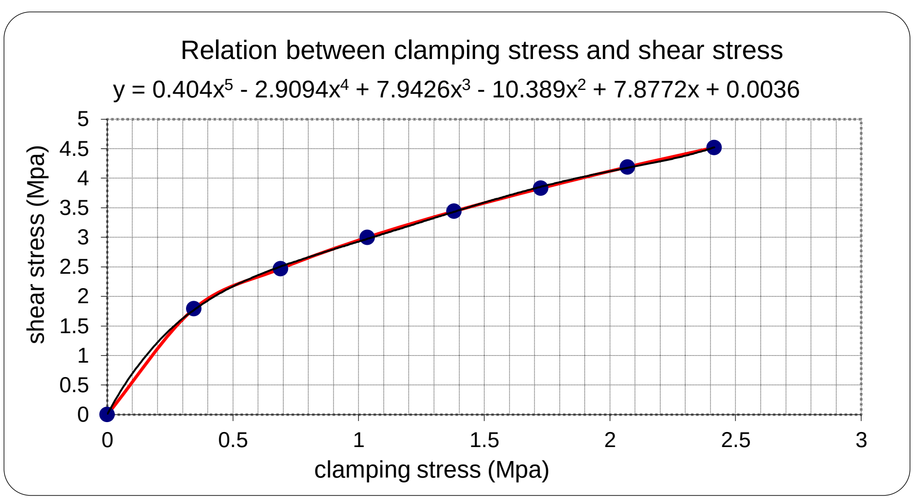


Figure (8): Relation between clamping stress $\rho_v f_y$ and shear strength v_n for f_c'=35 MPa.

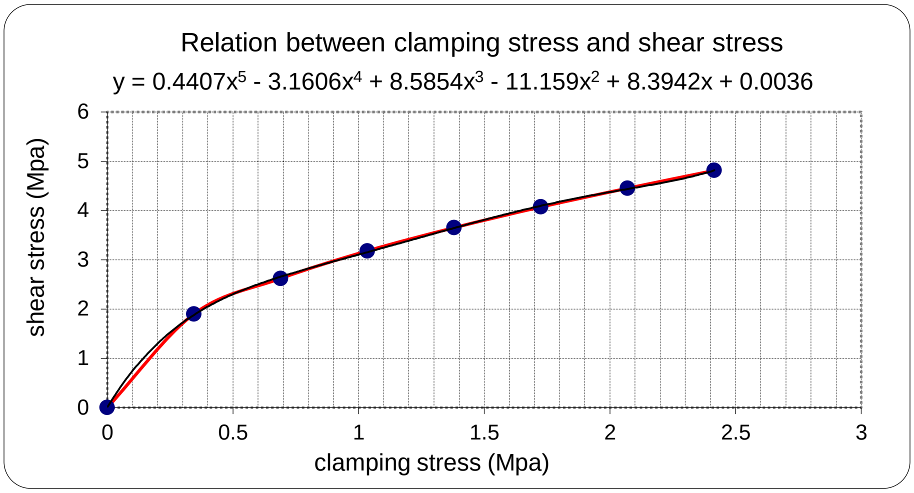


Figure (9): Relation between clamping stress $\rho_v f_y$ and shear strength v_n for f_c' =40 MPa.

