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WAVELET-BASED SOLUTION FOR VIBRATIONS OF A BEAM ON A NONLINEAR VISCOELASTIC FOUNDATION DUE TO MOVING LOAD

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The paper presents a new analytical solution for the dynamic response of an infinitely long Timoshenko beam resting on a nonlinear viscoelastic foundation. Vibrations of the beam are analysed by using Adomian's decomposition method combined with wavelet based approximation alleviating difficulties related to Fourier analysis and numerical integration. The developed approach allows various parametric analyses leading to full characteristics of the investigated dynamic system.

Key words: Timoshenko beam, Adomian's decomposition, coiflet expansion, moving load

1. Introduction

The development of modern high speed trains and continuously growing influence of train transportation on the surrounding environment, especially buildings and road constructions, have led to the necessity of better understanding and modelling of dynamic phenomena related to beam-soil-loads interactions (Bogacz and Frischmuth, 2009; Bogacz and Krzyzynski, 1991). The model of a beam on a nonlinear foundation is of great theoretical and practical significance in railway engineering as, in practice, the foundation can be strongly nonlinear. One can find in the literature many interesting results concerning moving load problems in mostly linear cases (Fryba, 1999), whereas the nonlinear approach is still open for investigation.

The lack of effective methods prevents analytical solving of nonlinear problems that can be used for parametric analysis of systems. Many results obtained by using numerical simulations have already been published showing better modelling possibilities when using nonlinear cases. Dahlberg (2002) obtained some partial results and found that the nonlinear model simulates the beam deflection fairly well, compared with measurements, whereas the linear model did not. More recently, Wu and Thompson (2004) used a similar nonlinear model and studied the dynamic response using FEM. A relatively small number of papers regarding analytical approaches can be found, highlighting their importance for the subject. In the paper by Kargarnovin *et al.* (2005), the governing nonlinear equations of motion are solved by using a perturbation method in conjunction with the Fourier transform and Cauchy's residue theorem.

The present paper shows a new approach for dynamic analysis of the Timoshenko beam on a nonlinear viscoelastic foundation. This new method is based on Adomian's decomposition method (Adomian, 1989) combined with wavelet-based expansion of functions (Koziol, 2010; Wang *et al.*, 2003) that allows one to omit numerical integration, leading to effective evaluation of approximating functions. The presented analytical solution, together with the developed innovative method, is the main novelty of the article. The question of stability for the obtained solution is left as an open problem and it is not discussed in the paper.

The wavelet-based method of the system dynamic solution, using the coiflet expansion of functions, allowed one to formulate a special methodology for vibration analysis of beam-foundation structures subject to moving loads. The basics of the developed methodology are presented in Koziol (2010), along with a number of mathematical examples showing main features of the coiflet-based method. In that monograph, a few models related to moving load problems were analysed. A model of the Euler-Bernoulli beam placed inside a viscoelastic layer (or on its surface) shows how the applied semi-analytical method improves efficiency and effectiveness of the parametric analysis compared to numerical computations (Koziol *et al.*, 2008; Koziol, 2010; Koziol and Mares, 2010).

The wavelet-based expansion combined with Bourret's approximation gave a new solution in the case of a dynamic response of the Euler Bernoulli and Timoshenko beam resting on a viscoelastic foundation with random variations modelled by a stochastic subgrade reaction. The obtained result extended a class of solutions for problems described by stochastic differential equations with differentiable correlation functions (Koziol and Hryniewicz, 2006; Hryniewicz and Koziol, 2009).

The analysis of the coiflet-based solution for the Euler-Bernoulli beam placed inside a viscoelastic medium subject to moving point load carried out in the frequency domain allowed one to estimate critical velocities for specific sets of parameters (Koziol and Mares, 2010). The Timoshenko beam was analyzed as a better representation of the track response to moving load (Hryniewicz, 2009; Hryniewicz and Koziol, 2009).

The present paper extends a class of models analyzed by using the developed coiflet-based methodology for the analysis of dynamic models. The wavelet expansion combined with Adomian's decomposition is proposed as one of possible semi-analytical approaches for the nonlinear model solution. The nonlinear term included in the Timoshenko beam equation leads to complexity of computations, and a relatively small number of analytical solutions for this model can be found in the literature so far. These solutions are usually insufficiently exact (Kargarnovin *et al.*, 2005) and may give wrong results for some sets of parameters.

2. Formulation of the system

The Timoshenko beam is considered by many authors as a more relevant modelling approach for moving load problems analysis due to its specific form taking into account both the shear deformations and the rotary inertia of cross sections. The dynamic equations for the beam on a nonlinear viscoelastic foundation can be written as follows

$$
\begin{aligned}
& m_b\frac{\partial^2 W}{\partial t^2} + C_d\frac{\partial W}{\partial t} - S\frac{\partial^2 W}{\partial \widetilde{x}^2} + S\frac{\partial \varPsi}{\partial \widetilde{x}} + k_L W + k_N W^3 = -P(\widetilde{x},t)\mathrm{e}^{\mathrm{i}\varOmega t} \\
& J\frac{\partial^2 \varPsi}{\partial t^2} - EI\frac{\partial^2 \varPsi}{\partial \widetilde{x}^2} + S\varPsi - S\frac{\partial W}{\partial \widetilde{x}} = 0
\end{aligned}
\tag{2.1}
$$

where W is the transverse displacement of the neutral axis, $\varPsi$ is the angular rotation of the cross-section, $\widetilde{x}$ is the space coordinate in the direction along the beam, t represents time, k_N is the nonlinear part of the foundation stiffness, k_L is the linear coefficient of the foundation stiffness, $k_L = \widetilde{k}(1 + 2\zeta\mathrm{i})$, $\mathrm{i} = \sqrt{-1}$ and ζ is the hysteretic damping ratio. The used parameters are: Young's modulus E, shear modulus G, mass density ρ, viscous damping of the foundation C_d, cross-section area A, moment of inertia I, shear correction factor κ, mass distribution per unit length $m_b = \rho A$, shear rigidity $S = \kappa AG$, beam flexural rigidity EI, mass moment of inertia $J = \rho I$, load frequency $\varOmega = 2\pi f_\varOmega$ and the load velocity V.

The cubic representation of the nonlinear term in Eq. $(2.1)_1$ was already used in previous papers (e.g. Kargarnovin *et al.*, 2005; Sapountzakis and Kampitsis, 2011). Some authors investigate theoretically more complex representations of the nonlinear factor. An analysis of how the order of polynomial representing the nonlinear spring force influences the solution for vibrations of the infinite Euler-Bernoulli beam resting on a nonlinear elastic foundation was carried out by

Jang *et al.* (2011). The authors assumed that the nonlinear restoration is analytic and therefore can be expanded by the Taylor series. Their newly developed iterative method allowed them to obtain the nonlinear deflection of the beam and to analyse the solution parametrically. It is shown that the contribution from the nonlinear spring becomes smaller when the order of nonlinearity increases.

In this paper, the distributed moving load is considered, being a more realistic representation of loading related to train motion

$$P(\widetilde{x},t)=\frac{P_0}{2a}\cos^2\frac{\pi}{2a}(\widetilde{x}-Vt)H[a^2-(\widetilde{x}-Vt)^2] \tag{2.2}$$

where $H(\cdot)$ is the Heaviside step function and $2a$ is the span of moving load.

In order to analyze the steady-state response of the beam, the Galilean co-ordinate transformation $x=\widetilde{x}-Vt$ can be used. In the case of an infinitely long beam, the boundary conditions must reflect the fact that the displacement, the slope of the beam curvature, the shear force and the bending moment tend to zero when the variable x tends to the infinity.

The following representation of the response can be assumed for analytical solution of the system

$$W(x,t)=w(x)\mathrm{e}^{\mathrm{i}\Omega t} \qquad \Psi(x,t)=\psi(x)\mathrm{e}^{\mathrm{i}\Omega t} \tag{2.3}$$

The use of the chain rule applied to these equations leads to following expressions

$$\begin{aligned}
&\frac{\partial W}{\partial\widetilde{x}}=\frac{dw}{dx}\mathrm{e}^{\mathrm{i}\Omega t} \qquad \frac{\partial^2 W}{\partial\widetilde{x}^2}=\frac{d^2w}{dx^2}\mathrm{e}^{\mathrm{i}\Omega t} \qquad \frac{\partial W}{\partial t}=\Big(-V\frac{dw}{dx}+\mathrm{i}\Omega w\Big)\mathrm{e}^{\mathrm{i}\Omega t}\\
&\frac{\partial^2 W}{\partial t^2}=\Big(V^2\frac{d^2w}{dx^2}-2\mathrm{i}\Omega V\frac{dw}{dx}-\Omega^2 w\Big)\mathrm{e}^{\mathrm{i}\Omega t}
\end{aligned} \tag{2.4}$$

and similar formulas can be obtained for the function Ψ. Substitution of Eqs. (2.4) to system (2.1) yields

$$\begin{aligned}
&a_0\frac{d^2w}{dx^2}+a_1\frac{dw}{dx}+a_2w+S\frac{d\psi}{dx}=-P(x)-k_Nw^3\mathrm{e}^{2\mathrm{i}\Omega t}\\
&b_0\frac{d^2\psi}{dx^2}+b_1\frac{d\psi}{dx}+b_2\psi-S\frac{dw}{dx}=0
\end{aligned} \tag{2.5}$$

where new coefficients are introduced:

$$\begin{aligned}
&a_0=m_bV^2-S && a_1=-V(2\mathrm{i}\Omega m_b+C_d) && a_2=k_L-m_b\Omega^2+\mathrm{i}C_d\Omega\\
&b_0=JV^2-EI && b_1=-2\mathrm{i}\Omega VJ && b_2=S-\Omega^2 J
\end{aligned} \tag{2.6}$$

3. Adomian's decomposition method

Adomian's decomposition assumes that the solution can be represented by an infinite series with one linear term, and the rest of them as a set of functions related to the nonlinear factor. Thus, the nonlinear part of Eq. $(2.5)_1$ can be described by a series

$$w^3(x)=\sum_{j=0}^{\infty}A_j(x) \tag{3.1}$$

with the Adomian polynomials (Pourdarvish, 2006)

$$A_j(x)=\frac{1}{j!}\Big[\frac{d^j}{d\lambda^j}\Big(\sum_{k=0}^{\infty}\lambda^k w_k(x)\Big)^3\Big]_{\lambda=0} \qquad j=0,1,2,\ldots \tag{3.2}$$

Adomian's decomposition assumes the specific form of solution

$$w(x) = \sum_{j=0}^{\infty} w_j(x) \qquad \psi(x) = \sum_{j=0}^{\infty} \psi_j(x) \tag{3.3}$$

and the explicit forms of the Adomian polynomials for the term $w^3(x)$ can be found as follows (Pourdarvish, 2006)

$$\begin{aligned} &A_0 = w_0^3 \qquad A_1 = 3w_0^2 w_1 \qquad A_2 = 3(w_0 w_1^2 + w_0^2 w_2) \\ &A_3 = w_1^3 + 6w_0 w_1 w_2 + w_0^2 w_3 \qquad A_4 = 3(w_1^2 w_2 + w_0 w_2^2 + 2w_0 w_1 w_3 + w_0^2 w_4) \quad \dots \end{aligned} \tag{3.4}$$

Equations $(2.5)_1$ can be rewritten by using the operator notation as follows

$$L_w w + S\frac{d\psi}{dx} = -P(x) - k_N w^3(x) \mathrm{e}^{2\mathrm{i}\Omega t} \qquad L_\psi \psi - S\frac{dw}{dx} = 0 \tag{3.5}$$

with the differential operators

$$L_w = a_0 \frac{d^2}{dx^2} + a_1 \frac{d}{dx} + a_2 \qquad L_\psi = b_0 \frac{d^2}{dx^2} + b_1 \frac{d}{dx} + b_2 \tag{3.6}$$

The convergence condition for series (3.1) can be formulated as the system of inequalities $0 \leqslant \alpha_j < 1$, $(j = 0, 1, 2, \dots)$ for the parameter α_j defined as follows

$$\alpha_j = \begin{cases} \dfrac{\|w_{j+1}\|}{\|w_j\|} & \text{for} \quad \|w_j\| \neq 0 \\ 0 & \text{for} \quad \|w_j\| = 0 \end{cases} \tag{3.7}$$

with the norm $\|w_j\| = \max_x |\mathrm{Re}[w_j(x)]|$. One can show (Adomian, 1989) that

$$\lim_{n\to\infty} S_n^w(x) = \lim_{n\to\infty} \sum_{j=0}^{n} w_j(x) = w(x) \tag{3.8}$$

and the effective analysis of solution can be carried out by using the n-th order approximation $S_n^w(x)$. The same assumptions can be used when dealing with the terms $\psi_j(x)$. The classical Fourier transforms are used for derivation of the solution

$$\widehat{w}(\omega) = F[w(x)] = \int_{-\infty}^{\infty} w(x)\mathrm{e}^{-\mathrm{i}\omega x}\, dx \qquad \widehat{\psi}(\omega) = F[\psi(x)] = \int_{-\infty}^{\infty} \psi(x)\mathrm{e}^{-\mathrm{i}\omega x}\, dx \tag{3.9}$$

and

$$\begin{aligned} w(x) &= F^{-1}[\widehat{w}(\omega)] = \frac{1}{2\pi} \int_{-\infty}^{\infty} \widehat{w}(\omega)\mathrm{e}^{\mathrm{i}\omega x}\, d\omega \\ \psi(x) &= F^{-1}[\widehat{\psi}(\omega)] = \frac{1}{2\pi} \int_{-\infty}^{\infty} \widehat{\psi}(\omega)\mathrm{e}^{\mathrm{i}\omega x}\, d\omega \end{aligned} \tag{3.10}$$

Formulas (3.1) and (3.2) lead to the following system of equations

$$L_w w_0 + S\frac{d\psi_0}{dx} = -P(x) \qquad L_\psi \psi_0 - S\frac{dw_0}{dx} = 0 \tag{3.11}$$

and

$$
\begin{aligned}
&L_w w_j + S\frac{d\psi_j}{dx} = -k_N A_{j-1}(x)\mathrm{e}^{2\mathrm{i}\Omega t} \\
&L_\psi \psi_j - S\frac{dw_j}{dx} = 0 \qquad\qquad j = 1,2,\dots
\end{aligned}
\tag{3.12}
$$

where the forms of $w_j(x)$ and $\psi_j(x)$ can be found using recursively the above system of formulas. Applying the Fourier transform to both sides of Eqs. (3.11), one obtains

$$
(-a_0\omega^2 + \mathrm{i}a_1\omega + a_2)\widehat{w}_0 + \mathrm{i}S\omega\widehat{\psi}_0 = -\widehat{P}(\omega) \qquad (-b_0\omega^2 + \mathrm{i}b_1\omega + b_2)\widehat{\psi}_0 - \mathrm{i}S\omega\widehat{w}_0 = 0 \tag{3.13}
$$

where

$$
\widehat{P}(\omega) = P_0 \sin\frac{a\omega}{2} a\omega\Big[1 - \Big(\frac{a\omega}{\pi}\Big)^2\Big]
$$

Solving the system of equations (3.13) leads to the transformed solution for w_0 and ψ_0

$$
\widehat{w}_0 = \frac{\widehat{P}H_\psi}{S^2\omega^2 - H_w H_\psi} \qquad \widehat{\psi}_0 = \frac{\mathrm{i}S\omega\widehat{P}}{S^2\omega^2 - H_w H_\psi} \tag{3.14}
$$

where $H_w = -a_0\omega^2 + \mathrm{i}a_1\omega + a_2$, $H_\psi = -b_0\omega^2 + \mathrm{i}b_1\omega + b_2$. In order to find the solution in the physical domain, it is enough to derive the inverse Fourier transforms (3.10) of Eqs. (3.14).

4. Coiflet-based wavelet approximation

The specific form of integrands (3.14) prevents effectively analytical evaluation of the integrals. On the other hand, numerical evaluations lead to computational difficulties resulting in inexact solutions. Therefore, one must apply an alternative method of derivation, giving results precise enough that can be used for further approximation of the higher order (i.e. calculation of w_1, ψ_1, w_2, ψ_2 and so on). One of the analytical methods whose efficiency for analysis of dynamic systems has already been proved before, is the approximation which uses the wavelet expansion of functions (Koziol, 2010; Wang *et al.*, 2003). This approach is based on approximation of multiresolution coefficients that can be relatively easily done in the case of wavelet functions defined by coiflets (Monzon *et al.*, 1999)

$$
\Psi_C(x) = \sum_{k=0}^{3N-1} (-1)^k p_{3N-1-k}\Phi_C(2x-k) \qquad \Phi_C(x) = \sum_{k=0}^{3N-1} p_k\Phi_C(2x-k) \tag{4.1}
$$

Ψ_C and Φ_C are the wavelet and scaling function, respectively, and N is treated as a degree of accuracy of the coiflet filter (p_k). The property of vanishing moments (Koziol, 2010; Mallat, 1998; Monzon *et al.*, 1999; Wang *et al.*, 2003) of coiflets for both, wavelet and scaling functions, allows one to estimate analytically the wavelet coefficients (Wang *et al.*, 2003), leading to formulas used in an effective approximation of the Fourier integrals. Using the refinement equations of Ψ_C and Φ_C in the transform domain (Mallat, 1998; Monzon and Beylkin, 1999; Wang *et al.*, 2003) and assuming $\widehat{\Phi}_C(0) = 1$ leads to an efficient algorithm for the approximation of the Fourier transform and the inverse Fourier transform

$$
\begin{aligned}
&\widehat{f}(\omega) = \lim_{n\to\infty} \widehat{f}_n(\omega) = \lim_{n\to\infty} 2^{-(n+1)} \prod_{k=1}^{\infty}\Big(\sum_{k=0}^{3N-1} p_k \mathrm{e}^{\mathrm{i}k\omega 2^{-(n+k)}}\Big) \sum_{k=-\infty}^{+\infty} f((k+M)2^{-n})\mathrm{e}^{-\mathrm{i}\omega k 2^{-n}} \\
&f(x) = \lim_{n\to\infty} f_n(x) = \lim_{n\to\infty} \frac{1}{2^{n+2}\pi} \prod_{k=1}^{\infty}\Big(\sum_{k=0}^{3N-1} p_k \mathrm{e}^{-\mathrm{i}kx 2^{-(n+k)}}\Big) \sum_{k=-\infty}^{+\infty} \widehat{f}((k+M)2^{-n})\mathrm{e}^{\mathrm{i}xk 2^{-n}}
\end{aligned}
\tag{4.2}
$$

The high order of coiflets accuracy, although giving better regularity of the generated wavelet function, leads to computational difficulties (Monzon *et al.*, 1999), and therefore, a good balance between the efficiency and cost-effectiveness must be found. The criterion for the choice of the approximation order is the stabilisation of solutions above some value of the parameter n (Koziol, 2010).

5. Wavelet-based solution

The coiflet expansion applied to formulas (3.14) gives the following representation of the solution for the first terms of sought series (3.8)

$$w_0(x) = \frac{1}{2^{n+1}\pi}\varphi_C(-x2^{-n}) \sum_{k=k_{min}}^{k_{max}} \widehat{w}_0((k+M)2^{-n})\mathrm{e}^{\mathrm{i}kx2^{-n}} \tag{5.1}$$

where

$$\varphi_C(-x2^{-n}) = \prod_{k=1}^{k_p} \sum_{j=0}^{3N-1} p_j \mathrm{e}^{jx2^{-(n+k)}} \tag{5.2}$$

and the approximated solution for $\psi_0(x)$ can be written similarly. In order to obtain the next terms of series (3.8), one must apply the Fourier transform to Eqs. (3.12). Assuming $j=1$, one obtains

$$\widehat{w}_1(\omega) = \frac{k_N H_\psi \widehat{A}_0(\omega)}{S^2\omega^2 - H_w H_\psi}\mathrm{e}^{2\mathrm{i}\Omega t} \qquad \widehat{\psi}_1(\omega) = \frac{\mathrm{i}S\omega k_N \widehat{A}_0(\omega)}{S^2\omega^2 - H_w H_\psi}\mathrm{e}^{2\mathrm{i}\Omega t} \tag{5.3}$$

where the term $\widehat{A}_0(\omega)$ is the Fourier transform of the Adomian polynomial $A_0(x) = [w_0(x)]^3$, and it can be derived by using the inverse coiflet expansion $(4.2)_1$. The inverse Fourier transform applied to $\widehat{w}_1(\omega)$ and $\widehat{\psi}_1(\omega)$ gives terms $w_1(x)$ and $\psi_1(x)$. Applying the same procedure for evaluation of terms with arbitrarily chosen index j, leads to a general form of the Adomian polynomials A_j and the solution represented by series (3.8)

$$\begin{aligned}
\widehat{A}_j(\omega) &= 2^{-n}\widehat{\varphi}_C\Big(\frac{\omega}{2^n}\Big) \sum_{k=k_{min}}^{k_{max}} A_j\Big(\frac{k+M}{2^n}\Big)\mathrm{e}^{-\mathrm{i}k\omega 2^{-n}} \\
\widehat{w}_j(\omega) &= \frac{k_N H_\psi \widehat{A}_{j-1}(\omega)}{S^2\omega^2 - H_w H_\psi}\mathrm{e}^{2\mathrm{i}\Omega t} \qquad \widehat{\psi}_j(\omega) = \frac{\mathrm{i}S k_N \omega \widehat{A}_{j-1}(\omega)}{S^2\omega^2 - H_w H_\psi}\mathrm{e}^{2\mathrm{i}\Omega t} \\
w_j(x) &= \frac{1}{2^{n+1}\pi}\varphi_C\Big(-\frac{x}{2^n}\Big) \sum_{k=k_{min}}^{k_{max}} \widehat{w}_j\Big(\frac{k+M}{2^n}\Big)\mathrm{e}^{\mathrm{i}kx2^{-n}} \\
\psi_j(x) &= \frac{1}{2^{n+1}\pi}\varphi_C\Big(-\frac{x}{2^n}\Big) \sum_{k=k_{min}}^{k_{max}} \widehat{\psi}_j\Big(\frac{k+M}{2^n}\Big)\mathrm{e}^{\mathrm{i}kx2^{-n}}
\end{aligned} \tag{5.4}$$

The range of summation $k_{min}(n) = \omega_{min}2^n - 3N - 2$, $k_{max}(n) = \omega_{max}2^n - 1$ must be found on the basis of information about the transformed function (Koziol, 2010; Wang *et al.*, 2003). The interval $[\omega_{min}, \omega_{max}]$ must cover the set of variable ω having strong influence on the behaviour of the original function.

6. Numerical examples

The observation point x can be chosen arbitrarily and the point $\widetilde{x} = 0$ (i.e. $x = -Vt$) is assumed for numerical examples presented in this paper.

The following set of parameters (Kargarnovin *et al.*, 2005; Kim, 2005) is adopted for numerical simulations

$$
\begin{array}{lll}
\zeta = 0.02 & k_N = 5 \cdot 10^{11}\,\mathrm{N/m^4} & \widetilde{k} = 2 \cdot 10^{7}\,\mathrm{N/m^2} \\
P_0 = 5 \cdot 10^{4}\,\mathrm{N/m} & EI = 3 \cdot 10^{5}\,\mathrm{N\,m^2} & a = 0.075\,\mathrm{m} \\
C_d = 0 & J = \rho I = 0.24\,\mathrm{kg\,m} & S = 2 \cdot 10^{8}\,\mathrm{N} \\
m_b = 300\,\mathrm{kg/m} & &
\end{array}
$$

The values used for numerical examples are chosen on the basis of analyses of systems presented in the literature and are accordingly adjusted in order to present important features of the developed method of solution. One should note that the present paper shows theoretical investigations. The aim of the authors is to present a new semi-analytical method of solution for the considered nonlinear model. The performed simulations show strong potential of the coiflet-based method for the analysis of nonlinear models. However, further more detailed investigations should be carried out for better physical interpretation of the solution and determination of stability domains.

Significant changes in the shape of plots can be observed for some systems of parameters compared with the linear model, especially in the area of critical velocities. Figure 1 shows the vertical displacement of the beam for the linear case and fourth order Adomian's approximation $w \approx w_0 + w_1 + \ldots + w_4$ for sub-critical ((a) and (b)) and super-critical velocities ((c) and (d)) (Hryniewicz, 2009).

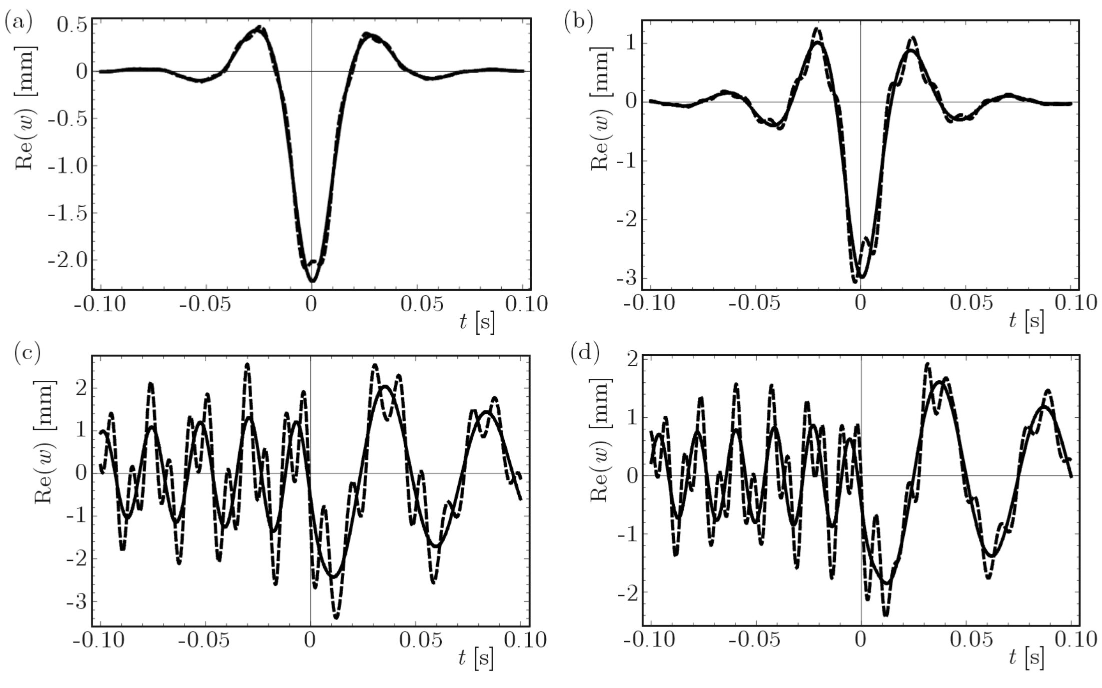

Fig. 1. The vertical displacement in the case of the linear (solid) and nonlinear (dashed) system ($f_\Omega = 22\,\mathrm{Hz}$): (a) $V = 55\,\mathrm{m/s}$; (b) $V = 65\,\mathrm{m/s}$; (c) $V = 85\,\mathrm{m/s}$; (d) $V = 105\,\mathrm{m/s}$

Figure 2 indicates that the effect of higher order terms of Adomian's series (3.8) on the vibratory characteristics of the beam vanishes and they can be neglected above some value of n. The influence of nonlinear terms of Adomian's estimation on the obtained solution becomes stronger along with increasing velocity, with the strongest impact in the case of velocities near critical values (Fig. 3).

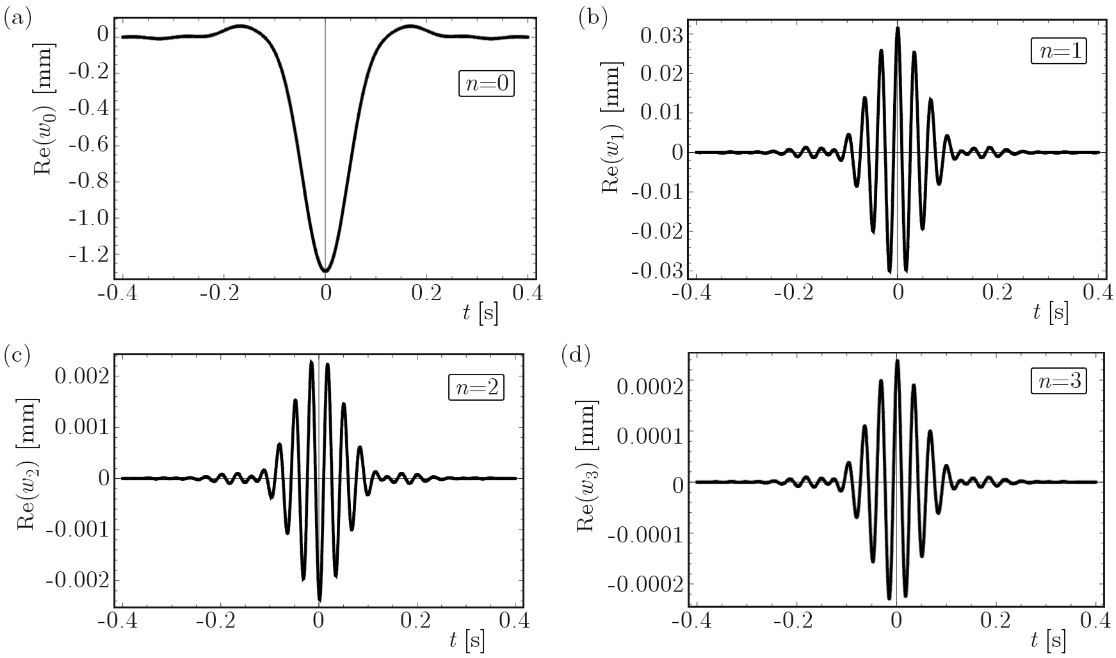

Fig. 2. The first four terms of Adomian's decomposition for the vertical displacement w ($V = 10\,\mathrm{m/s}$ and $f_\Omega = 22\,\mathrm{Hz}$)

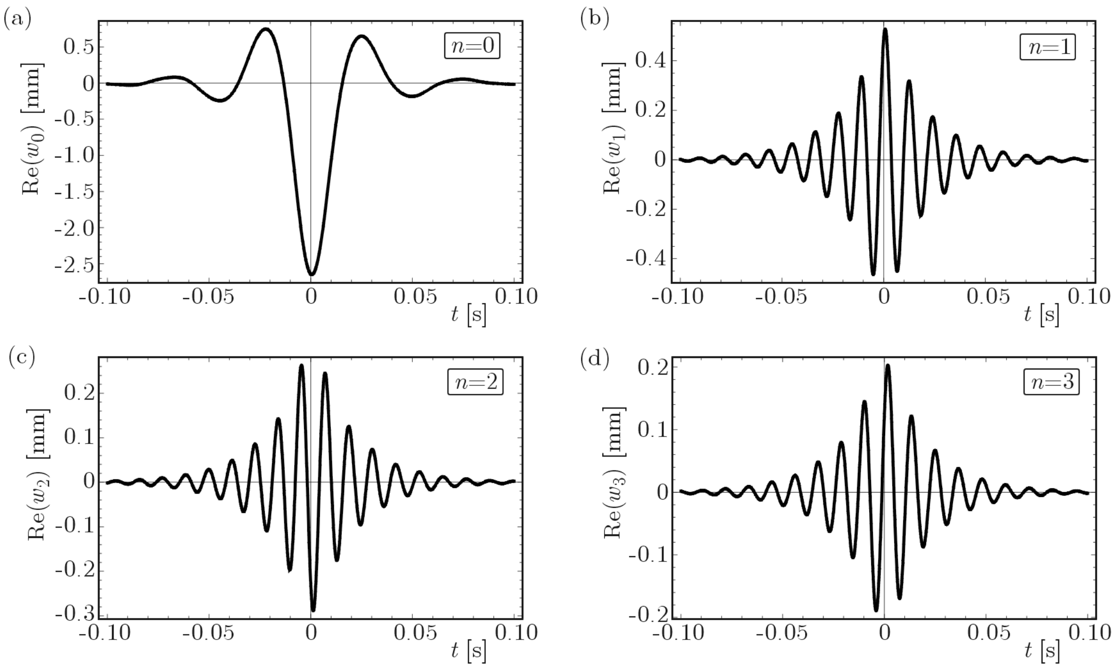

Fig. 3. The first four terms of Adomian's decomposition for the vertical displacement w ($V = 62\,\mathrm{m/s}$ and $f_\Omega = 22\,\mathrm{Hz}$

7. Conclusions

The new solution for vibrations of an infinite Timoshenko beam resting on a nonlinear viscoelastic foundation as a result of moving load is obtained. Adomian's decomposition method combined with the wavelet approximation is adopted in order to obtain an effective solution allowing parametric analysis of the dynamic system. The performed simulations show strong efficiency of the developed approach compared with methods applied by other authors. The presented method can be used for further investigations in order to carry out deeper analysis of physical properties of this model and similar ones.

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Falkowe rozwiązanie problemu drgań belki spoczywającej na nieliniowym lepkosprężystym podłożu generowanych przez poruszające się obciążenie

Streszczenie

Artykuł prezentuje nowe analityczne rozwiązanie problemu dynamicznej odpowiedzi nieskończenie długiej belki Timoshenki spoczywającej na nieliniowym lepkosprężystym podłożu, poddanej ruchomemu obciążeniu rozłożonemu na odcinku i harmonicznemu w czasie. Analiza drgań belki została przeprowadzona przy użyciu dekompozycji Adomiana połączonej z aproksymacją falkową pozwalającą ominąć trudności związane z numerycznym całkowaniem oraz zminimalizować niedogodności analizy Fouriera. Uzyskane rozwiązanie daje możliwość parametrycznej analizy badanego układu dynamicznego prowadzącej do opisu jego fizycznych własności. Opracowana metoda wykorzystująca filtry falkowe typu coiflet może być zastosowana do rozwiązania nieliniowych równań różniczkowych opisujących inne układy dynamiczne typu belka-podłoże.

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