OPTIMAL SELECTION OF HEAT TRANSFER SURFACES FOR PLATE-FIN HEAT EXCHANGERS

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Abstract

The surface selection method is the performance evaluation criteria (PEC) to reach an optimum surface selection. The method is not limited to surfaces found in the literature, but will accommodate any type of the heat transfer surfaces. The capability is demonstrated by the surface of selection of gas /gas plate-fin heat exchanger. A general methodology for plate-fin heat exchangers surface selection has been shown and applied for a specific illustration problems. The objective of the present work can be summarized as the selection of the plate-fin surfaces (high performance surfaces) depending on the qualitative and quantitive consideration for low Re.

Keyword: Heat exchangers, Compact, Surfaces, Fins, Optimization.

الاختيار الأمثل لسطوح انتقال الحرارة لمبادلات صفيحة-زعنفة الحرارية

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الخلاصة

إن طريقة الاختيار المثلى لسطوح انتقال الحرارة في هذا البحث هي طريقة تقييم الأداء للوصول إلى امثل اختيار. هذه الطريقة غير محددة بالسطوح الموجودة بالبحث فحسب, بل هي ملائمة لكل سطوح انتقال الحرارة. تم تطبيق هذة الطريقة على اختيار سطوح انتقال الحرارة لمبادل غاز-غاز الحراري ذو الصفيحة-زعنفة. كذلك تم عرض أسلوب وطريقة الاختيار المثلى بمسائل توضيحية. هذا البحث يعتمد على الاعتبارات الكمية والنوعية للوصول للاختيار الأمثل للسطوح الحرارية لمبادلات الصفيحة-زعنفة الحرارية.

Nomenclature

A : Total heat transfer area, m^2

A_{fr} : Frontal or face area on one side of the exchanger, m^2

A_f : Fin or extended surface on one side of the exchanger, m^2

A_0 : Minimum free flow area on one side of the exchanger, m^2

b : Distance between two plates (fin height)in a plate-fin exchanger, m

C : Flow stream heat capacity rate, $\dot{m}C_p, W/^oC$

C_p : Specific heat of fluid at constant pressure, J/kg.^oC

D_h : Hydraulic diameter of flow passages, $4\,A_O L/A, m$

E_{std} : pumping power per unit of heat transfer surface area

f : Fanning friction factor, dimensionless

G : Exchanger flow-stream mass velocity, $k_g/m^2.s$

h : Heat transfer coefficient, $W/m^2.^oC$

j : Colburn factor, St $\Pr^{2/3}$, dimensionless

L : Fluid flow (core) length on one side of the exchanger, m

$\dot{m}$: Fluid mass flow rate, , kg/s

NTU : Number of heat transfer units, dimensionless

Pr : Prandtl number, dimensionless

Q : Heat transfer, W

Re : Reynolds number, dimensionless

P : Pimping Power, W

p : Pressure, N/m^2

p_f : Fin Pitch, mm

St : Stanton number, h/GC_p, dimensionless

α : Ratio of free-flow area to the frontal area

β : Ratio of total heat transfer area on one side of a plate-fin heat exchanger to the volume between the plates on that side

η_0 : Total surface effectiveness

μ : Viscosity coefficient, Pa.s

ρ : Density, kg/m^3

Δp : Pressure drop, Pa

δ : Fin thickness, mm

Subscripts

f : fin

max : Maximum

min : Minimum

T : Triangular

R : Rectangular

s : Reference surface in performance evaluation criteria (PEC)

std : Standard temperature and pressure

Introduction

A proper selection of surface is one of the most important considerations in plate-fin heat exchanger design. There is no such thing as surface that is best for all applications. The particular application strongly influences the selection of the surface to be used. The objective of the system in which the heat exchanger is to be used also influences the surface selection. Both qualitative and quantitative considerations will be presented for surface selection. Comprehensive heat exchanger optimization is a formidable task, complicated by many qualitative and quantitive considerations affecting the selection of a surface pattern (Shah, 1978). The heat exchanger in the present work represents a one type of the plate-fin heat exchangers. The design of a plate-fin heat exchanger is a complex task requiring the examination and optimization of a wide variety of heat transfer surfaces. Studies have shown that a poor choice of either the heat transfer surfaces or design parameters can be more than double the costs chargeable to a heat exchanger . The selection method, in the present study, is based on the Performance Evaluation Criteria (PEC) of Webb (1981). The qualitative considerations for surface selection are the operating temperatures, pressure, fluid contamination, cost, maintenance, and ruggedness. The problem of heat exchanger design is very intricate. Because of a large number of qualitative judgments, trade-offs and compromises, the heat exchanger design is more of an art at this stage. In general, no two engineers will come up with the same heat exchanger design for a given application. Most probably a "better" design will be arrived at experienced engineer. Somewhat arbitrarily, a plate-fin surface will be specified that has an area density β greater than 700 m^2/m^3 (Shah, 1983). The uniqueness of plate-fin and enhanced exchangers are: (1) many surfaces available having different orders of magnitude of surface area density; (2) flexibility in distributing the area on the hot and cold sides as desired by design considerations; and (3) generally substantial cost, weight, or volume savings (Shah and Webb, 1983). Fins are attached to the plates by mechanical fit, gluing, soldering, welding, or extrusion.