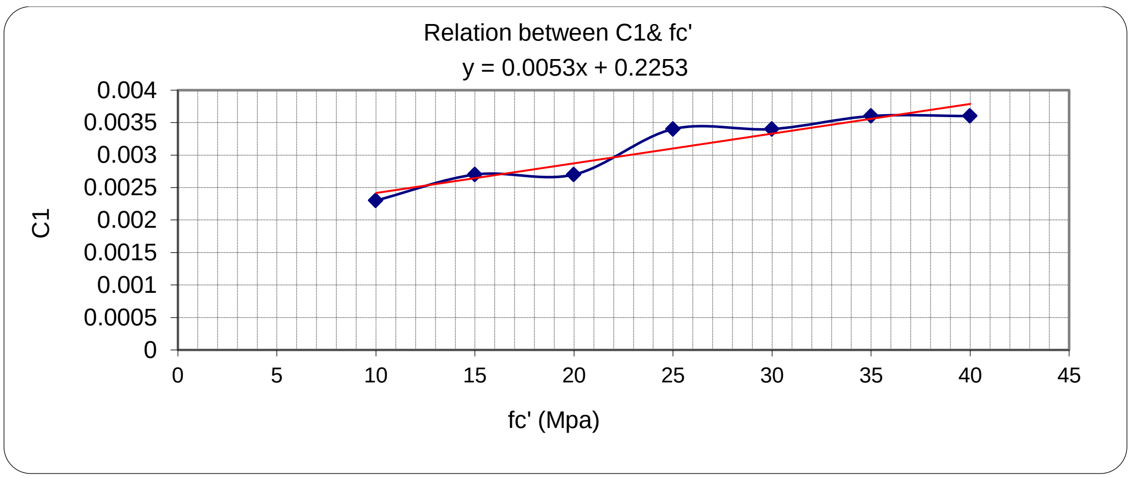


Figure (10): Relation between C1 and f_c'.

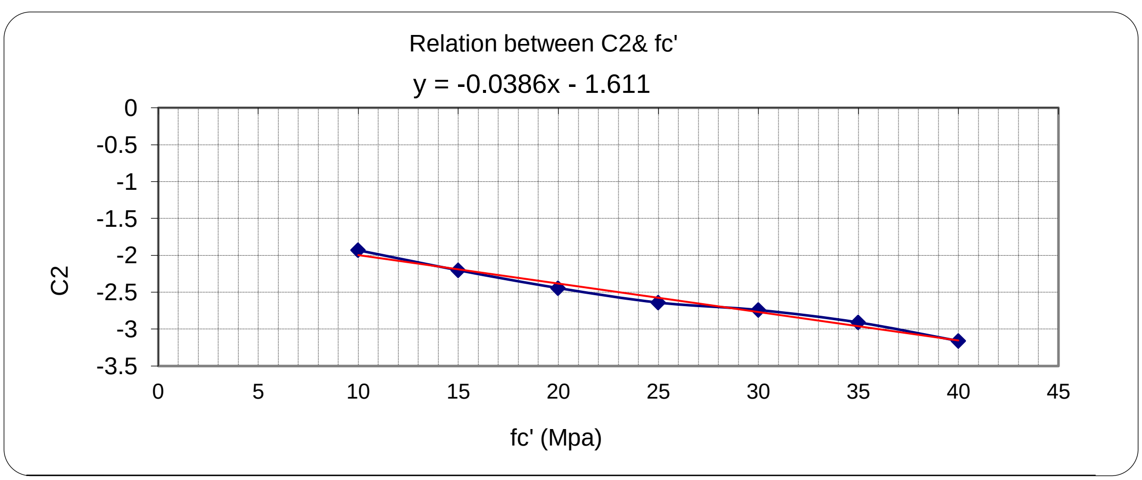


Figure (11): Relation between C2 and f_c'.

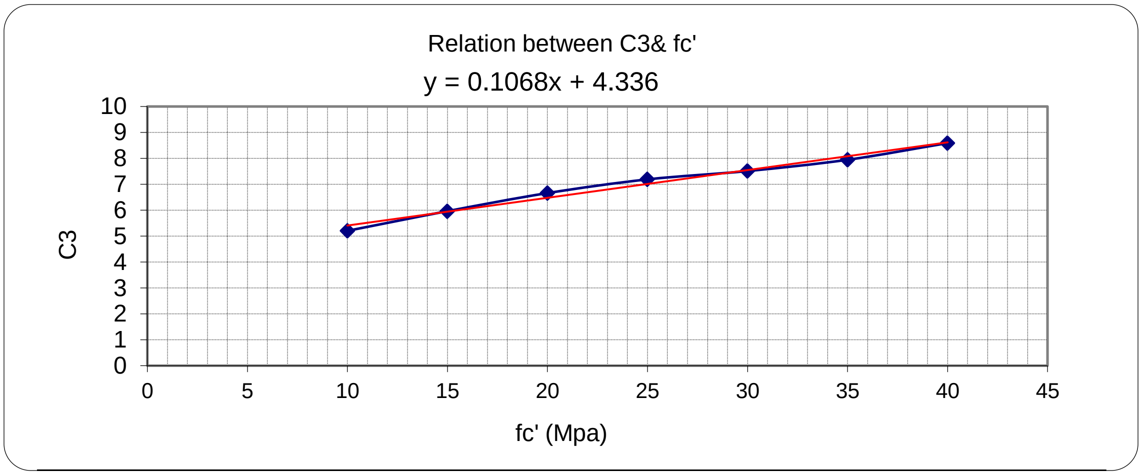


Figure (12): Relation between C3 and f_c'.

