FREE VIBRATION ANALYSIS OF A SYMMETRIC AND ANTI-SYMMETRIC LAMINATED COMPOSITE PLATES WITH A CUTOUT AT THE CENTER

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Abstract

The natural frequencies of composite laminates plate with effect of various plate parameters have been studied using ANSYS5.4 program. Laminate composites are increasing used in various mechanical structures and industrial applications, due to their higher stiffness and higher strength-to-weight ratio. The effects of number of layers, angle of fiber orientation, boundary conditions, width to thickness ratio and laminate arrangement with the natural frequencies of plate having cutout at the center are studied. The non-dimensional fundamental frequency of vibration is found to increase with increase in width to thickness ratio and angle of fiber orientation. The natural frequencies of plate depend on size and shape of the cutout, with increasing values from the plate without cutout because the mass of the plate decrease. The effect of number of layers is found to be insignificant beyond four layers and the laminate arrangement show different results between symmetric and anti-symmetric laminate plate. Some of the results compared with M.K.Pandit et al. [2], that have various size of rectangular cutout at the center, with good agreement results.

Keywords: free vibration, laminate composite plate, symmetric, anti-symmetric, cutout.

تحليل الاهتزاز الحر للسطوح المركبة المتناظرة والغير متناظرة بوجود ثقب في الوسط

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الخلاصة

تم دراسة الترددات الطبيعية للسطوح المركبة باختلاف عدة متغيرات باستخدام البرنامج ANSYS 5.4.إن السطوح المركبة تستخدم بشكل واسع في مختلف الأجزاء الميكانيكية والتطبيقات الصناعية لاحتوائها على صلابة ومقاومة عاليتين نسبة إلى الوزن. لذلك تم تقديم دراسة عن الترددات الطبيعية للسطوح التي تحتوي على مقطوعات في الوسط تحت تأثير التغيرات المتمثلة بعدد السطوح، زاوية الألياف، الظروف المحيطة، نسبة العرض إلى السمك وكذلك ترتيب الألياف. تبين إن التردد الطبيعي اللااتجاهي للاهتزاز يزداد مع زيادة نسبة العرض إلى السمك ومع زيادة زاوية الألياف. وان التردد الطبيعي يعتمد على حجم وشكل الفجوة الموجودة مع زيادة في الترددات للسطوح التي تحتوي على ثقوب عن التي لا تملك. ان تاثير عدد الطبقات يصبح غير مجدي بعد زيادته عن الأربع طبقات وان ترتيب الصفائح يعطي نتائج مختلفة للسطوح المتناظرة والغير متناظرة. تم مقارنة بعض النتائج المستحصلة مع M.K.Pandit et al. [2], باستخدام ثقوب مستطيلة الشكل وبإحجام مختلفة وكانت النتائج متقاربة.

Nomenclature

u : flexural displacement (m).
h : thickness of plate (m).
E : Young modulus (N/m^2).
U(x,y) : mode shapes.
m,n : mode numbers.
M : bending moment (N.m).
V : volume fraction.
D : bending rigidity (N.m).
ω : natural frequency (rad/sec).
ώ : non-dimensional natural frequency.
ρ : density (kg/m^3).
υ : Possion ratio.
α : angle of fiber orientation.

Introduction

Laminated plates and shells have been used in many engineering applications in recent years because of their many beneficial properties. Researches are produced to design safe and more economic thick laminated composite materials through number of layers, fiber angles, laminated distribution,…etc. The one of the main practical consideration inevitable in structures are cutouts, which are used as access ports for mechanical and electrical systems, damage inspection, altering the resonant frequency of the structures and to serve as doors and windows. The undesirable vibrations may cause sudden failure due to resonance in the presence of cutouts. So to avoid the resonant behavior of the structures, the results of the free vibration analysis for the laminated composite structures using in the structural design are very important. Also most structures whether they are used in civil, marine or aerospace are subjected to dynamic loads during their operation. Therefore , free vibration and transient analysis of laminated composite plates and shells must be consideration.

Abdullah Secgin and A.Saide [1] study presents a detailed procedure for the implementation of a discrete singular convolution (DSC) approach to the free vibration analysis of composite plates based on classical laminated plate theory (CLPT). The approach performs a numerical solution of differential equation of motion by using a grid discretization based on distribution theory and wave lets. The solution of DSC method give good agreement compared with the exact results of simply supported isotropic thin beams, fully simply supported one layer isotropic and specially orthotropic plates, and also some symmetrically laminated thin composite plates orientated to become specially orthotropic. And prediction for laminated composite plates with different boundary conditions and ply numbers.

free vibration analysis of laminated composite rectangular plate was discussed in the work of M.K.Pandit et al. [2] using finite element method. A nine- nodded isoparametric plate-bending element has been used for the analysis of free undamped vibration of isotropic and fibre reinforced laminated composite plates. Two types of an effective mass lumping scheme with rotary inertia has been recorded. Numerical examples of isotropic and composite rectangular plates having different fiber orientations angles, thickness ratio and aspect ratio have been solved.

vibration analysis of simply supported rectangular plates with unidirectionally, arbitrarily varying thickness is proposed by Sang Wook and Sang-Hyun [3]. The plate is divided into a number of regions with constant thickness and the close-form frequency function that yields the eigenvalues of the plate is extracted by considering the condition of continuity in displacement and shape between the regions and by considering the simply supported boundary condition of plate.

S. Latheswary [4] study the linear and non-linear free vibration analysis of laminated composite plates using a finite element model, based on third-order shear deformation theory. This study has been motivated by the lack of open literature on large amplitude dynamic analysis of laminated plates based on higher-order shear deformation theory. The effect of various plate parameters on the linear and non-linear fundamental frequencies of vibration is brought out.

S.S. Akavci [5] examined buckling and free vibration analysis of simply supported symmetric and anti-symmetric cross-ply thick composite plates on elastic foundation using a new hyperbolic displacement model. In this new model, in plane displacements vary as a hyperbolic function across the plate thickness, so account for parabolic distributions of transverse shear stresses and satisfy zero shear stress condition at the top and bottom surfaces of plate. The closed form solutions are obtained by using Navier technique, and then buckling loads and fundamental frequencies are found by solving the results of eignvalue problems.

The solution of the free and forced vibration analysis of laminated composite plates and shells using a 9-node assumed strain shell element have been given by Won-Long Lee and Sung-Cheon Han [8]. The natural frequencies of isotropic laminates and the forced vibration analysis of laminated composite plates and shells subjected to arbitrary loading are investigated. The effect of damping on the forced vibration analysis is studied.

In the literature, study of free vibration analysis of laminated composite plates is very important for damage inspection and altering the resonant frequency. In the present work, free vibration analysis of symmetric and anti-symmetric laminated composite plates with multi shapes of cutout at the centre produced using a well known computer program ANSYS 5.4. The effect of various plate parameters such as number of layers, thickness ratio, fiber orientation and boundary conditions have been considered. The results obtained in the form of natural frequencies with the various parameters. Some of results compared with those available in the literature to show the accuracy of present analysis.

Problem Formulation

A square plate used with cutout at the center has a uniform structure and constant thickness, while the cross section is varied at the lines through the cutout. Differential equation of harmonic bending vibration for laminated thin composite plate with natural frequency (ω) having side length (a and b), thickness (h), average mass density (ρ) and Poisson ratio (υ) can be written in Cartesian co-ordinates (x, y) in terms of flexural displacement (u) as follows [7]:

$$D\left(\frac{\partial^4 u}{\partial x^4} + 2\frac{\partial^4 u}{\partial x^2 \partial y^2} + \frac{\partial^4 u}{\partial y^4}\right) + \rho h \frac{\partial^2 u}{\partial t^2} = 0 \qquad (1)$$

Substituting

$$u(x, y, t) = U(x, y)e^{j\omega t} \qquad (2)$$

gives

$$D\left(\frac{\partial^4 U}{\partial x^4} + 2\frac{\partial^4 U}{\partial x^2 \partial y^2} + \frac{\partial^4 U}{\partial y^4}\right) - \rho h \omega^2 U = 0 \qquad (3)$$

For simply supported plate from all edges the boundary conditions are:

$$\begin{array}{ll} u(0,y,t)=0 & u(x,0,t)=0 \\ u(a,y,t)=0 & u(x,b,t)=0 \\ M_{xx}(0,y,t)=0 & M_{yy}(x,0,t)=0 \\ M_{xx}(a,y,t)=0 & M_{yy}(x,b,t)=0 \end{array} \tag{4}$$

and for clamped plate from all edges, the boundary conditions are:

$$\begin{array}{ll} u(0,y,t)=0 & u(x,0,t)=0 \\ u(a,y,t)=0 & u(x,b,t)=0 \\ \frac{\partial u}{\partial x}(0,y,t)=0 & \frac{\partial u}{\partial y}(x,0,t)=0 \\ \frac{\partial u}{\partial x}(a,y,t)=0 & \frac{\partial u}{\partial y}(x,b,t)=0 \end{array} \tag{5}$$

Assumed mode shapes to be:

$$U(x,y)=\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b} \tag{6}$$

This satisfies the partial differential equation and the boundary conditions. So, the natural frequency of orthotropic simply supported plate is:

$$\omega_{mn}=\pi^2[(\frac{m}{a})^2+(\frac{n}{b})^2]\sqrt{\frac{D}{\rho h}} \tag{7}$$

m, n = 1,2,3,…

The constitutive relationship for a homogenous orthotropic lamina in a state of plane stress as shown in fig.(1-a)is [6]:

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{bmatrix}=\begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & Q_{33} \end{bmatrix}\begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{bmatrix} \tag{8}$$

Where

$$Q_{11} = \frac{E_{xx}}{1-\upsilon_{xy}\upsilon_{yx}}$$
$$Q_{22} = \frac{E_{yy}}{1-\upsilon_{xy}\upsilon_{yx}}$$
$$Q_{12} = \frac{\upsilon_{yx}E_{xx}}{1-\upsilon_{xy}\upsilon_{yx}} \qquad (9)$$
$$Q_{21} = \frac{\upsilon_{yx}E_{yy}}{1-\upsilon_{xy}\upsilon_{yx}}$$
$$Q_{33} = G_{xy}$$

Because of requirement that $Q_{12} = Q_{21}$, it can obtain:

$$\upsilon_{yx}E_{xx} = \upsilon_{xy}E_{yy} \qquad (10)$$

In terms of volume fractions where $V_f + V_m = 1$, it can be seen that :

$$E_{xx} = E_f V_f + E_m V_m$$
$$E_{yy} = E_m \frac{[E_f + E_m + (E_f - E_m)V_f]}{E_f + E_m - (E_f - E_m)V_f}$$
$$\upsilon_{xy} = \upsilon_f V_f + \upsilon_m V_m \qquad (11)$$
$$G_{xy} = G_m \frac{[G_f + G_m + (G_f - G_m)V_f]}{G_f + G_m - (G_f - G_m)V_f}$$

Finite Element Modeling

The ANSYS 5.4 finite element program [9] was used to study free vibration analysis of laminated plates with multi shapes of cutout. For this purpose, the key points were first created and then the segments were formed. The lines were combined to create an area. Fine meshes of element type shell 99 with element size 0.01m as shown in **Figure (1).** Modal analysis with subspace method have been used to extract modes number with boundary conditions as simply supported or clamped plate.

Results and Discussions

In order to demonstrate the accuracy and applicability of the present simulation, laminated composite square plates have been analyzed and compared with the published results. A simply supported laminate composite plate with different size of cutout at the plate center have been studied to compared with Pandit et al. [1] for the first six minimum frequencies. So, a square laminate plate (a*a), (0/90) and (a/h=100) with different size of cutout considered. The results have been compared for frequency parameter of $\lambda = \omega a^2 \sqrt{(\rho / E_2)/h}$ show good agreement results as shown in table(1). The first six mode shapes of for laminate composite plate (0/90) without cutout shown in **Figure (2),** given by present study of a deformation in z-direction. Then the simulation include different thickness ratio, fiber orientation angle, number of layers and boundary condition with multi shapes of cutout and for symmetric and anti-symmetric laminate composite plate. The effect of these various parameters on the fundamental frequency of vibration is studied by considering square laminates of side (a=25cm) and having the following material properties: $E_1 = 25*10^{10}$ N/m^2, $E_2 = 1*10^{10}$ N/m^2, $G_{12} = G_{13} = 0.5*10^{10}$ N/m^2, $G_{23} = 0.2*10^{10}$ N/m^2, $\upsilon_{12} = \upsilon_{23} = \upsilon_{13} = 0.25$ and $\rho = 1*10^{10}$ kg/m^3. The results are expressed in non-dimensional form as:

$$\varpi = \frac{\omega a^2}{h}\sqrt{\frac{\rho}{E_2}} \tag{12}$$

1- Effect of Number of Layers:

The cross-ply and angle-ply laminates with symmetric and anti-symmetric arrangement of layers having width to thickness ratio (10 and 100) with multi shapes of cutout are analyzed to study the effect of number of layers on the fundamental frequency. The results for angle-ply laminates is shown in **Figure (3),** as the number of layers increase the fundamental frequency increase. The presence of cutout give high natural frequency, different with cutout because of less mass of the plate. It is also seen for all shapes that for laminates with symmetric lay-up, there is a gradual increase in the value with increase in the number of layers. But for anti-symmetric lay-up, there is a sadden increase in the value of (ώ) from two layers to four layers and thereafter the increase is at a slow rate. The fundamental frequencies for symmetric lay-up have a low value from that of anti-symmetric laminates.

2- Effect of Fiber Orientation:

Four-layers symmetric (α/-α/-α/α) and anti-symmetric (α/-α/α/-α) laminates with angle of fiber orientation varying from (0^o to 45^o) with (a/h=10 and 100) are analyzed having different shape of cutout. A change in fiber orientation angle from (0^o to 45^o) leads to an increase in the fundamental frequency of vibration in the case of both thick (a/h=10) and thin (a/h=100) plates as shown in **Figure (4).** The fundamental frequency of vibration for symmetric arrangement is less than that for anti-symmetric arrangement, the difference being more for higher values of (α). It can also seen that the fundamental frequency of vibration for rectangular, square and circular cutout is higher than for plate without cutout.

3- Effect of Boundary Conditions:

The variation of non-dimensional fundamental frequency with a/h ratio for a simply supported and clamped edges are shown in **Figure (5),** by analyzing four-layers plate with symmetric (0/90/90/0) and anti-symmetric (0/90/0/90) laminates. In case of simply-supported edges, there is a sudden increase in the non-dimensional fundamental frequency up to (a/h=20), beyond which frequency remain practically constant for both symmetric and anti-symmetric laminate plate with an increase between them. But for laminates with clamped edges, the values are high than first and goes on increasing with a/h ratio. Anti-symmetric laminate arrangement has low value from symmetric arrangement for both edge supported.

Conclusions

It is important to predict the natural frequencies of laminate composite plate with cutout at the center because cutouts are commonly used as access ports for mechanical and electrical systems. The undesirable vibrations may cause sudden failures due to resonance in the presence of cutouts. The present models lead to the following conclusions:

1. Natural frequencies depend on size and shape of cutout.
2. Natural frequency increase with increasing the cutout size because of less mass of the plate.
3. Non-dimensional fundamental frequency of vibration is found to be increase with increasing width to thickness ratio and angle of fiber orientation.

4. The effect of number of layers is found to be insignificant beyond four layers.
5. The non-dimensional fundamental frequency for symmetric arrangement is less than for anti-symmetric arrangement as a number of layers and angle of fiber changed, on the converse with a/h changed.
6. A change in angle of fiber orientation from 0 to 45 leads to an increase in the fundamental frequency of vibration in both thick (a/h=10) and thin (a/h=100).
7. The variation of non-dimensional fundamental frequency increase as a/h increases, but the increase is small beyond a/h=20 (as a thick plate).
8. The edge conditions of the plate play an important role in the frequency of vibration of the system.
9. Non-dimensional fundamental frequency of vibration of clamped plate is higher than for simply supported plate, and farther increasing beyond (a/h=20).

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Table (1): Frequency parameter $\lambda = \omega a^2 \sqrt{(\rho / E_2)/h}$ of simply supported cross ply (0/90), square laminate plate having rectangular cutout.

Cut-out size	Reference	First six minimum frequencies					
		1	2	3	4	5	6
0.2a*0.2a	Ref.[1]	9.071	25.057	25.057	37.643	53.154	59.856
	Present study	9.11	25.63	25.80	38.11	54.23	60.64
0.4a*0.4a	Ref.[1]	9.061	19.93	19.93	35.07	42.866	60.27
	Present study	9.12	20.25	20.34	35.67	44.76	61.81
0.6a*0.6a	Ref.[1]	11.085	18.173	18.173	31.56	33.92	51.712
	Present study	11.31	18.69	18.71	32.87	34.34	53.11
0.4a*0.2a	Ref.[1]	8.765	20.57	24.35	36.57	50.83	60.81
	Present study	8.85	21.31	27.82	37.12	51.22	62.14
0.8a*0.4a	Ref.[1]	9.603	11.542	27.03	30.63	49.42	59.27
	Present study	9.72	11.81	27.36	31.15	50.63	60.37
0.6a*0.2a	Ref.[1]	8.49	15.27	25.08	34.02	50.24	59.21
	Present study	8.54	15.87	25.45	34.87	51.26	60.13

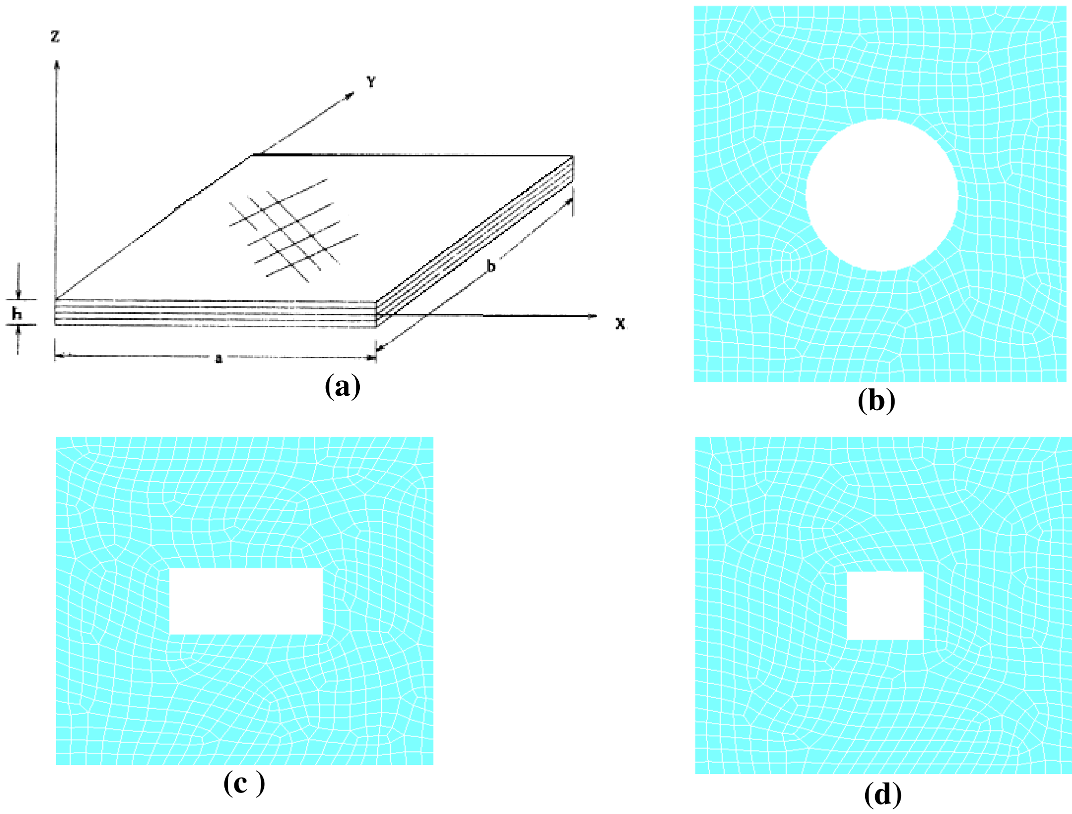


Figure (1): Geometry and Mesh of the Models (a)Laminate plate without cutout. (b)Mesh with circular cutout (r =0.2a). (c)Mesh with rectangular cutout (0.2a*0.2a). (d)Mesh with square cutout (0.2a*0.2a)

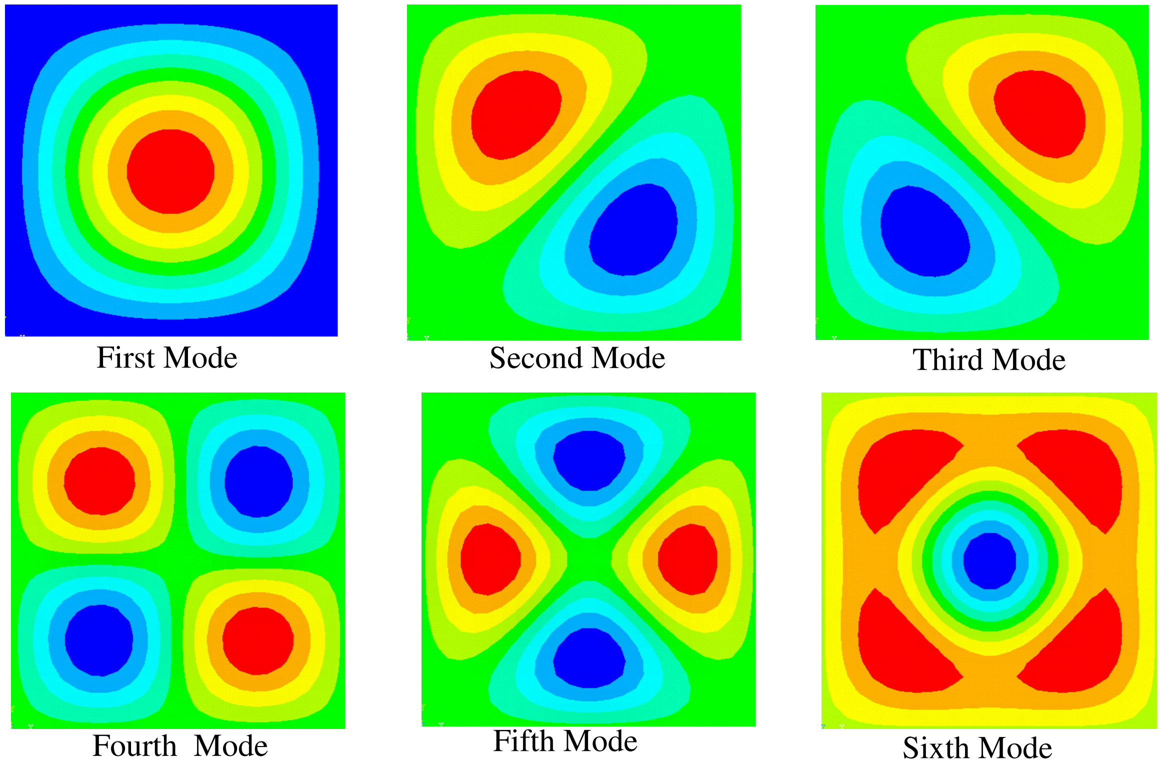


Figure (2): The First Six Mode of laminate composite plates (0/90) as solved in ANSYS Program.

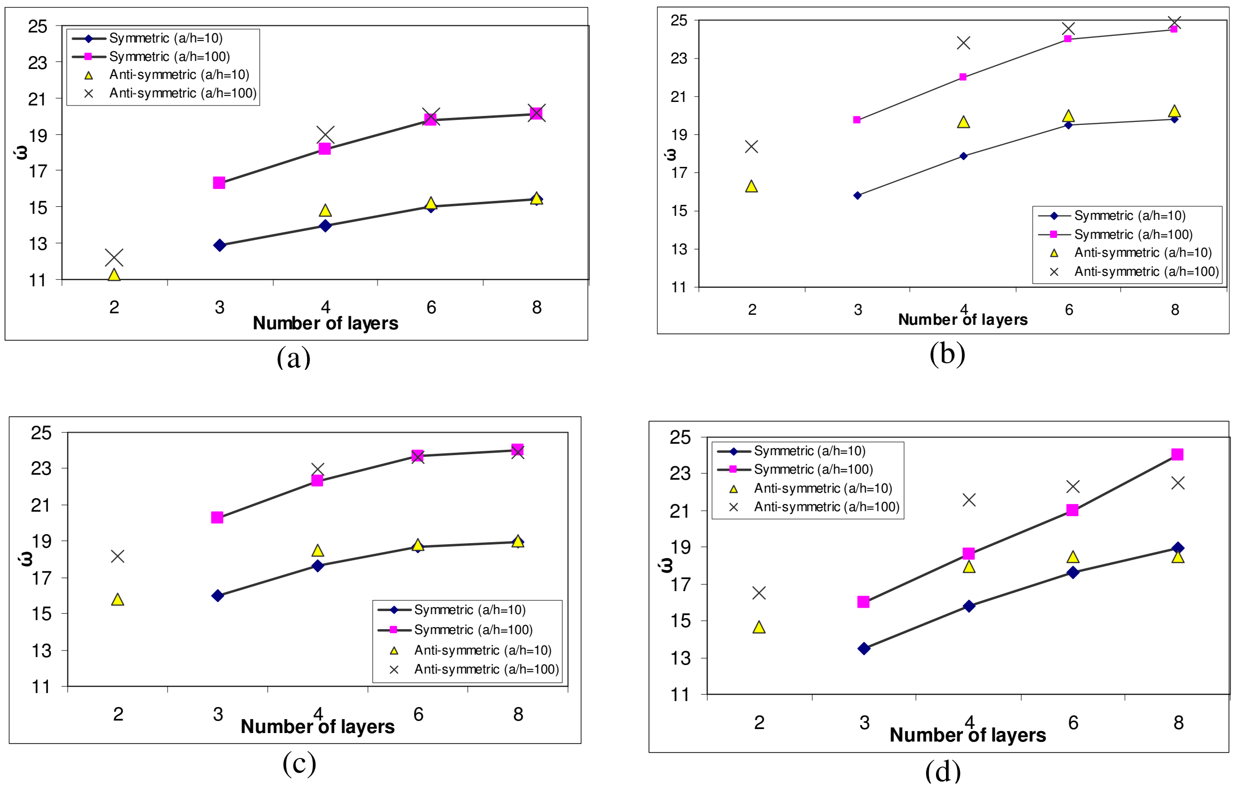
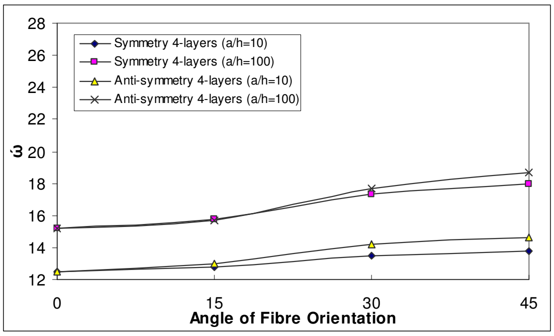


