SIMULATION OF HEAT TRANSFER HARACTERISTICS FOR FLOW OF NON NEWTONIAN POWER LAW FLUIDS IN TUBE

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Abstract:

Heat transfer behavior for non-Newtonian power law fluids flow in circular duct with laminar flow, fully developed, constant heat flux. And constant wall temperature was studied. A mathematical model which is capable of accurately predicting temperature and velocity profiles and heat transfer rates for power law non-Newtonian fluids was obtained. The theoretical Nussult number was compared with previously published works where good agreement was noticed, which can be easily evaluated using theoretical model as a function of power law index only for constant heat flux and as a function of power law index and Graets number for constant wall temperature. It was found find that Nussult Number decreases when values of power index increases.

Keywords: Heat transfer; non-notonian; power law fluid; power law exponent.

محاكاة خصاص انتقال الحرارة لجريان الموائع غير النيوتينية ذات قانون القدرة في الانبوب

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الخلاصة

تم دراسة سلوك انتقال الحرارة للموائع غير النيوتينية ذات قانون القوة و في حالة الجريان الطبقي .تم وضع و تطوير موديل رياضي الذي له القدرة على تحديد و بصورة دقيقة درجة الحرارة و السرعة الجانبية و معدل انتقال الحرارة للموائع المذكورة .رقم نسلت النظري قورن بأعمال منجزة سابقا وقد وجد هناك توافق جيد ،و يمكن بسهولة حساب رقم نسلت باستخدام الموديل الرياضي النظري كدالة لمعامل قانون القوة و لمعدل التدفق الحراري الثابت و كدالة لمعامل القوة و رقم كريتس بثبوت درجة حرارة الجدار .وجد بأنه رقم نسلت يتناقص عند تزايد قيم معامل قانون القوة تدريجيا.

Nomenclature :

c_p Heat capacity of fluid, J/kg.k

$(k_p)_b$ Consistency coefficient for pipe flow at temperature of bulk fluid

$(k_p)_w$ Consistency coefficient for pipe flow at temperature of wall

m Mass flow rate of fluid,Kg/s

n Power law index

r Radial coordinate

R Duct radius,M
T Fluid temperature, K
T_b Bulk temperature, K
T_c Center temperature, K
T_w Wall temperature, K
ρ Density of fluid, Kg/m^3
α Thermal diffusivity of fluid
τ Shear stress
Coefficient of volume expansion β
G_z Graetz number
Nu Nusselt number
u Velocity,M/s
Mean velocity $\bar{u}$
P Pressure, N/m^2

Introduction:

Fluids treated in the classical theory of fluid mechanics and heat transfer rate are the ideal (perfect) fluid and Newtonian fluid. The former is completely frictionless so that shear stress is absent. The latter simply has a linear relationship between the shear stress and shear rate. Since world ware II, the study of real fluids used in the mechanical and chemical industries has become increasingly important; mainly because of sever limitations in the application of ideal and Newtonian theories to real situations. Most real fluids exhibit so called non-Newtonian behavior, which means that the shear stress is no longer linearly proportional to the velocity gradient (popovaska, 1977).

An understanding of the heat transfer behavior of these non-Newtonian fluids is important in as much as most of the industries chemical and many fluids in the food processing and biochemical industries are visco-elastic in nature and undergo heat exchanger processes either application (Gottifredi, 1985).

The Graetz-Nussult problem in heat transfer theory involved the finding of the temperature profile and heat transfer rate in a fully developed laminar flow of Newtonian and non-Newtonian fluids inside circular ducts. Several works (Richardson, 1979),(Rohsenow, 1985)and (Shah, 1987) investigation this problem both experimentally and theoretically, unfortunate none of them presented complete analytical solution therefore the presents work attempt to:

- Developing a mathematical model, this is capable of accurately predicting temperature and velocity profiles and heat transfer rates for power law model non-Newtonian fluids.

Previous Work:

In view of its industrial importance heat transfer in tubes become the subject of considerable study of the main contribution have recently been comprehensive reviewed by (Holland,1970) and only the brief summary of the various types of approaches will be given here.

For constant heat flux condition, solution is available by (Rohsenow, 1973) for circular ducts:

$$Nu = 4.36\left(\frac{3n+1}{4n}\right)^{1/3} \text{..(1)}$$

(Gottifredi, 1985) modified the Leveque correlation for non-Newtonian fluids, by suggesting the following modified version of the relationship between the mean Nussult number, Nu, and the Graetz number, Gz, for constant wall temperature conditions in tube flow:

$$Nu = 1.75\delta^{1/3}Gz^{1/3}\left[\frac{(kp)_w}{(kp)_b}\right]^{0.14} \text{.................................(2)}$$

Where $\delta = \frac{3n+1}{4n}$

Although, these correlations have little theoretical basis, but they have been used for engineering design purpose (Bird, 1977).

Mathematical Model:

The governing equation for the Graetz-Nusslt problems is obtained by applying an energy balance for circular duct. The model was based on the following assumptions:

1. The flow is laminar and steady.
2. Conduction of heat transfer in the axial direction is negligible relative to radial conduction, which is justified when Re Pr $\approx$ 100.
3. Physical properties are constant.
4. No distribution for momentum and velocity in axial direction.
5. The tube wall temperature or heat flux is constant.

On the basis the simplified equation of motion, energy and continuity are as follows:

Equation of motion

$$\frac{\partial p}{\partial x}=\frac{1}{r}\frac{\partial}{\partial r}(r\tau)\text{.......................................}(3)$$

Equation of energy

$$\rho c_p u\frac{\partial T}{\partial x}=\frac{k}{r}\frac{\partial}{\partial r}(r\frac{\partial T}{\partial r})\text{..................................}(4)$$

Equation of continuity

$$m=2\pi\rho\int_0^R rudr\text{..}(5)$$

1- Velocity profile in fully developed Laminar Flow:

An analysis of laminar heat transfer for non-Newtonian fluids requires an understanding of the hydrodynamic behavior of these fluids. The analytical procedure to obtain the velocity profile for non-Newtonian fluids with the power-law model is exactly the same as for Newtonian fluids except for the specification of the shear stress in the momentum equation. The assumption of the power-law fluid particularly in the laminar flow regime is a good approximation for most non-Newtonian fluids.

With the power law equation, shear stress in a circular tube is,

$$\tau_{rx}=k\left(\frac{du}{dr}\right)^n\text{...}(6)$$

Using this equation and the equation of motion, equation (3). Since the pressure is independent of r,the equation of motion can be integrated directly twice with respect to r and the boundary conditions:

B.C.1 $\frac{\partial u}{\partial r}=0$ *at r = 0*

B.C.2 *u = 0* *at r = R*

Yielding the fully developed velocity profile

$$u=(-\frac{dp}{dx})\frac{R^2}{4k}\left[1-(\frac{r}{R})^{\frac{n+1}{n}}\right]\text{...}(7)$$

However, it is more useful to express the velocity in terms of mean velocity $\bar{u}$ rather than the pressure gradient $(-\frac{dp}{dx})$. If we now substituted velocity in to equation of continuity, eqn. (5), and integrate to obtains:

$$\frac{u}{\bar{u}}=\frac{3n+1}{n+1}\left[1-\left(\frac{r}{R}\right)^{\frac{n+1}{n}}\right] \quad \text{......(8)}$$

Where

$$\bar{u}=\left(\frac{n}{3n+1}\right)\left[\left(-\frac{\partial p}{\partial x}\right)\left(\frac{1}{2k}\right)\right]^{1/n} R^{\frac{n+1}{n}} \quad \text{......(9)}$$

Or expressions the results in term of the maximum velocity $u_\circ$ by using B.C.3 $u=u_\circ$ at $r = 0$

$$\frac{u}{u_\circ}=\left[1-\left(\frac{r}{R}\right)^{\frac{n+1}{n}}\right] \quad \text{......(10)}$$

Where

$$\frac{u_\circ}{\bar{u}}=\left[\frac{(3n+1)}{(n+1)}\right]$$

For n less than 1, this gives a velocity flatter than the parabolic profile of Newtonian fluids. As n approaches zero, the velocity profile predicted by this equation approaches plug flow profile.