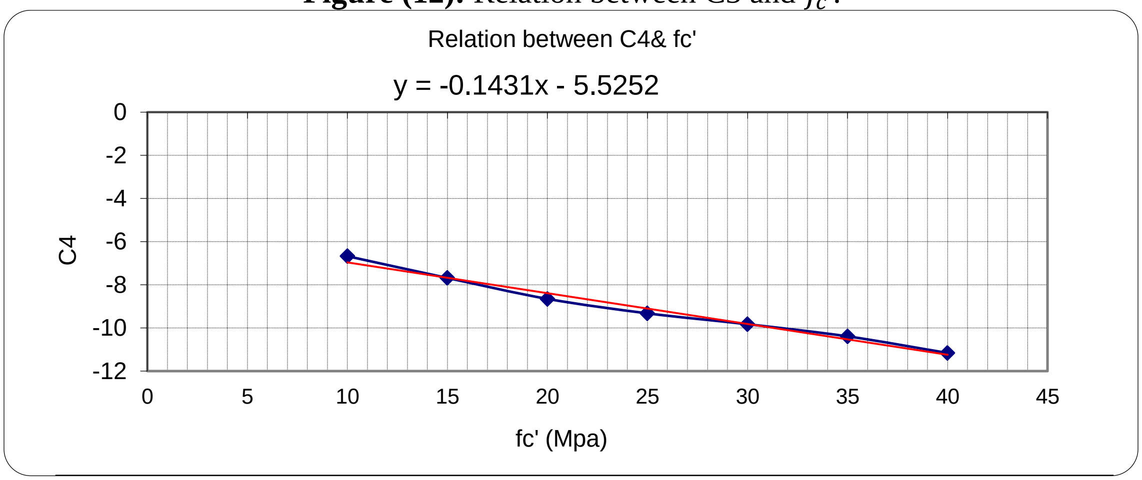


Figure (13): Relation between C4 and f_c'.

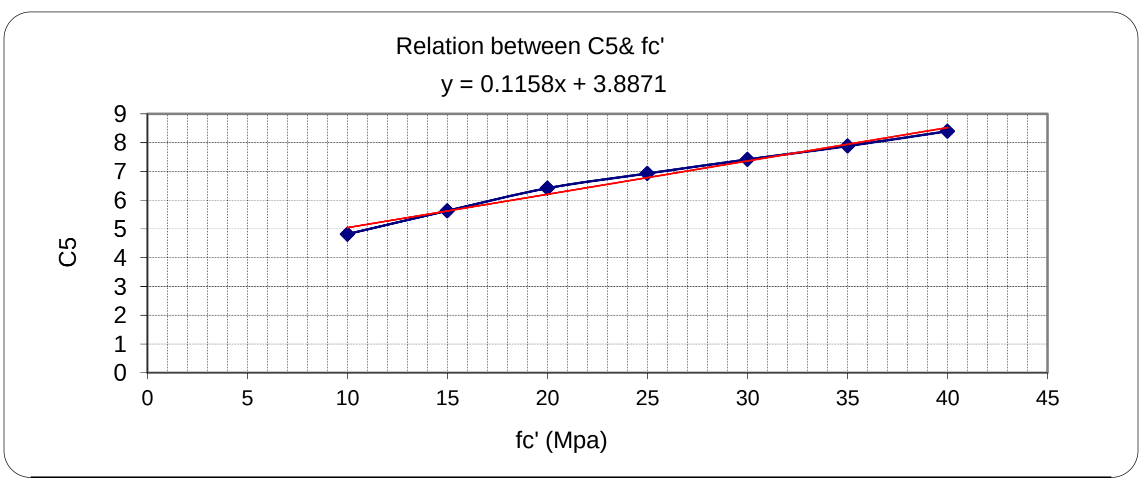


Figure (14): Relation between C5 and f_c'.