In the automotive industry, fins in the plate-fin heat exchangers unit is referred to as centers in order to distinguish them from fins outside of the tubes in a tube- fin exchanger. The latter are simply referred to as fins. Fins used in a plate-fin heat exchangers exchanger are categorized as follows: (1) plain (uncut surfaces) and straight fins; (2) plain but wavy fins; and (3) interrupted fins such as strip, louver and perforated fins. The velocity and temperature boundary layers thicken on plain surfaces resulting in both a lower heat transfer coefficient and lower friction factor. Plain fins are used when the pressure drop is critical and interrupted or wavy fins cannot meet the pressure drop requirement together with a flow area constraint. Plain fins are made such that the flow passages have triangular, rectangular or other noncircular shapes. When the plain fin is formed such that it has a wavy surface in the flow direction, the boundary layers are either thinned or interrupted when the flow is turned resulting in both higher heat transfer coefficient and a higher friction factor. Boundary layers can be more completely discontinuous. Examples are strip fins, and perforated fins. Strip fins are also referred to as also offset– fins, lance– offset fins, serrated fins or segmented fins (Shah, 1981b). Specific qualitative considerations for Plate- Fin Surfaces (Shah, 1983). A number of heat- exchanger design methods have been proposed to determine the heat exchanger design. Bergles et al. (1974) performed an evaluation of different objective functions for plate-fin heat exchangers with different heat- transfer surfaces, but with the same specifications. The method did not include any actual optimization techniques, but results did show that a great improvement in heat exchanger performance could be made by proper selection of design parameters.

Quantitative Selection

Surface selection is made by comparing performance of various heat exchanger surfaces and choosing the best under some specified criteria for a given heat exchanger application. The performance evaluation criteria used in the comparison were those recommended by Shah (1978). These criteria all required the j factor, friction factor f, and Reynolds number, together with the geometry specification. Where the gas- side properties were required at a standard temperature and pressure, these were taken to be for dry air at $25\ ^{o}C$ and 1.01325 bar, respectively. The ratio of j factor to the friction factor, against Reynolds number, generally known as the "flow area goodness factor," suggested by London (1964), where

$$j/f = \frac{1}{A_0^2}\left[\frac{\Pr^{2/3}\ NTU\ \dot{m}^2}{2\Delta p}\right] \quad (1)$$

where $NTU = UA/C_{\min}$

The standardized heat transfer coefficient against the pumping power per unit of heat transfer surface area suggested by London and Ferguson (1949), where

$$h_{std} = \frac{j\cdot \mathrm{Re}\cdot \mu_{std}\cdot C_{p,std}}{D_h\cdot \Pr_{std}^{2/3}} \quad (2)$$

And

$$\mathrm{E}_{std} = \frac{f\cdot \mathrm{Re}^3\cdot \mu_{std}^3}{2\cdot \rho_{std}^2\cdot D_h^3} \quad (3)$$

The performance of the heat exchanger per unit volume, the criteria Suggested by Shah (1978). This method includes the effect of the fin effectiveness, Which is an important factor in heat exchanger evaluation. A good performance-using criterion gives the best heat exchanger to use where the size of the unit is an important consideration.

$$\eta_o\cdot h_{std}\cdot \beta = \frac{j\cdot \mathrm{Re}\cdot 4\cdot \sigma\cdot \eta_o\cdot C_{p,std}\cdot \mu_{std}}{\Pr_{std}^{2/3}\cdot D_h^{\ 2}} \quad (4)$$

And

$$\mathrm{E}_{std}\cdot \beta = \frac{f\cdot \mathrm{Re}^3\cdot 4\cdot \sigma\cdot \mu_{std}^3}{2\cdot \rho_{std}^2\cdot D_h^4} \quad (5)$$

For a given PEC, the ratio of the design objective for surface of interest to a reference is then calculated as function of a similar ratio of a design variable. Equationds (6) and (2.7) are the generalized equations necessary for calculation of the performance improvement afforded by the enhanced surface.

$$\frac{P}{P_s} = \left(\frac{f}{f_s}\right)\left(\frac{\mathrm{A}}{\mathrm{A}_s}\right)\left(\frac{G}{G_s}\right)^3 \quad (6)$$

$$\frac{U\mathrm{A}}{U_s\mathrm{A}_s} = \frac{1}{\frac{St_s}{St}\left\{\frac{f}{f_s}\cdot\frac{P}{P_s}\cdot\left(\frac{A}{A_s}\right)^2\right\}^{1/3}} \quad (7)$$

The advantage of this comparison method for plat-fin heat exchangers are: (1) the designer can select his own criteria for comparison; (2) he can then compare performance of a surface to that of a reference surface directly; and (3) he does not need to evaluate the fluid properties since they drop out in computing the performance ratios. However, the performance comparison is considered only for one side of a compact heat exchanger. Surfaces on each side of an exchanger may be selected by one of the methods of the preceding section. When such surfaces are incorporated in a heat exchanger, the resultant exchanger may not be optimum since criteria other than those related to j and f may play an important role. For example, in addition to prescribed heat transfer and pressure drop, the core frontal area may have been restricted on one side of the exchanger. If the surface selected by one of the proceeding methods requires larger than specified frontal area, it will not meet the design specifications and hence it will not be considered in the final selection (Shah, 1983). In general heat exchangers are designed for many varied applications, and hence may involve many different performance criteria. Some of these criteria may be minimum initial and operating costs, minimum weight or material, minimum volume or heat transfer surface area, minimum mean temperature difference, maximum heat transfer rate, and so on. When a single performance measure has been defined qualitatively and is to be minimized or maximized, it is called an “objective function” in design optimization. A particular design also be subjected to certain requirements such as required heat transfer, allowable pressure drop, limitation on height, width and/or length of the exchanger, and so on These requirements are called “constraints” in a design optimization.

Method of Optimal Selection

We can use the tables for the surface parameters such as tables in Kays and London(1964). If the parameters were not found in the literature we can use the following procedure:
Using the nomenclature of **Fig. 1**

$$D_h = \frac{4\,flow\ area}{wetted\ \ perimeter} \tag{8}$$

$$D_h = \frac{2(p_f-\delta)(b-\delta}{(b-\delta)+(p_f-\delta)} \quad \text{(for plain rectangular fin)} \tag{9}$$

$$D_h = \frac{2(p_f-\delta)(b-\delta)}{(p_f-\delta)+\dfrac{(b-\delta)}{\sin\theta}} \quad \text{(for triangular fin)} \tag{9a}$$

$$\sigma_f = \frac{flow\ area}{frontal\ area} = \frac{(p_f-\delta)(b-\delta)}{(p_f+\delta)(b)} \quad \text{(for plain rectangular fin)} \tag{10}$$

$$\sigma_f = \frac{(p_f-\delta)(b-\delta)}{(p_f)(b)} \quad \text{(for triangular fin)} \tag{10a}$$

$$\beta = \frac{4\sigma}{D_h} \tag{11}$$

$$j = \frac{3}{\mathrm{Re}},\ f = \frac{14}{\mathrm{Re}} \quad \text{(for plain rectangular fin)(Shah and London, 1978)} \tag{11a}$$

$$j = \frac{Nu.k/D_h}{GC_p\,\mathrm{Pr}^{-2/3}},\ Nu = 4.439,\ f = \frac{18.233}{\mathrm{Re}} \text{(for triangular fin)} \tag{11b}$$

also we will derive two cases(case 1 and case 6) only to illustrate the procedure. The other criteria derived by the same procedure.

We will use the following optimization cases according to the aims of a customer

<u>Case1:</u> Max Q subject to

$$\frac{A_{fr}}{A_{fr,s}}=1,\ \frac{L}{L_s}=1,\ \frac{\dot{m}}{\dot{m}_s}=1,\ \frac{\mathrm{Re}}{\mathrm{Re}_s}=1,\ \frac{P}{P_s}>1,\ \frac{Q}{Q_s}>1,\frac{\Delta T_i}{\Delta T_{is}}=1$$

$$\left(\frac{P}{P_s}\right)=\left(\frac{f}{f_s}\right)=\left(\frac{A}{A_s}\right)=\left(\frac{G}{G_s}\right) \tag{12}$$

since $\left(\frac{G}{G_s}\right)=\left(\frac{\mathrm{Re}}{\mathrm{Re}_s}\right)=1$ (given),

eq. (12) becomes

$$\left(\frac{P}{P_s}\right)=\left(\frac{f}{f_f}\right) \tag{13}$$

$$\left(\frac{A_0}{A_{0s}}\right)=\left(\frac{h/P}{h_s/P_s}\right)=\left(\frac{f/f_s}{h_s/P_s}\right) \tag{14}$$

$$\left(\frac{hA}{h_sA_s}\right)=\left(\frac{Q}{Q_s}\right) \tag{15}$$

since same heat transfer area eq. (15) becomes

$$\left(\frac{h}{h_s}\right)=\left(\frac{Q}{Q_s}\right) \tag{16}$$

eq. (14) becomes

$$\left(\frac{h}{h_s}\right)=\left(\frac{P}{P_s}\right)\left(\frac{j/j_s}{f/f_s}\right) \tag{17}$$

consolidating eqs. (13), (16), and (17) yields

$$\left(\frac{Q}{Q_s}\right)=\left(\frac{h}{h_s}\right)=\left(\frac{j}{j_s}\right) \tag{18}$$

<u>Case2</u>: Min ΔT_i subject to

$$\frac{A_{fr}}{A_{fr,s}}=1,\ \frac{L}{L_s}=1,\ \frac{\dot{m}}{\dot{m}_s}=1,\ \frac{\mathrm{Re}}{\mathrm{Re}_s}=1,\ \frac{P}{P_s}=1,\ \frac{Q}{Q_s}=1,\frac{\Delta T_i}{\Delta T_{is}}<1,\text{ yields }\frac{\Delta T_i}{\Delta T_{is}}=\frac{\left(f/f_s\right)^{1/3}}{j/j_s}$$

<u>Case3</u>: Max Q subject to

$$\frac{A_{fr}}{A_{fr,s}}=1,\ \frac{L}{L_s}=1,\ \frac{\dot{m}}{\dot{m}_s}<1,\ \frac{\mathrm{Re}}{\mathrm{Re}_s}<1,\ \frac{P}{P_s}=1,\ \frac{Q}{Q_s}>1,\ \frac{\Delta T_i}{\Delta T_{is}}=1,\text{ yields}$$

$$\left(\frac{\mathrm{Re}}{\mathrm{Re}_s}\right)=\frac{W}{W_s}=\left(f/f_s\right)^{-1/3},\ \left(\frac{Q}{Q_s}\right)=\left(\frac{j/j_s}{f/f_s}\right)\left(\frac{W}{W_s}\right)^{-2}$$

<u>Case4:</u> Min ΔT_i subject to

$$\frac{A_{fr}}{A_{fr,s}}=1,\ \frac{L}{L_s}=1,\ \frac{\dot{m}}{\dot{m}_s}<1,\ \frac{\mathrm{Re}}{\mathrm{Re}_s}<1,\ \frac{P}{P_s}=1,\ \frac{Q}{Q_s}>1,\frac{\Delta T_i}{\Delta T_{is}}=1, \text{ yields}$$

$$\frac{\mathrm{Re}}{\mathrm{Re}_s}=\frac{W}{W_s}=\left(\frac{f}{f_s}\right)^{-1/3},\ \ \frac{\Delta T_i}{\Delta T_{is}}=\frac{\left(f/f_s\right)^{1/3}}{\left(j/j_s\right)}$$

<u>Case5</u>: Min P subject to

$$\frac{A_{fr}}{A_{fr,s}}=1,\ \frac{L}{L_s}=1,\ \frac{\dot{m}}{\dot{m}_s}<1,\ \frac{\mathrm{Re}}{\mathrm{Re}_s}<1,\ \frac{P}{P_s}<1,\frac{Q}{Q_s}=1\ ,\frac{\Delta T_i}{\Delta T_{is}}=1, \text{ yields}$$

$$\frac{P}{P_s}=\frac{\left(j/j_s\right)^3}{\left(f/f_s\right)},\ \frac{W}{W_s}=\frac{\mathrm{Re}}{\mathrm{Re}_s}=\left(\frac{P}{P_s}\right)^{1/2}\left(\frac{j/j_s}{f/f_s}\right)^{1/2}$$

<u>Case6</u>: Min L subject to

$$\frac{A_{fr}}{A_{fr,s}}=1,\ \frac{L}{L_s}<1,\ \frac{\dot{m}}{\dot{m}_s}<1,\ \frac{\mathrm{Re}}{\mathrm{Re}_s}<1,\ \frac{P}{P_s}=1,\ \frac{Q}{Q_s}=1,\frac{\Delta T_i}{\Delta T_{is}}=1$$

Using eq. (6) with the constraint of case 6 yields

$$1=\left(\frac{f}{f_s}\right)=\left(\frac{A}{A_s}\right)=\left(\frac{G}{G_s}\right)^3$$

then

$$\left(\frac{A}{A_s}\right)=\left(\frac{f_s}{f}\right)\left(\frac{G_s}{G}\right)^3 \tag{19a}$$

$$1=\left(\frac{\dot{m}}{\dot{m}}\right)^2\left(\frac{f/f_s}{j/j_s}\right) \tag{19b}$$

then

$$\left(\frac{\dot{m}}{\dot{m}_s}\right)=\left(\frac{j/j_s}{f/f_s}\right)^{1/2} \tag{20}$$

$$\frac{L}{L_s}=\frac{V}{V_s}=\frac{\beta_s}{\beta}.\frac{A}{A_s} \tag{21}$$

eqs. (19), (20), and (21) represents the performance evaluation criteria for the case 6, which used for any plate-fin surfaces(fins).

<u>Case7</u>: Min L subject to

$$\frac{A_{fr}}{A_{fr,s}}=1,\ \frac{L}{L_s}<1,\ \frac{\dot{m}}{\dot{m}_s}<1,\ \frac{\mathrm{Re}}{\mathrm{Re}_s}=1,\ \frac{P}{P_s}<1,\frac{Q}{Q_s}=1\ ,\frac{\Delta T_i}{\Delta T_{is}}=1, \text{ yields}$$

$$\left(\frac{P}{P_s}\right)=\left(\frac{f/f_s}{j/j_s}\right),\ \frac{A}{A_s}=\frac{\left(P/P_s\right)}{\left(f/f_s\right)},\ \frac{L}{L_s}=\left(\frac{A}{A_s}\right)\left(\frac{\beta}{\beta_s}\right)^{-1}$$

Case8: Min P subject to

$$\frac{A_{fr}}{A_{fr,s}}=1,\ \frac{L}{L_s}<1,\ \frac{\dot{m}}{\dot{m}_s}=1,\ \frac{\mathrm{Re}}{\mathrm{Re}_s}=1,\ \frac{P}{P_s}<1,\frac{Q}{Q_s}=1\ ,\frac{\Delta T_i}{\Delta T_{is}}=1,$$

$$\frac{P}{P_S}=\left(\frac{f/f_S}{j/j_S}\right),\ \frac{A}{A_s}=(P/P_s)(f/f_s)^{-1},\ \frac{L}{L_s}=\left(\frac{A}{A_s}\right)\left(\frac{\beta}{\beta_s}\right)^{-1}$$

Case9: Min A subject to

$$\frac{A_{fr}}{A_{fr,s}}>1,\ \frac{L}{L_s}<1,\ \frac{\dot{m}}{\dot{m}_s}<1,\ \frac{\mathrm{Re}}{\mathrm{Re}_s}<1,\ \frac{P}{P_s}<1,\ \frac{Q}{Q_s}=1,\frac{\Delta T_i}{\Delta T_{is}}>1,\ \text{yields}$$

$$\frac{A_o}{A_{os}}=\left(\frac{f/f_s}{j/j_s}\right)^{1/2},\ \frac{A}{A_s}=\frac{(f/f_s)^{1/2}}{(j/j_s)^{3/2}},\ \frac{\mathrm{Re}}{\mathrm{Re}_s}=\left(\frac{j/j_s}{f/f_s}\right)^{1/2}$$

Case10: Max Q subject to

$$\frac{A_{fr}}{A_{fr,s}}>1,\ \frac{L}{L_s}<1,\ \frac{\dot{m}}{\dot{m}_s}=1,\ \frac{\mathrm{Re}}{\mathrm{Re}_s}<1,\ \frac{P}{P_s}=1,\frac{Q}{Q_s}=1\ ,\frac{\Delta T_i}{\Delta T_{is}}=1,\ \text{yields}$$

$$\left(\frac{A}{A_s}\right)=\frac{(j/j_s)^{3/2}}{(f/f_s)^{1/2}},\qquad \frac{\mathrm{Re}}{\mathrm{Re}_s}=(f/f_s)^{-1/3}(A/A_s)^{-1/3},\qquad \frac{A_o}{A_{os}}=\left(\frac{\mathrm{Re}}{\mathrm{Re}_s}\right)^{-1},\qquad \frac{Q}{Q_s}=(A_o/A_{os})^2\left(\frac{j/j_s}{f/f_s}\right),$$

$$\frac{L}{L_s}=\left(\frac{A}{A_s}\right)\left(\frac{\beta}{\beta_s}\right)^{-1}$$

Case11: Min ΔT_i subject to

$$\frac{A_{fr}}{A_{fr,s}}>1,\ \frac{L}{L_s}<1,\ \frac{\dot{m}}{\dot{m}_s}=1,\ \frac{\mathrm{Re}}{\mathrm{Re}_s}<1,\ \frac{P}{P_s}=1,\frac{Q}{Q_s}=1\ ,\frac{\Delta T_i}{\Delta T_{is}}<1,\ \text{yields}$$

$$\left(\frac{A_o}{A_{os}}\right)=\left(\frac{f/f_s}{j/j_s}\right)^{1/2},\ \frac{A}{A_s}=(A_o/A_{os})^3/(f/f_s),\ \frac{\Delta T_i}{\Delta T_{is}}=(A/A_s)^{2/3}(f/f_s)^{1/3}(j/j_s)^{-1},\ \frac{L}{L_s}=\left(\frac{A}{A_s}\right)\left(\frac{\beta}{\beta_s}\right)^{-1}$$

Case12: Min P subject to

$$\frac{A_{fr}}{A_{fr,s}}>1,\ \frac{L}{L_s}<1,\ \frac{\dot{m}}{\dot{m}_s}=1,\ \frac{\mathrm{Re}}{\mathrm{Re}_s}<1,\ \frac{P}{P_s}<1,\frac{Q}{Q_s}=1\ ,\frac{\Delta T_i}{\Delta T_{is}}=1,$$

$$\frac{A}{A_s}=(f/f_s)^2(j/j_s)^{-2},\ \frac{P}{P_s}=(j/j_s)^3(f/f_s)^{-1}(A/A_s)^{-2},\ \frac{A_o}{A_{os}}=\left(\frac{f/f_s}{j/j_s}\right)^{1/2}(P/P_s)^{-1/2},\ \frac{\mathrm{Re}}{\mathrm{Re}_s}=\left(\frac{A_o}{A_{os}}\right)^{-1}$$

Results and Discussions

In the present work, the air/air will be taken for the plate-fin heat exchanger. Hence the objective function for PEC at gas side is to minimize the pumping power. But, the objective function for the air side is to minimize the heat transfer area for the same heat recovery. However, the selection procedure will be illustrated by a specific examples for each side.

1-Air-Side Surface Selection

Offset Strip Fin (OSF)and plain fins for the air side to be compared as a reference and augmented surfaces. The two surfaces to compared are the surface 101 (London and Shah, 1968)as shown in **Table(1)**, and the surface 10-27 (Kays and London, 1964)as shown in **Table(2)**. The case 9 was chosen is which minimizing the surface area. The flow cross-sectional area required to obtain hA for operation at fixed friction power (P) and flow ($\dot{m}$) is given by, see eq. (1)

$$A_o^2 = \frac{\Pr^{2/3} hA\dot{m}^2}{2C_p\rho}\frac{f}{j} \tag{22}$$

By setting $hA/h_sA_s = P/P_s = 1$ for $\dot{m}/\dot{m}_s$ =1, eq. (22) may be written in the nondimensional form

$$\left(\frac{A_o}{A_{os}}\right)^2 = \frac{G_s^2}{G^2} = \frac{f/f_s}{j/j_s} \tag{23}$$

Equation (23) defines the ratio of mass velocities required to satisfy the constraints for case 9. A/A_s Ratio is calculated from eq. (6) as follow

$$(A/A_s) = \frac{(f/f_s)^{1/2}}{(j/j_s)^{3/2}} \tag{24}$$

Before applying these equations, the surface 101 must be scaled to give same D_h of the plain-fin surface. Surface 101 (London and Shah, 1968) represents OSF surfaces, and surface 10-27 (Kays and London, 1964) represents the plain fin surfaces. After scaling the dimensions, which is found, the following geometries for the surface 101 will be obtained:

b=12.2 mm, D_h=3.51 mm, δ=0.2mm , β=1024 m^2/m^3 , and σ_f=0.889

The plain fin surface is assumed to operate at Re = 900. Since both geometries have the same D_h, (Re_s /Re)^2 is given by eq. (23). Using the j and f vs Re data, see **Table(1)**, for both surfaces. eq. (23) will be solved as follow.

$$\left(\frac{\mathrm{Re}_s}{\mathrm{Re}}\right)^2 = \frac{f/f_s}{j/j_s} = \frac{0.061/0.0253}{0.01535/0.0074} = 1.1623 \text{ Then } \left(\frac{\mathrm{Re}_s}{\mathrm{Re}}\right) = 1.078$$

Where the reference surfaces s is the plain fin surface. Then, by evaluating eq. (23) for Re_s=900 and Re=834.88 we obtain

$$\left(\frac{A}{A_s}\right) = 0.49\,, \left(\frac{V}{V_s}\right) = \frac{A}{A_s}\frac{\beta_s}{\beta} = (0.49)\left(\frac{1024}{1003}\right) = 0.5$$

using eq. (4) yields

$$\frac{A_{fr}}{A_{fr,s}}=\frac{A_o/\sigma_f}{A_{os}/\sigma_{f,s}}=\frac{A_o}{A_{os}}\cdot\frac{\sigma_{f,s}}{\sigma_f}\ ,\ \frac{A_o}{A_{os}}=\frac{\dot{m}/G}{\dot{m}/G_s}=\frac{G_s}{G}\qquad \text{Then}\ \frac{A_{fr}}{A_{fr,s}}=\frac{G_s}{G}\cdot\frac{\sigma_{f,s}}{\sigma_f}=1.078\left(\frac{0.899}{0.88}\right)=1.1\ \frac{A_{fr}}{A_{fr,s}}$$

Therefore, the offset strip-fin will provide hA as the plain fin using 51% less surface area but will require 10% greater flow frontal area (A_{fr}) to satisfy $P/P_s=\dot{m}/\dot{m}_s=1$ constraints.

2- Gas-Side Surface Selection

By the qualitative criteria, the following surfaces has been selected:
The triangular and rectangular plain fins for the gas side. One of the two surfaces will be a reference surface, and the other must be as an augmented surface for PEC.

This example will be presented using case 8, which use to minimize the pumping power. Two plain fin geometries having the same fin pitch are compared: a triangular geometry (T) and a rectangular (R) with 0.2 mm fin pitch, 8 mm plate spacing, and 0.2 mm fin thickness. The case 9 constrains the two geometries to operate at the same mass flow rate with equal frontal velocity and hA values. Since the velocities are known, the j and f values are directly calculable. The calculations are made for air flow at 25^oC and 4 m/sec frontal velocity. The j factor is given by fully developed laminar solution for constant wall temperature. After scaling the triangular fin dimension to give the same D_h of the rectangular fin we obtain the following dimension
Using the calculated j and f values we obtain *

$$\left(\frac{P}{P_s}\right)=\left(\frac{P_T}{P_R}\right)=\left(\frac{f_T/f_R}{j_T/j_R}\right)=\frac{0.0185/0.024}{0.004/0.00646}=1.24\ ,\ \left(\frac{A}{A_s}\right)=\frac{A_T}{A_R}=\left(\frac{P}{P_s}\right)\left(\frac{f_s}{f}\right)=\left(\frac{P_T}{P_R}\right)\left(\frac{f_R}{f_T}\right)$$

$$=(1.24)\left(\frac{0.024}{0.0185}\right)=1.6$$

Then, for the same frontal area, flow area, and heat transfer rate, case 9 shows that the triangular geometry requires 60% more surface area and 24% greater pumping power.

Fig. 2 shows the results of the 12 cases of PEC for the triangular and rectangular fins basing on the results of previous section. Thermophysical properties for the air is calculated at $T=298K$ and $u_m=4m/s$. **Fig. 2a** represent the temperature difference ratio w.r.t. the previous 12 cases. From the above figure case 10 represent the best case for selection to increase the inlet temperature difference for plate-fin heat exchanger. **Fig. 2b** represents the mass flow rate ratio w.r.t. cases. From the above figure case 5 represent the best case for the surface selection. **Fig. 2c** represents the Re ratio w.r.t. cases. From the above figure case 5 represent the best case for the surface selection.

Fig. 2d represents the pumping power ratio w.r.t. cases. From the above figure case 5 represent the best case for the surface selection. **Fig. 2e** heat transfer ratio w.r.t. cases. From the above figure case 5 represent the best case for the surface selection. Fig. 2f represents the mass frontal area ratio w.r.t. cases. From the above figure case 7 represent the best case for the surface selection. **Fig. 2g** represents the length ratio w.r.t. cases. From the above figure case 7 represent the best case for the surface selection.

* PEC for gas- side, rectangular fins is the reference

From the above discussion cases 5 and 7 represents the best cases for the surface selection for the plate-fin heat exchangers. In the case of generating other new cases according to the procedure cited in this research, the cases 5 and 7 expected to remain best.

Conclusions

1-The performance evaluation criteria method, is constructed to be an archival source in years to come for selecting the reasonable case for optimization according to the designer's own criteria.
2-The twelve cases in this research is in the method as shown. This method can be used to generate a large number of other cases, i.e. , the method in this research is a general for the optimal selection of heat transfer surfaces for any plate-fin heat exchanger.
3-Cases 5 and 7 represents the best cases for the surface selection for the plate-fin heat exchangers.
4-A complete procedure for generating the cases for the optimal surface selection for the plate-fin surfaces.

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Table(1): Data of Surface 10-27
(Kays and London, 1964)

$p_f = 437 \quad fin/m$ $D_h = 3.5$

$b = 12.2 \quad mm$ $\beta = 1023 \quad m^2/m^3$

$\delta = 0.2 \quad mm$

Re	j	f	Re	j	f
500	0.0103	0.038	2000	0.00375	0.0119
600	0.0089	0.0319	2500	0.00373	0.0112
800	0.00704	0.0243	3000	0.00368	0.0105
1000	0.00586	0.0198	4000	0.00353	0.00958
1200	0.00505	0.0166	5000	0.00338	0.00900
1500	0.0042	0.0137			

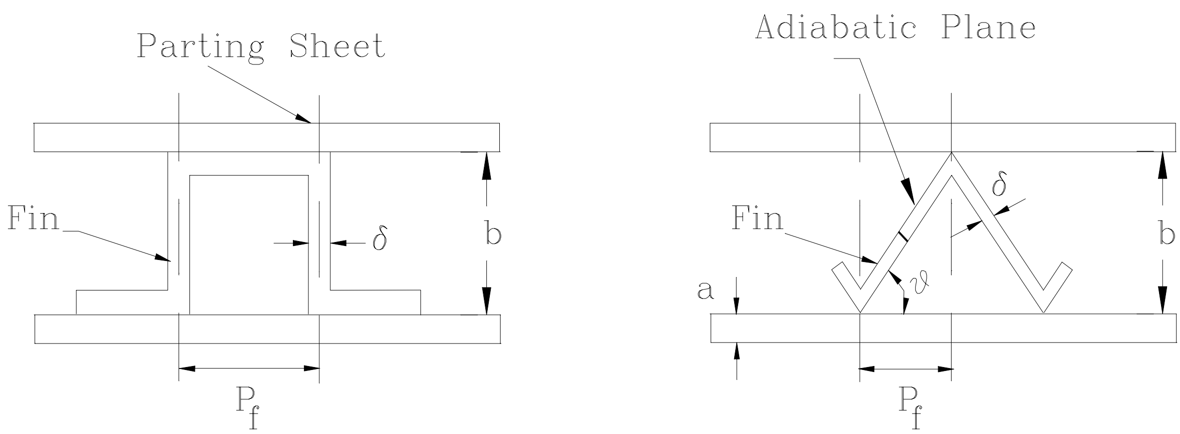
Table(2): Data of Surface 101
(Kays and Shah, 1964)

$p_f = 984 \quad fin/m$ $D_h = 1.48$

$b = 5 \quad mm$ $\beta = 2358 \quad m^2/m^3$

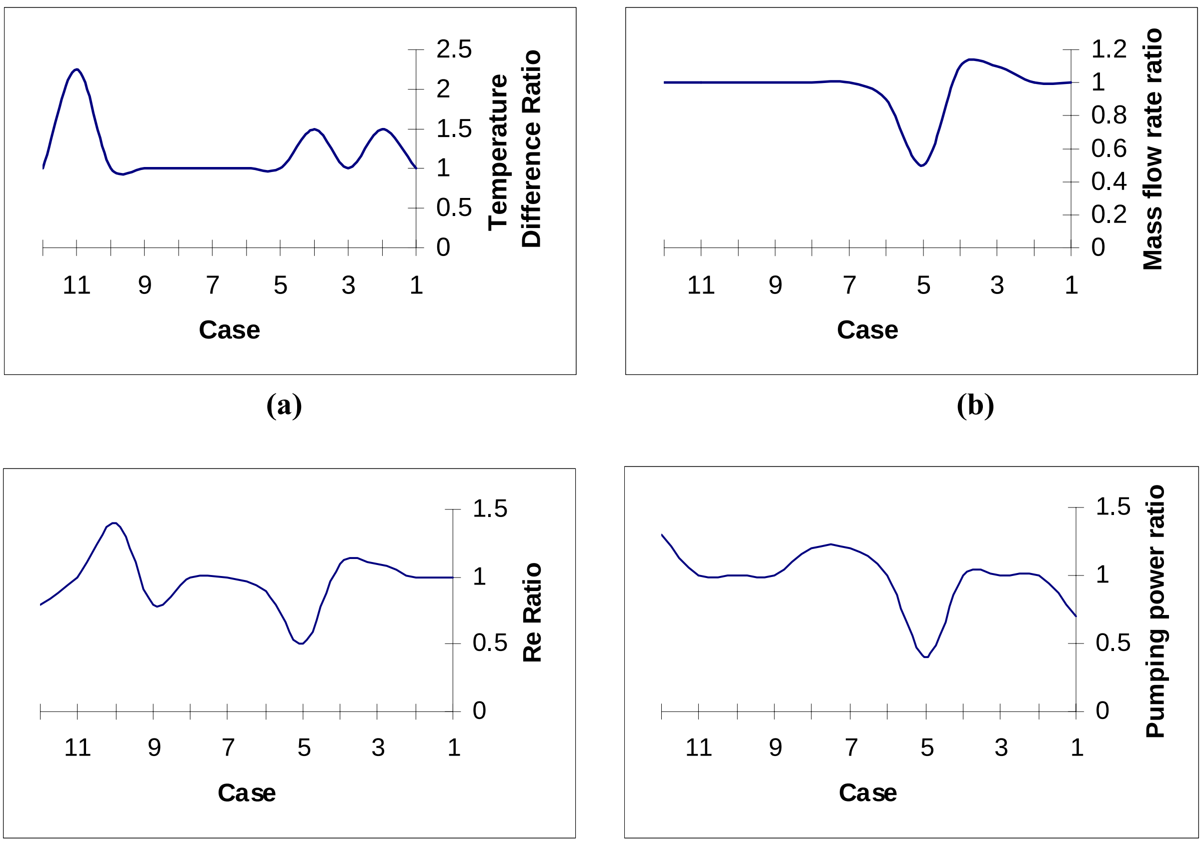
$\delta = 0.2 \quad mm$

Re	j	f	Re	j	f
500	0.0207	0.0883	1200	0.0133	0.0528
600	0.0187	0.0772	1500	0.0121	0.0.0483
800	0.0162	0.0647	2000	0.0107	0.0437
1000	0.0145	0.0573	3000	0.0095	0.0398



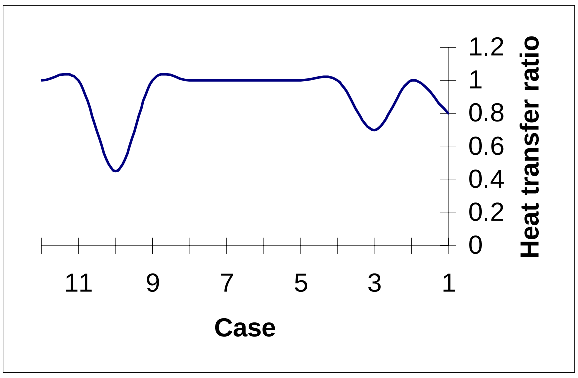
(a) (b)

Fig. 1: (a) Plain rectangular fin, and (b) plain triangular fin (Shah, 1981 b).

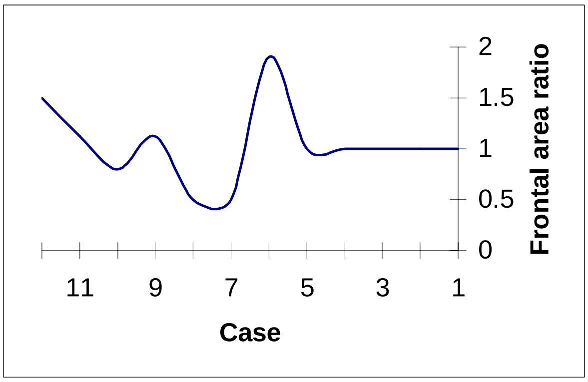


(a) (b)

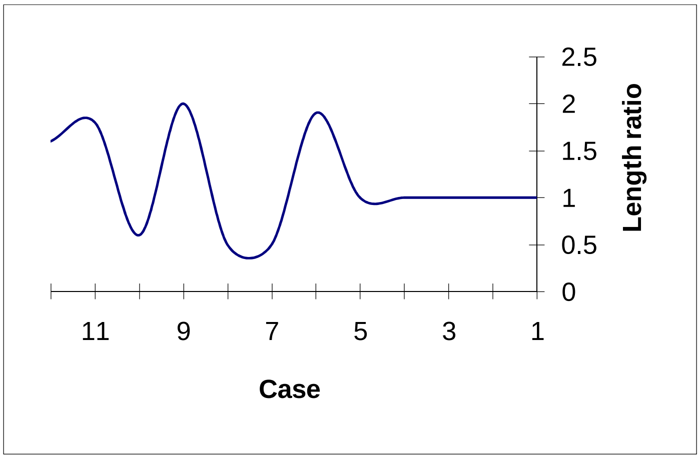
(c) (d)



(e)



(f)



(g)

Fig. 2: Effects of the surface selection cases on the selected surface for plate-fin heat exchanger.