2- Temperature profile in fully developed Laminar Flow:

There are at least two boundary conditions, for which it will be possible to integrate the energy equation directly with respect to r, Treat it as an ordinary differential equation, the possible conditions constant heat flux and constant wall temperature:

Case I. constant heat flux:

Technically, constant heat flux problems arises in a number of situation; electric resistance heating, radiant heating, nuclear heating, and in counter current heat exchangers where fluid capacity rates are the same. Therefore,

The term $\frac{\partial T}{\partial x}$ is constant

The applicable boundary conditions are:

B.C.1 r = 0 $T = T_c$

B.C.2 $r = 0$ $\frac{\partial T}{\partial r}=0$

Substituting into equation (4) the velocity profile, Eqn. (10) to get :

$$\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial T}{\partial r})=\frac{u_\circ}{\alpha}\frac{\Delta T}{\Delta x}\left[1-(\frac{r}{R})^{\frac{n+1}{n}}\right]......................................(11)$$

This equation can now be directly integrated twice with respect to r,

$$T=T_c+\frac{u_\circ}{\alpha}\frac{\Delta T}{\Delta x}\frac{R^2}{4}\left[(\frac{r}{R})^2-(\frac{2n}{3n+1})^2(\frac{r}{R})^{\frac{3n+1}{n}}\right].......................(12)$$

For convenience Tb is defined as the mixed mean fluid temperature, this temperature is sometimes referred to as the mass averaged temperature bulk fluid temperature, or mixing cup temperature. It is the temperature which characteristic defined the average thermal energy state of the fluid. This may be calculated from:

$$T_b=\frac{\int_0^R 2\pi r\rho ucTdr}{\int_0^R 2\pi r\rho ucdr}..(13)$$

If equation (10) and equation (12) are substituted for u and T, then this equation can be readily integrated to yield:

$$T_b=T_c+\frac{u_\circ}{\alpha}\frac{\Delta T}{\Delta X}\frac{R^2}{4}\frac{\left[\frac{1}{4}-(\frac{n}{3n+1})-\frac{1}{4}(\frac{n}{5n+1})(\frac{n}{3n+1})^2+2(\frac{n}{3n+1})^3\right]}{\left[\frac{1}{2}-(\frac{n}{3n+1})\right]}....(14)$$

Because $q=hA(T_w-T_b)=KA(\frac{\partial T}{\partial r})_{r=R}$..(15) the heat transfer coefficient is calculated from:

$$h=\frac{k(\frac{\partial T}{\partial r})_{r=R}}{T_w-T_b}..(16)$$

The temperature gradient is given by:

$$\frac{\partial T}{\partial r}\ |r=R=\frac{u_\circ}{\alpha}\frac{\Delta T}{\Delta x}\frac{R^2}{2}\left[1-(\frac{2n}{3n+1})\right]$$

Using B.C.3 r = R T = Tw

$$T_w=T_c+\frac{u\circ}{\alpha}\frac{\Delta T}{\Delta x}\frac{R^2}{2}\left[1-(\frac{2n}{3n+1})^2\right]$$

Substituting the two previous Eqns. in to Eqn. (16) gives:

$$h=\frac{2k\left[1-\frac{2n}{3n+1}\right]}{R\left\{1-\left[(\frac{2n}{3n+1})^2\right]-\frac{\frac{1}{4}-(\frac{n}{5n+1})-\frac{1}{4}(\frac{n}{5n+1})(\frac{n}{3n+1})+2(\frac{n}{3n+1})^3}{\left[\frac{1}{2}-(\frac{n}{3n+1})\right]}\right\}}\text{.............(17)}$$

Or expressions the results in term of the Nussult number as following :

$$Nu=\frac{8(3n+1)(5n+1)}{31n^2+12n+1}\text{...(18)}$$

Case II : constant wall temperature :

It occurs in such heat exchanger as evaporators, condensers, and in fact, any heat exchanger where one fluid has a very much higher capacity rate than the other.

The term $\frac{\partial T}{\partial x}$ is not constant

Introduce the dimensionless quantities as followings

$$\theta=\frac{T-Tw}{Tb-Tw};\eta=\frac{r}{R};\xi=\frac{\alpha x}{\bar{u}R^2}=\frac{1}{G_z}\quad\text{And}\quad \bar{n}=n+1$$

The dimensionless partial differential energy equation, Eqn. (4) , can be written as :

$$(1-\eta\bar{n}+1)\frac{\partial\theta}{\partial\xi}=\frac{1}{\eta}\frac{\partial}{\partial\eta}(\eta\frac{\partial\theta}{\partial\eta})$$

With boundary condition

1. $\theta=1 \quad at \quad \xi=0$

2. $\theta=0 \quad for \quad \eta=1$

3. $\frac{\partial\theta}{\partial\eta}=0 \quad at \quad \eta=0$

By applying the separation of variable technique and letting :

$$\theta(\eta,\xi)=X(\xi)R(\eta)$$

The substitution result two ordinary equation differential equation :

$$\frac{dx}{d\xi}=-\beta^2X$$

$$\frac{1}{\eta}\frac{d}{d\eta}(\eta\frac{dR}{d\eta})+\beta^2(1-\eta^{\bar{n}+1})R=0$$

The complete solution be a linear combination of products of the form :

$$\theta(\eta,\xi)=\sum_{i=1}^{\infty}C_i R(\eta)e-\beta_i^2\xi \qquad (19)$$

Where Ci is arbitrary constant , which determined by using the first boundary condition and the orthogonality of the eigen - functions can be evaluated from the following :

$$Ci=\frac{\int_0^1 \eta(1-\bar{\eta}^{n+1})R(\eta)d\eta}{\int_0^1 \eta(1-\bar{\eta}^{n+1})R^2(\eta)d\eta} \qquad (20)$$

The integral in this equation can be numerically evaluated . We now turn to the evaluation of the Nusselt number as :

$$Nu=\frac{2hR}{k}=-2R\frac{(\frac{\partial T}{\partial r})_{r=R}}{T_b-T_w}=-2\frac{(\frac{\partial\theta}{\partial\eta})^{\eta=1}}{\theta_b} \qquad (21)$$

The temperature gradient as :

$$\frac{\partial\theta}{\partial\eta}\Big|_{\eta=1}=\sum_{i=1}^{\infty}C_i R_i(1)e-\beta_i^2\xi \qquad (22)$$

$$\theta b=\frac{\int_0^1 \phi(\eta)\theta(\xi,\eta)\eta d\eta}{\int_0^1 \phi(\eta)\eta d\eta} \qquad (23)$$

Substitution the velocity and temperature profiles give :

$$\theta_b=-2\sum\frac{C_i}{\beta_i^2}R_i(1)e-\beta_i^2\xi \qquad (24)$$

The Nussult number is then given by :

$$Nu=\frac{\sum_{i=1}^{\infty}C_i R_i(1)e-\beta_i^2\xi}{\sum_{i=1}^{\infty}(\frac{C_i}{\beta_i^2})R_i(1)e-\beta_i^2\xi} \qquad (25)$$

For large ξ we need only the term in each sum so that :

$$\lim_{\xi\to\infty}Nu=\beta_i^2$$

$$i.e Nu=\beta_i^2 \qquad (26)$$

Results and Discussions:

A theoretical treatment is present which allows redial temperature and velocity profiles to be predicted when power law (Ostwald equation) non-Newtonian fluid in laminar flow with fully developed, constant heat flux, and constant wall temperature to exit in circular duct. For n (power law exponent) less than 1, this gives a velocity flatter than the parabolic profile of Newtonian fluid. As n approaches zero, the velocity profile predicted by this equation approach plug flow profile .Also for increasing value of n gradually, Nu is decreases with constant heat flux and constant wall temperature

The rate of heat transfer and the fluid exit temperature is simply evaluated by using temperature profile in equation (12) for constant heat flux and in equation (19) for constant wall temperature.

It can easily be evaluated using theoretical model as a function of power law index only, for constant heat flux as in equation (18), and as a function of power law index and Graets number for constant wall temperature as in equation (26). **Table(l)** shows the results for different n values, the results can be compared with the corresponding predictions, where the asymptotic, downstream, Nussult number for both the isothermal wall condition (constant Wall temperature) and the uniform heat flux (constant heat flux) are shown in **Figure (1)**

Conclusions:

In conclusion, it is obvious from the above discussions that for n (power law exponent) less than 1, this give a velocity flatter than the parabolic profile of Newtonian fluid. As n approaches zero, the velocity profile predicted by this equation approach plug flow profile .Also for increasing value of n gradually, Nu is decreases with constant heat flux and constant wall temperature. This work finds many practical applications in petroleum drilling, manufacturing of foods, production of polymers and slurries. More importantly, the boundary layer concept of non – Newtonian power law fluid has application in the reduction of frictional drags in many engineering process.

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Table (1):Nussult Number for Two Boundary conditions

Constant Heat Flux (q_x,Uniform)		Constant Wall Temperature (T_w,Uniform)	
Power Index n	Nussult Number Nu	Eign value β_1^2	Nussult Number Nu
0.0	8.000	2.408	5.800
0.2	5.517	2.203	4.851
0.5	4.746	2.013	4.050
0.8	4.468	1.937	3.752
1.0	4.364	1.897	3.603
1.5	4.214	1.857	3.452
2.0	4.134	1.844	3.400
3.0	4.051	1.841	3.391
4.0	4.007	1.838	3.383

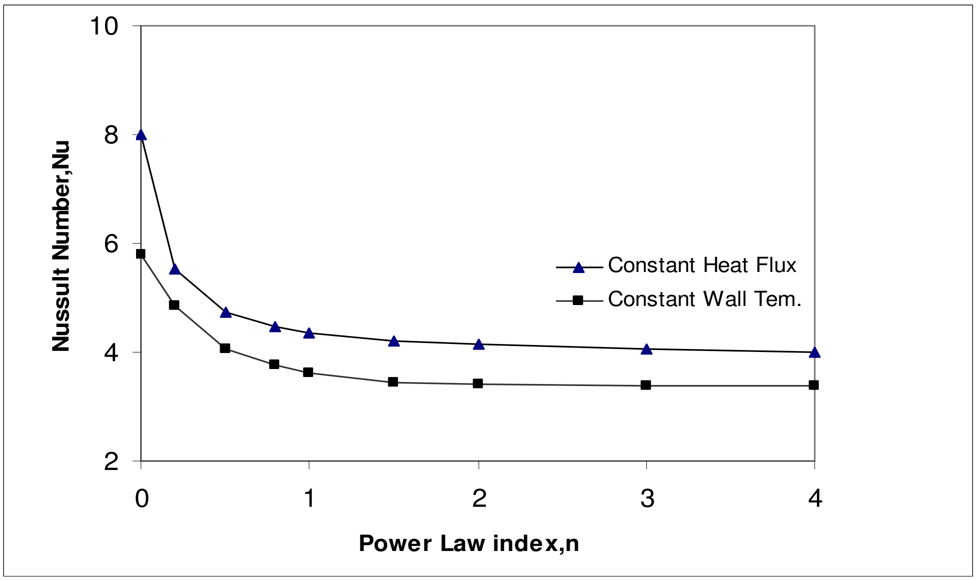

Figure (1): Fully Developed Nussult Number for Laminar flow of a power Law non Newtonian fluid in a tube.