Figure (3): Variation of Fundamental Frequency with Number of Layers for Symmetric and Anti-symmetric Laminated plate. (a) without cutout. (b) with circular cutout. (c) with square cutout. (d) with rectangular cutout



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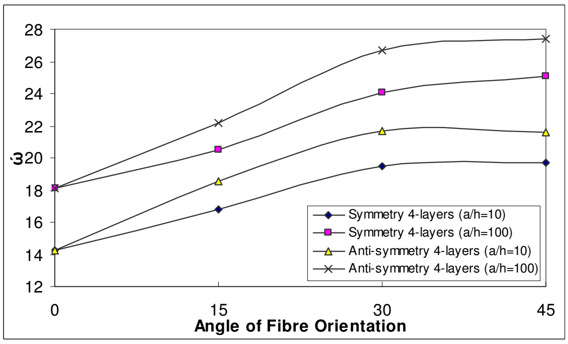

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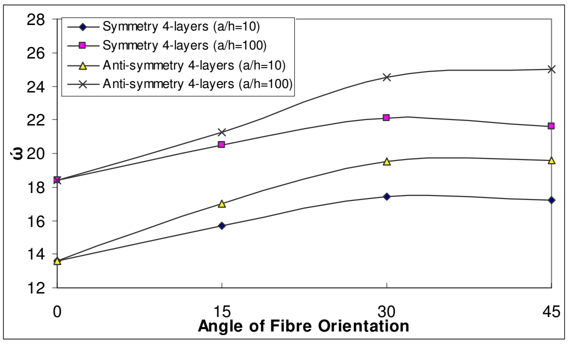

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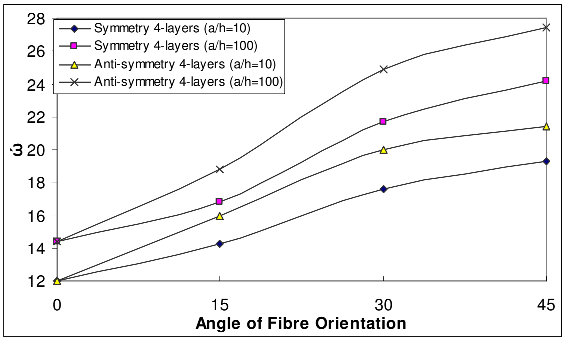
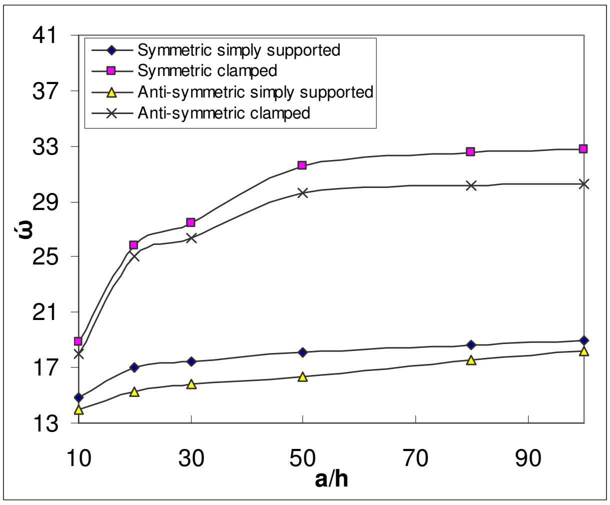


(d)

Figure (4): Variation of Fundamental Frequency with Fiber Orientation Angle for Four Layers Symmetric (α/-α/-α/α) and Anti-symmetric (α/-α/α/-α) laminated plate. (a) without cutout. (b) with circular cutout. (c) with square cutout. (d) with rectangular cutout



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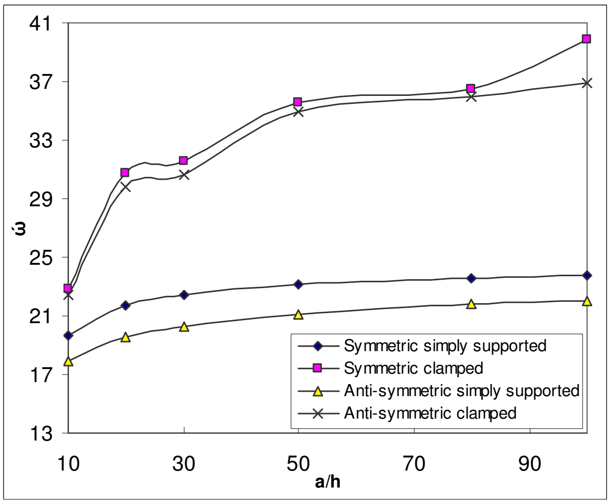

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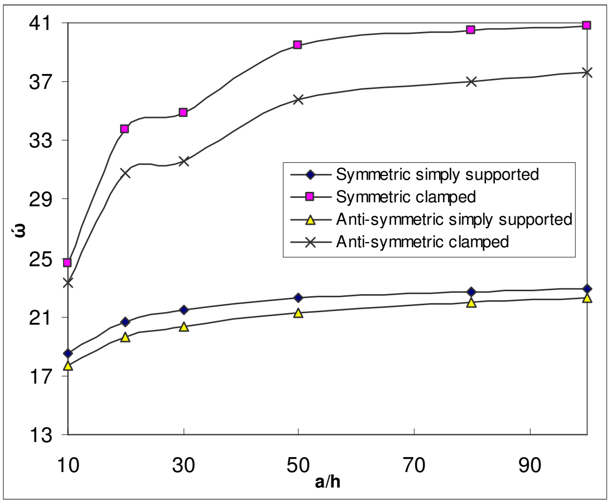

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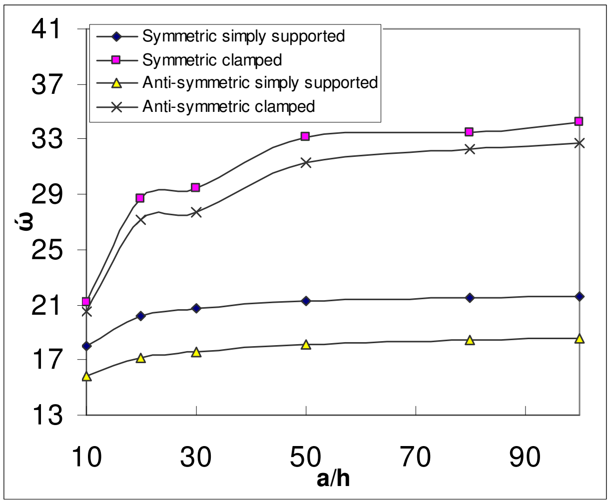

(d)

Figure (5): Variation of Fundamental Frequency With Edge Condition for Four Layers Symmetric and Anti-symmetric laminated Plate. (a) without cutout. (b) with circular cutout. (c) with square cutout. (d) with rectangular cutout