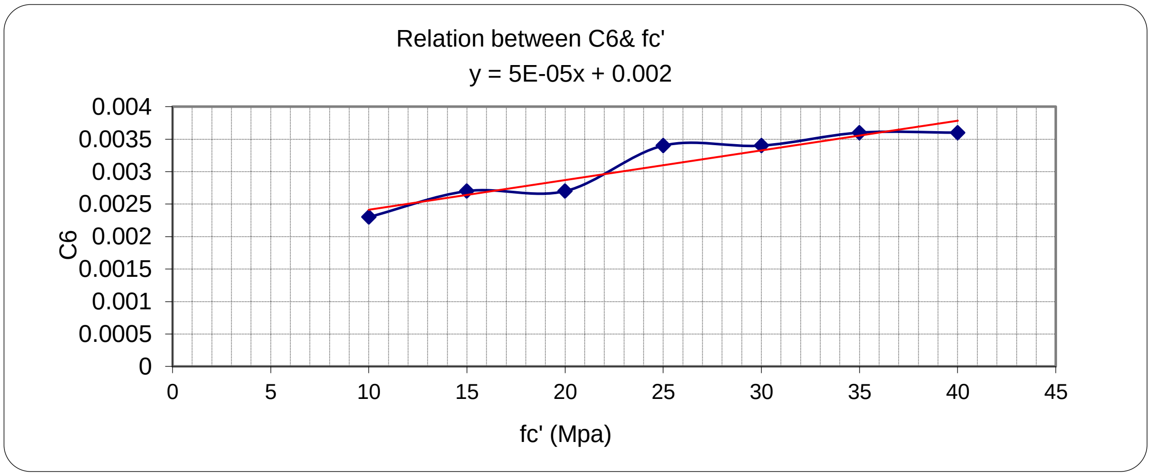


Figure (15): Relation between C6 and f_c'.

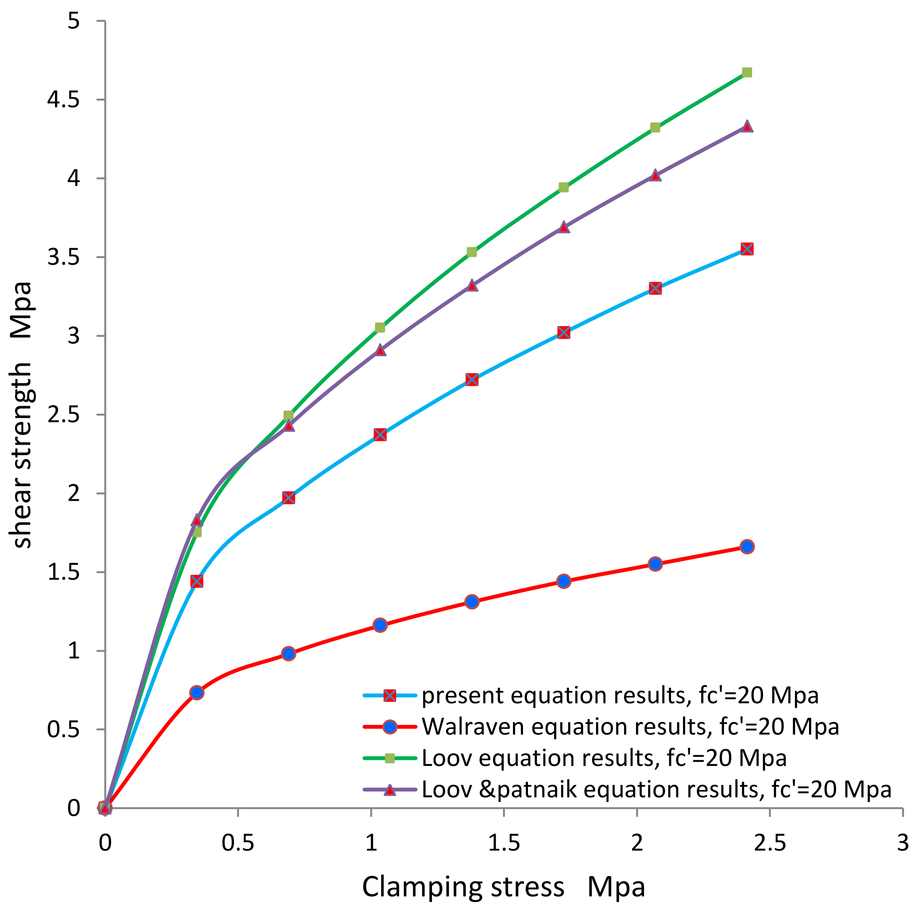


Figure (16): Comparison between results of shear strength governed by present equation and others.

From the results of comparison, the following conclusion points can be drawn: