ANALYSIS OF WATER HAMMER FOR AL-KUT WATER SUPPLY PROJECT USING THE METHOD OF CHARACTERISTICS

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Abstract

In this study, a mathematical model have been prepared for al – Kut water supply project depending upon the numerical methods such as Newton - Raphson method and Gaussian elimination method to solve non – linear simultaneous equations with known boundary condition and by using the characteristic method, the partial differential equations are transformed to ordinary differential equations.

In this mathematical model, the values of the generated pressures were calculated when using control devices or without, also, to find the influence of this devices on, the pressure values was found. In addition to, the suitable number of control devices required for each pipeline was found to keep the pressure values within a tolerable limits.

This model was applied on a published case study and the results were quite satisfactory.

Key words: Water hammers Method of characteristics

تحليل ظاهرة المطرقة المائية لمشروع إسالة ماء الكوت باستخدام طريقة المعادلات المميزه

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الخلاصة

تم في هذا البحث إعداد نموذج رياضي خاص لمشروع ماء الكوت وذلك بالاستعانة ببعض الطرق العددية كطريقة نيوتن – رافسون وطريقة كاوس للحذف لحل المعادلات الآنية غير الخطية التي تتحكم بالشروط الحدودية. وباستخدام **طريقة المعادلات المميزة** تم تحويل المعادلات التفاضلية الجزئية الى معادلات تفاضلية اعتيادية.

لقد تم في هذا النموذج إيجاد قيم الضغوط المتولدة في حالة استخدام وفي حالة عدم استخدام اجهزة السيطرة وملاحظة مدى تأثيرها على قيم الضغوط واستخدام العدد المناسب من هذه الاجهزة لكي يمكن المحافظة على قيم الضغوط ضمن حدودها المسموحة. حيث تم التحقق من هذا النموذج بتطبيقه على مسألة منشورة حيث جاءت النتائج متطابقة.

Nomenclature

A	area of pipe
a	Velocity of propagation of fluid transients
c_1	parameter that describes the effect of pipe constraint condition on the wave speed
C_n, C_p	Constants in characteristic equation
D	diameter of pipe
E	modulus of elasticity (pipe wall)
E	pipe wall thickness
F()	pressure wave travelling in the -x direction in the pipe
f()	pressure wave travelling in the +x direction in the pipe
F	Darcy-Weisbach friction factor
G	gravitational acceleration
H	instantaneous pressure head
H_0	steady-state pressure head
H_t	$\partial H/\partial t$; partial derivative
H_x	$\partial H/\partial x$; partial derivative
h_v	head losses
K	bulk modulus of elasticity (fluid)
L	length of pipe
P	instantaneous pressure
p_0	steady-state pressure
p_t	$\partial p/\partial t$; partial derivative
p_x	$\partial p/\partial x$; partial derivative
Q	Steady state Pipe line discharge
T	Time
V	instantaneous velocity of flow
V_0	steady-state velocity of flow
V_t	$\partial V/\partial t$; partial derivative
V_x	$\partial V/\partial x$; partial derivative
X	distance along the pipe measured from upstream end
Z	elevation above datum
A	pipe slope
M	Poisson's ratio
P	density of fluid
Z	head loss coefficient (valves, orifices)

Introduction

Early work of several mathematicians and physicists has been concerned with the speed of propagation of pressure waves in two distinctive fields by Thorley in 1976 [Streeter (1993)]: one was an elastic medium in a rigid pipe, the other of an incompressible fluid in an elastic pipe. Korteweg in 1878 was the first to develop an equation for the wave speed in a compressible fluid contained in an elastic pipe. But this early work was not concerned with pressure and velocity variations in pipes. Probably the first reported analytical treatment of pressure- and flow-variations was that of Frizell in1898, a consulting engineer for a hydroelectric power development. Joukowsky in 1900 conducted the first comprehensive series of experiments on a large-scale system, which he supported with an analytical treatment. The experiments were made in cast iron pipes (diameters 2", 6", and 24") of the Moscow Water Works in 1897/1898. The work of Joukowsky represents a milestone in the history of water hammer. His conclusions formed the basis of much later work. They were:

1. The pressure wave is transmitted through the pipe at a constant speed, the value of which depends upon the pipe dimensions, the elastic modulus of the pipe wall material, and the bulk modulus and density of the fluid.
2. The amplitude of the pressure wave traveling along a uniform pipe remains constant.
3. The concept of transmitted and reflected waves completely explains the periodic nature of the pressure recordings.
4. A pressure wave is reflected from an open-ended pipe with constant pressure.
5. Pressure changes are doubled by reflection at a dead end.
6. Air chambers are an effective method of preventing excessive pressure rise.
7. The pressure diagram can be used as a source of information about the conditions in the pipe, e.g. sites and mass of entrapped air, sites of heavy

In Iraq, Reyad Zuhair was one of the first researchers who work on the water hammer using the characteristics method at one pipe at Baiji Water Pipe Line (Zuhair, R, 1987). Then, Abd Al-Abbas came later and work on the water hammer using the same method but at four pipes at Najaf –Kufa water supply project (Abd Al-Abbas, F, 2000).

Water-hammer/ transient flow

The intermediate – stage flow, when the flow conditions are changed from one steady conditions to another steady state is called transient state flow, in other words, the transient conditions are initiated whenever the steady – state condition are disturbed. Common examples of the cases of transients in engineering system are: opening or closing of valves in a pipeline, power failure to pumps, action of reciprocating pumps, starting up a hydraulic turbine, vibrations of the vans of a runner or the blades of a fan, and waves on a reservoir water surface [Roberson et. al (1998), Streeter (1971), and Streeter and Wylie (1993)].

The terms water hammer or transient flow refer to those unsteady flow situations where the change in the motion of the fluid is comparatively rapid and the time for the forcing conditions to change is short compared to the time taken for a wave of pressure to pass through the fluid column. The behavior of the pressure transients is governed by the inertia of the moving water and the combined elasticity of the water and the pipe system. In general both, inertia and elasticity, must be considered, which requires compressible flow theory. Depending on the complexity of the task, the graphical method or characteristics method may be used [Larreteguy et. al,(2005)].

1- Basic equations for pressure transients

The general derivation of the partial differential equations for the unsteady flow of liquids in slightly elastic pipes is based on the principle of conservation of mass and momentum. Both, the equation of motion and the continuity equation are developed regarding an elemental control volume. Once a transient has been generated and propagated along a pipe, its subsequent behavior depends on the boundary conditions of the particular system. Therefore the actual boundary conditions are an important part for the investigation of pressure transients in hydraulic systems. There are different forms of both equations that are used when investigating pressure transients. One of the common forms will be presented at first and simplified forms of the equations will follow [Wylie (1983)].

1.1 Equation of motion (momentum equation)

This equation is governing the transient state condition. Applying the low of conservation of mass to a fluid element within a pipe of a cross sectional area (A) and length (*d*x) [White (1991)]

$$gH_x + VV_x + V_t + fV|V| /2D = 0 \qquad [2.1]$$

This equation must be valid also for steady flow, a special case of unsteady flow. By setting $V_x = 0$ and $V_t = 0$ it becomes

$$\Delta H = f(\Delta x/D)(v^2/2g)$$

which is the Darcy-Weisbach equation.

1.2 Continuity equation

It may be applied the low of conservation of mass to a control volume yields to device the continuity equation [White (1991)]

$$VH_x + H_t - V\sin\alpha + (a^2/g)V_x = 0 \qquad [2.2]$$

where the wave speed "a" is considered to be a constant depending on the properties of the fluid, the pipe and its means of support:

$$a^2 = (K/\rho)/(1 + ((K/E)(D/e))c_1 \qquad [2.3]$$

Three support situations for a thin-walled pipeline may be regarded:

I	Pipe anchored at its upstream end only	$c_1 = 1 - \mu/2$
Ii	Pipe anchored throughout against axial movement	$c_1 = 1 - \mu^2$
Iii	Pipe anchored with expansion joints throughout	$c_1 = 1$

The value of Poisson's ratio μ for steel is close to 0.3. It should be noticed that a very small amount of entrained gas in the fluid causes much greater change in the wave speed than the effect of μ.The two basic equations, which govern the behavior of pressure transients in liquids in slightly deformable pipes contain four variable quantities, (x, t) as independent variables and (H, V) as dependent variables. For unsteady flow situations this means that the pressure and the velocity of flow in a system depends on both, position in the system and time.

<u>Methods for solving continuity and momentum equations</u>

The momentum and continuity equations are quasi – linear, hyperbolic, partial deferential equations. A closed form for solution of these equations is not available. However, neglecting or linearizing the non-linear equations has developed linear terms, various graphical and analytical methods [Parmakian (1963) and Smith (1978)]. These methods are approximate and can not be used to analyze large systems or systems having complex boundary conditions.

The arithmetic method neglects the friction term and assumes that the pipe is horizontal. This method is consider simple theoretical development but has iterative calculations and tedious operations [Smith (1978)].

The impedance method is suitable for digital computer analysis only, because of the lengthy algebraic equations involved. The transfer matrix method has been used for analyzing structure and mechanical vibrations, and for analyzing the electric system. This method was introduced by many authors such as Chaudhry as a method for analyzing the transient state. The transfer matrix is derived by a numerical procedure is presented by Chaudhry [Chaudhry (1987) and Roberson et. al (1998)].

Similar to the impedance method, the transfer method is based on the linearized equations and on sinusoidal flow and pressure fluctuation. However, both methods are restricted for special case of the transient flow that called "steady – oscillatory flow". The algebraic method transforms the characteristics method equations into a from in which the time is subscript. This method has the advantage that the characteristics equations may be applied over one or more reach of a pipe are applied. The disadvantage of this method is that the friction term is not handled so accurately when more than one reach is covered by an equation. More details of the algebraic method may be found in Streeter [Streeter and Wylie (1988)].

Analytical methods are assumed that the true friction loss approximate to the first power of velocity, i.e., linearizing the non – linear term, or this term may be ignored. This method includes very difficult mathematical operation, hence it can not be used for a complex system. The graphical method assumed that the head loss along the pipe is approximated by placing hypothetical obstruction at selected locations along the pipeline, with the friction head loss along the pipe concentrated at these location. The accuracy of this method increases by the number of obstructions, leading to increase complexity. In the finite difference method, the partial derivatives are replaced by finite – difference approximations and the resulting equations are solved simultaneously [Chaudhry (1979), and Streeter and Wylie (1988)].

The analysis by finite difference method become more complex by increasing the complexity of the system. Because of computer, requirements become more and in a large system, a large number of equations must be solved which requires large amount of computing time. The method of characteristics utilized a special property of hyperbolic partial differential equations to find their numerical solutions. For a system having such equations, there are two characteristics directions in the (x-t) plane in which the integration of the partial differential equations is reduced to the integration of a system of ordinary differential equations. The advantages of this method are that, it is a method of solution, which allows the direct inclusion of friction losses, if, offers ease in handling the boundary conditions and in programming a complex system. It is a general method, i.e., the program, once written, may be used for analyzing any piping system having the same boundary conditions, and the transient state conditions obtained by using this method are close to the actual situation. The only restriction in this method is that flow must be one-dimensional state and the time increment chosen to satisfy the stability conditions. [Parmakian(1963) and Streeter (1993)]

This method above is used in this research and it is discussed briefly in the next paragraph.

1-Theory of the research

The theory of the research includes the characteristic equations are written to solve the problem and the simple and complex boundary conditions.

The characteristic equations may be written as:

$$Q_p = C_p - C_a H_p \qquad [4.1]$$

$$Q_p = C_p + C_a \ H_p \qquad [4.2]$$

$$\therefore C_p = Q_a + \frac{gA}{a} H_a - R\Delta t Q_a |Q_a| \qquad [4.3]$$

$$C_n = Q_b - \frac{gA}{a} H_b - R\Delta t Q_b |Q_b| \qquad [4.4]$$

and

$$C_a = \frac{gA}{a} \qquad [4.5]$$

Noting that equation [4.1] is valid along the positive characteristic line AP and equation [4.2] is valid along the negative characteristic line BP as shown in **Figure(1)**

The values of constants C_p and C_n are known for each time step and the constant C_a depends upon the conduit properties . Referring to Equation [4.1] and Equation [4.2] the values of two unknowns Q_p and H_p can be determined by simultaneous solving of these equations , i.e.

$$Q_p = 0.5 (C_p + C_n) \qquad [4.6]$$

and by substituting the values of Q_p at either equation [4.1] or equation [4.2], the value of H_p can be determined. However, at the boundaries, either equation [4.1] or equation [4.2] is available. Therefore, special boundary conditions to determine the conditions at the boundaries at time (to subscript Δt) are needed.

Figure (2) shows a pipeline divided into (n) reaches each having length (Δx). The ends of these reaches are called sections , nodes or grid points .

The end sections of each pipe are referred to as boundaries, and the sections excluding the boundaries are called interior sections, interior nodes or interior grids.

2- Boundary Conditions

In the last section, the required special boundary condition have been discussed to determine the transient state head and discharge at the boundaries. These conditions are in terms of special relationships that define, at the boundary, the discharge, the head, or a relationship between them. These boundary conditions may be classified into two categories as, simple and complex boundary conditions.

2.1. Simple Boundary Conditions

Simple boundary conditions are those which are simple to handle and can take the form of specifying either the head or the discharge as a constant (as at a dead end of a pipe), specifying relationship between them (as at a valve) or specifying either as a function of time.

2.2. Complex Boundary Conditions

Complex boundary conditions are usually difficult to handle than simple boundary condition. Pump failure, control devices and column separation are examples for this kind of boundary.

In Al- Kut water supply project, transients are caused by a disturbance introduction. This disturbance may be caused by opening or closing of a regulating valve that control reservoirs inlet, also sudden stopping of pumps, i.e, power failure.

Pressure changes during transient state depend upon the rate of change in the initial flow condition. The change in pressure is more dangerous, therefore, the pump starting is made on the condition that the delivery valve is closed, then opened slowly after 90 seconds after the motor starting. The basic characteristics of al - Kut water supply project are summarized at **Tables (1) and (2).**

<u>The Results</u>

The results of the transient state analysis obtained from the mathematical model may be summarized in Tables **(3) and (4).**

Conclusions

Analysis with two air vessels for western, eastern, southern pipelines and one air vessel for northern pipeline is very suitable to keep the transient state pressure within a tolerable limits. The tolerable limits of high and low pressures are between (0 – 100) m.

Many values of head loss through the vessel outlet were assumed to give the desirable limits for pressure head and minimum number are used of control devices at the same time.

Results also indicated that the water level at the main reservoir could substantially change the computed minimum pressure. the results showed that when the minimum operation level has been used, the worst case occurs. The properties of the vessel which was selected for the new Al – Kut project were:

1) Diameter = 2.51 m, 2) V11olume = 18 m3, 3) Height = 5.86 m,
4) Minimum Water Level = 2.75 m.

Recommendations

Several recommendations are given below. It is representing guidance for further improvements and as extension of this study:

1. Possibility of optimizing the effects of the air vessels by placing them at different location in the network and thus reduce the number of air vessels required.
2. Make a comparison between several kinds of control devices by applying them to a case study and then the results have been discussed and compared.

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Table (1) : Characteristics of Al-Kut water supply project

Pump Name	**Rated Discharge (m3 / min)**	**Rated head (m)**	**Power (KW)**	**Rated Speed (Nr)**	**Specific Speed (Ns)**	**Efficiency (%)**
Western P.L	22.02	36	150	1450	1.53	82
Eastern P.L	22.32	44	187	1450	1.53	85
Southern P.L	22.02	36	150	1450	1.53	82
Northern P.L	16.5	33	164	1450	1.53	75

Table (2) : Diameters and lengths of the pipes of Al-Kut water supply project

pipe name	**Diameter (mm)**	**Length (m)**
Western P.L.	900	4500
Eastern P.L	900	5000
Southern P.L	900	4000
Northern P.L	900	1000

Table (3): Minimum and maximum pressure of mathematical model without air vessels.

Pipe Name	Minimum Pressure (m)	Maximum Pressure (m)
Western	-25	118
Eastern	-20	111
Southern	-24	124
Northern	- 13	105

Table (4): Minimum and maximum pressure of mathematical model with air vessels

Pipe Name	Minimum Pressure (m)	Maximum Pressure (m)	No. of air vessels
western	25	58	2
Eastern	12	80	2
Southern	25	60	2
Northern	36	53	1

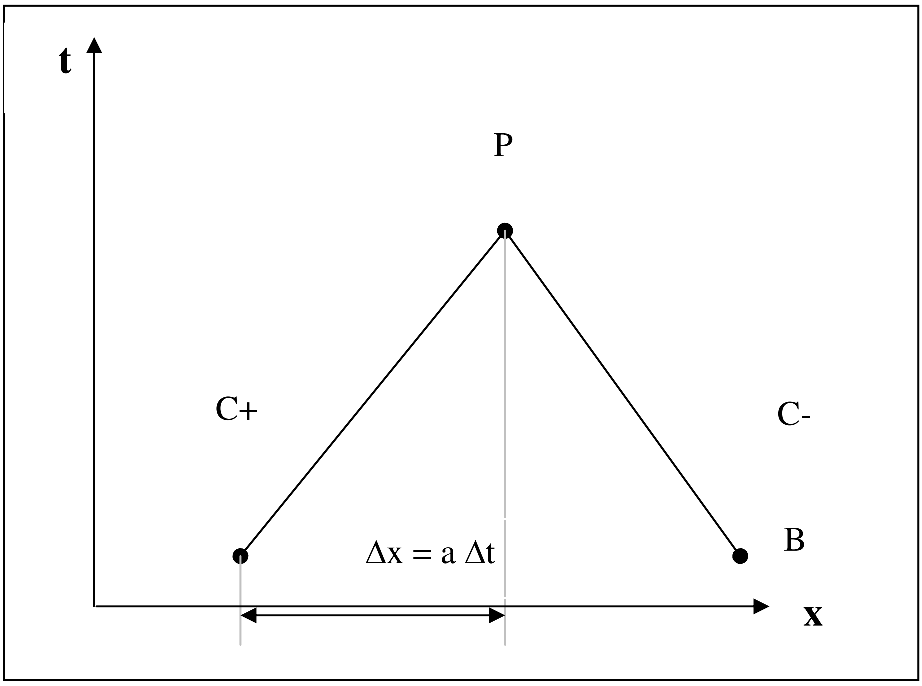

Figure (1): Characteristic lines in x-t Plane

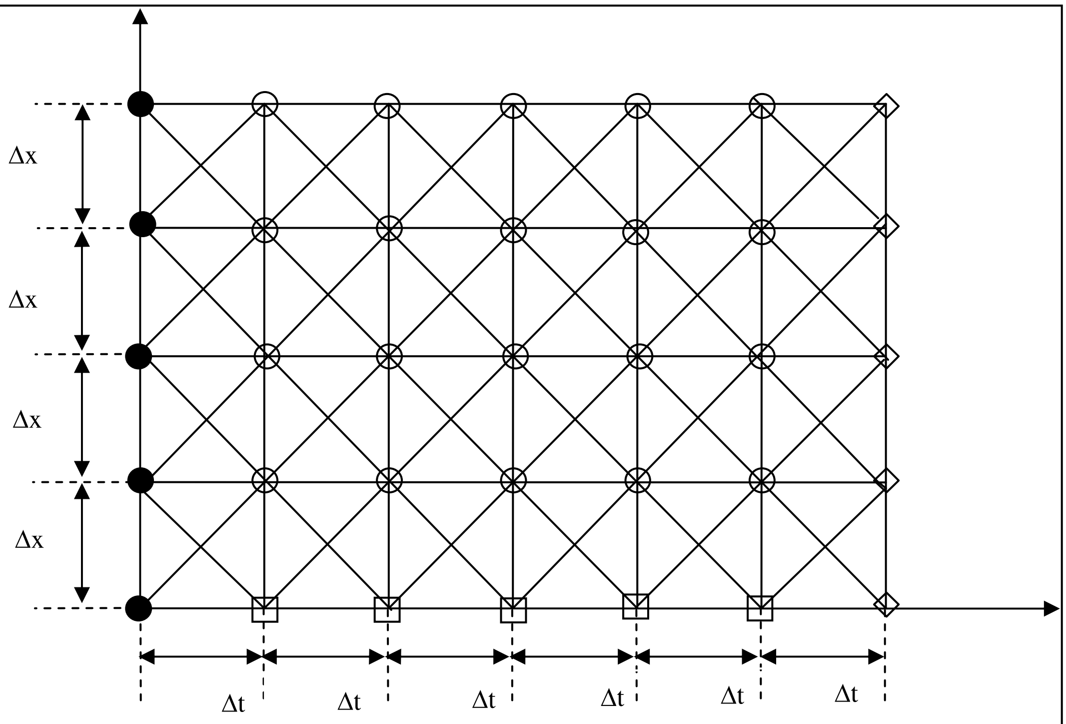

Figure (2): Characteristic Grids