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CHAOTIC VIBRATION OF A JOURNAL-BEARING SYSTEM

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The possibility of occurrence of chaotic phenomena in a symmetric journal-bearing system has been examined in the paper. The influence of transverse load applied to the system as well as the amplitude of kinematic excitation on the regions of chaos has been determined.

1. Introduction

A majority of investigations within the field of dynamics is concerned with the problem of existence of periodic solutions and their properties ensuing from mathematical descriptions that quantitatively come across a given phenomenon. Predominatly, the deterministic character of such descriptions is assumed; i.e., the appropriate differential equations are expected to yield the results considered as a future state of the system under imposed initial conditions.

At the end of the 19th century H. Poincaré discovered and described some irregularities in the behaviour of certain mechanical systems – now considered as chaotic manifestation of the system's response. However, the appearance of the new quality in mechanics did not enjoy special favour those days. Only the works by Lorenz (1963) and Landau (1965) tens of years later returned to the studies on chaotic phenomena. In 1979 Ueda observed chaotic vibration in an electric circuit governed by the Duffing's equation. The early 1980s enriched the contemporary knowledge of chaos with the works by Moon (1980 and 1987), Dowell (1984) and others. Also the contribution of Polish researchers to this freshly reborn domain cannot escape the notice – initiated by Szemplińska-Stupnicka (1986) and continued by Kapitaniak (1987) and the others.

Recent years have revealed the commonness of chaos having been successively found in different systems – not necessarily mechanical. Schuster (cf Kapitaniak (1987)) shows a few examples of chaotic phenomena in chemistry, biology, etc. The development of analytical tools and the availability of new generations of computational units were of significant importance in disclosing the omnipresence of chaos. Numerical simulations, though tedious, enable searching through various combinations of parameters and eventually determination of the regions where chaotic vibration occurs. For proving the chaotic character of those vibration most frequently the Poincaré maps are constructed.

This work is intended to examine the chaotic motion in a journal-bearing system. The detection of regions of the parameters leading to irregularities in the journal motion are here of the highest interest.

2. Considered model

The subject of study is posed by an undeformable rotor supported by slide bearings. The rotor is under external excitation through bearing housing and machine foundations from the base. The scheme of the system is shown in Fig.1.

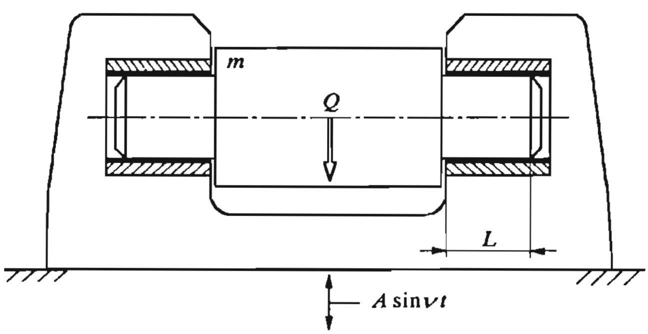


Fig. 1. Model of the considered system

A uniform pressure distribution along the bearing length is assumed. The cross-sections of the journal and the bearings are circular and the lubricating medium is incompressible. The in-plane motion of the journal is regarded. The above assumptions enable taking advantage of the Reynolds' equation in the simplified form

$$\left(h^3 p^*_{,\varphi}\right)_{,\varphi} = \omega h_{,\varphi} + 2h_{,t} \tag{2.1}$$

where

p^*	–	reduced pressure function, $p^* = p\delta^2/(6\mu)$
p	–	pressure
δ	–	relative clearance, $\delta = c/R$
c	–	clearance
R	–	journal radius
μ	–	absolute viscosity of oil
h	–	normalized oil thickness, $h = H/c$
e	–	eccentricity
ω	–	angular velocity of the rotor
φ, Θ	–	coordinates according to Fig.2
t	–	time.

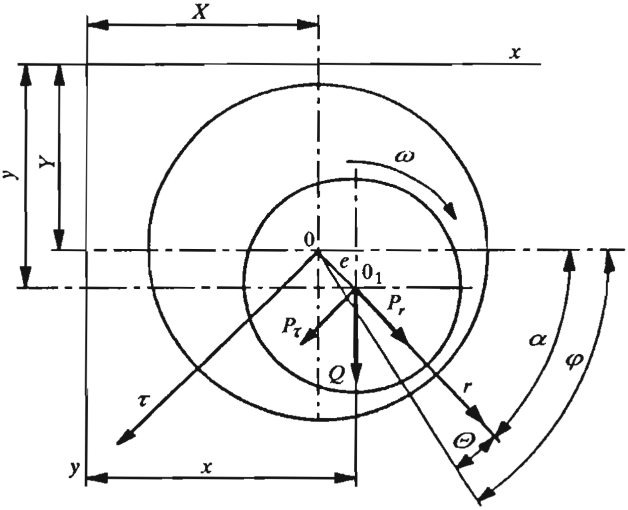


Fig. 2. Coordinate systems

An arbitrary motion of the journal can be decomposed into three components:

(a) rotation about the journal axis

(b) rotation about the bearing axis with the speed $\dot{\alpha}$

(c) transverse motion along the eccentricity radius (squeeze effect) with the velocity $\dot{\varepsilon}$, where ε is the relative eccentricity $(\varepsilon = e/c)$.

For the considered model the circumferential pressure distribution is described as follows (see Kurnik and Starczewski (1984))

$$p^*(\Theta) = \frac{2 - \varepsilon\cos\Theta}{(1 - \varepsilon\cos\Theta)^2}\Big[\frac{2\dot{\alpha} - \omega}{2 + \varepsilon^2}\varepsilon\sin\Theta + \dot{\varepsilon}\cos\Theta\Big] \tag{2.2}$$

Subsequent integration of Eq (2.2) within the positive pressure region for separated wedge and squeeze effects yields the formulas for radial and circumferential components of the hydrodynamic uplift force, P_r and P_τ

$$P_r = -\frac{12\mu RL}{\delta^2}\left[\frac{\varepsilon^2(\omega - 2\dot{\alpha})}{(1 - \varepsilon^2)(2 + \varepsilon^2)} + \frac{\varepsilon\dot{\varepsilon}}{1 - \varepsilon^2} + \frac{2\dot{\varepsilon}}{\sqrt{(1 - \varepsilon^2)^3}}\arctan\sqrt{\frac{1 + \varepsilon}{1 - \varepsilon}}\right] \tag{2.3}$$

$$P_\tau = \frac{6\pi\mu RL}{\delta^2}\frac{\varepsilon(\omega - 2\dot{\alpha})}{\sqrt{1 - \varepsilon^2}(2 + \varepsilon^2)}$$

where: L denotes the bearing length (see Fig.1).

3. Equations of motion

The equations of motion pertinent to symmetric rotor of mass m transversely loaded by a unidirectional force Q in the unmovable Cartesian coordinate system x, y are as follows

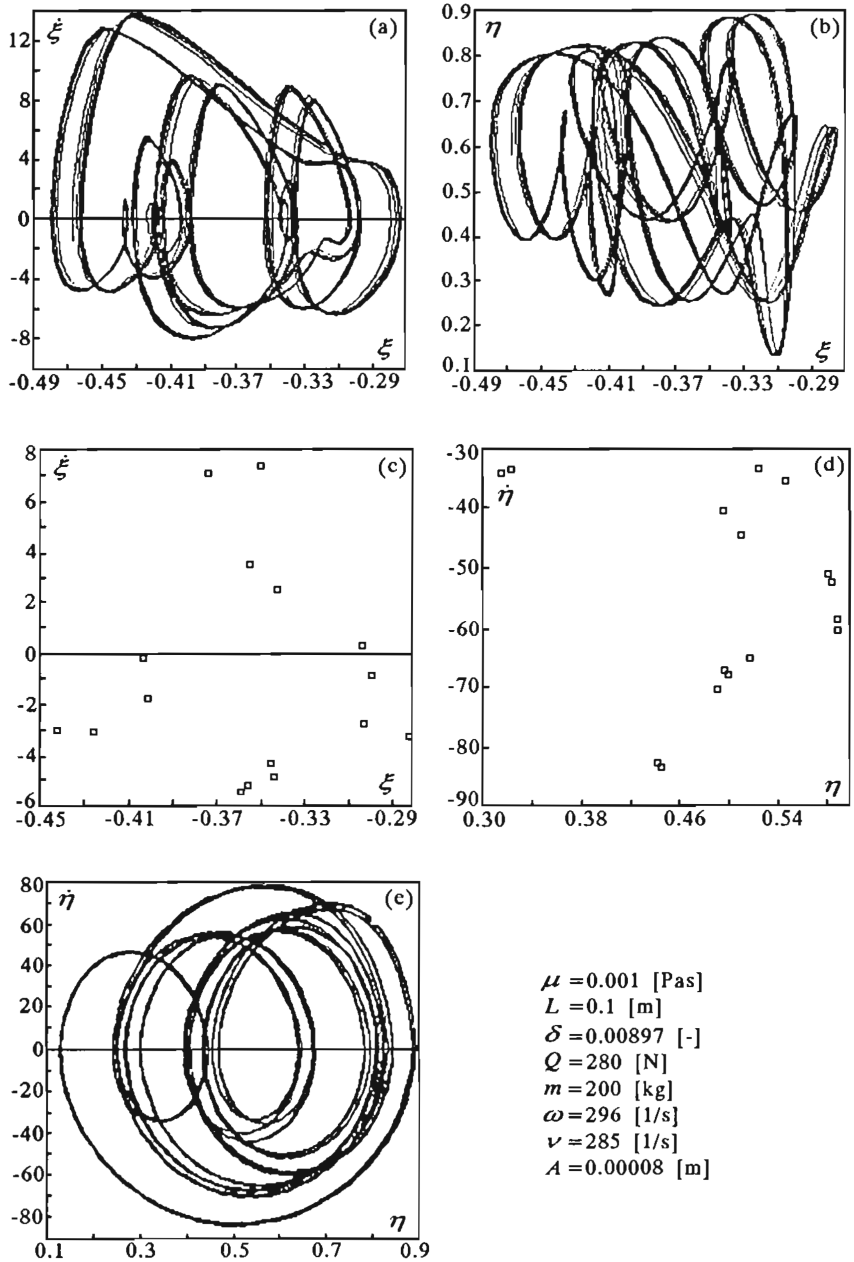
$$\begin{aligned} m\ddot{x} &= P_r\cos\alpha - P_\tau\sin\alpha \\ m\ddot{y} &= P_r\sin\alpha + P_\tau\cos\alpha \end{aligned} \tag{3.1}$$

Introducing new variables (according to the notation given in Fig.2) one obtains

$$\xi = \frac{x - X}{c} \qquad \eta = \frac{y - Y}{c} \tag{3.2}$$

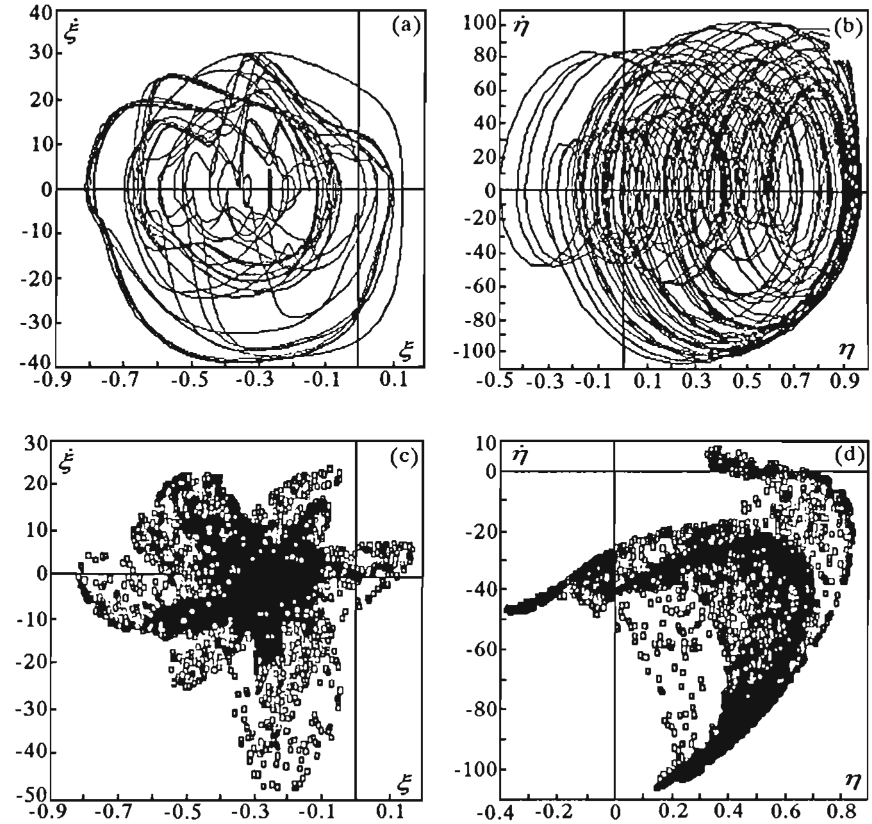
where X, Y are time-dependent coordinates of the bearing center (see Fig.2). Imposing kinematic constraints: $X(t) \equiv 0$, $Y(t) = A\sin\nu t$ one can rewrite Eq (3.1) in the form

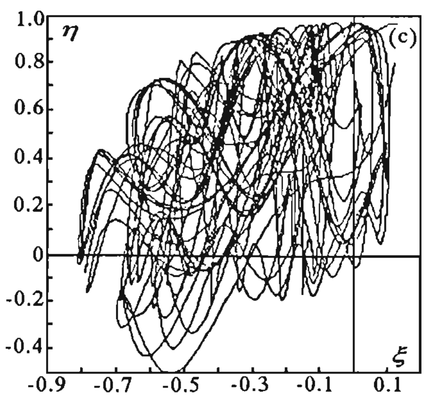
$$\begin{aligned} mc\ddot{\xi} &= P_r\cos\alpha - P_\tau\sin\alpha \\ mc\ddot{\eta} &= P_r\sin\alpha + P_\tau\cos\alpha + Q + mA\nu^2\sin(\nu t) \end{aligned} \tag{3.3}$$



μ =0.001 [Pas]
L =0.1 [m]
δ =0.00897 [-]
Q =280 [N]
m =200 [kg]
ω=296 [1/s]
ν =285 [1/s]
A =0.00008 [m]

Fig. 3.





$\mu = 0.001$ [Pas]
$L = 0.1$ [m]
$\delta = 0.00897$ [-]
$Q = 300$ [N]
$m = 200$ [kg]
$\omega = 296$ [1/s]
$\nu = 285$ [1/s]
$A = 0.00008$ [m]

Fig. 4.

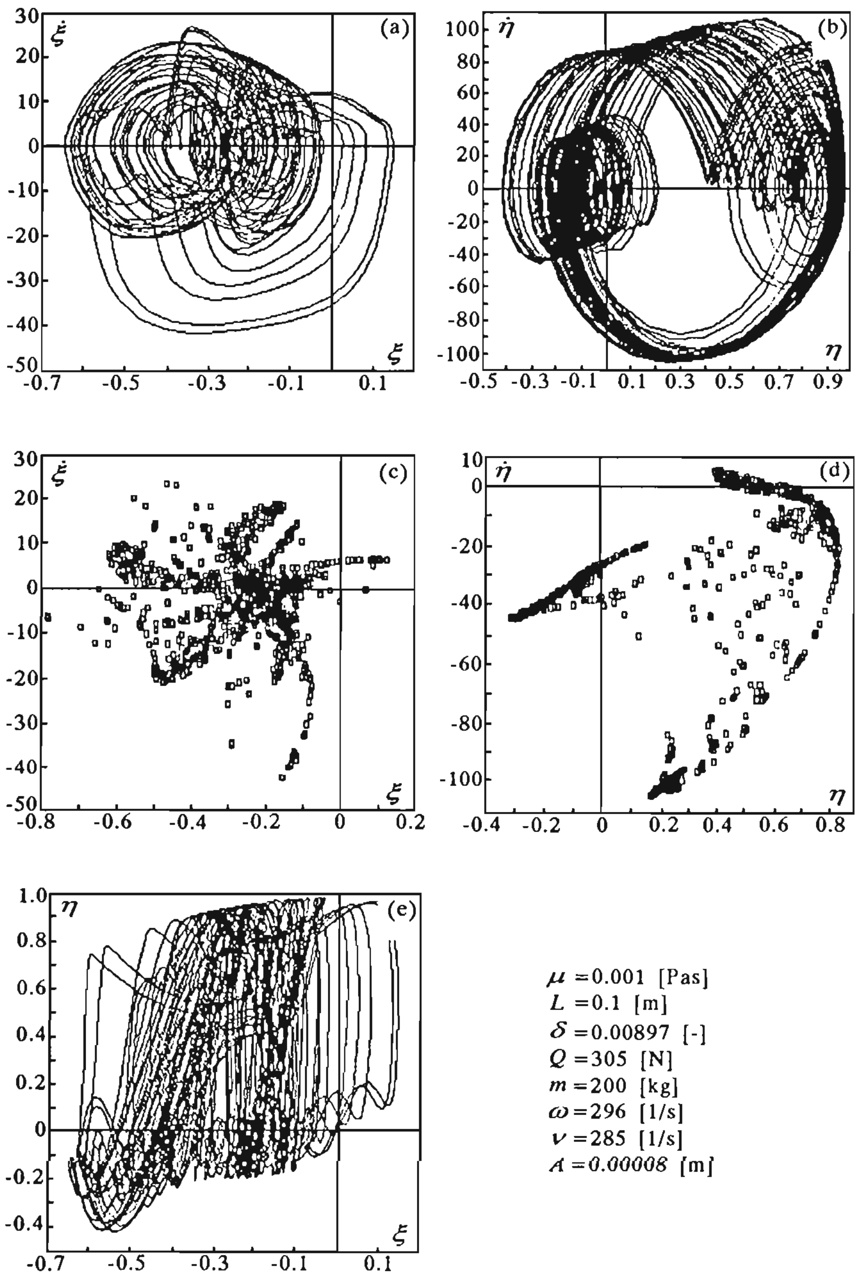

(a)
(b)
(c)
(d)
(e)
μ =0.001 [Pas]
L =0.1 [m]
δ =0.00897 [-]
Q =305 [N]
m =200 [kg]
ω =296 [1/s]
ν =285 [1/s]
A =0.00008 [m]

Fig. 5.

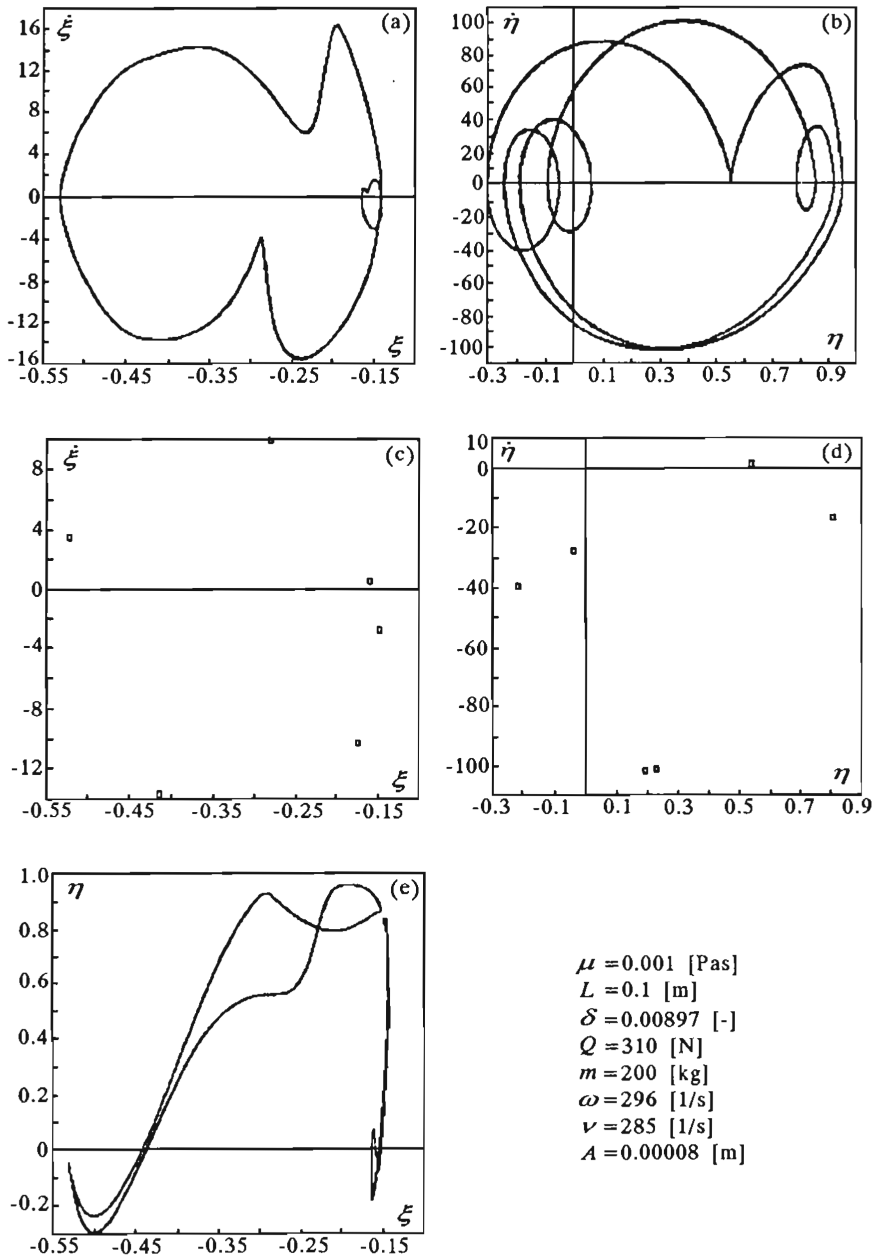

(a)
(b)
(c)
(d)
(e)
μ =0.001 [Pas]
L =0.1 [m]
δ =0.00897 [-]
Q =310 [N]
m =200 [kg]
ω =296 [1/s]
ν =285 [1/s]
A =0.00008 [m]

Fig. 6.

where ν denotes the frequency of the excitation, generally different from the angular velocity of the rotor ω and A stands for the amplitude of the base vibration.

4. Numerical simulation

Since Eqs (2.3) and (3.3) are coupled and, first of all, strongly non-linear with respect to displacements and velocities, the possibility of their purely analytical study is excluded and the numerical approach is proposed. The main problem consists in searching the structural and operating parameters within which the chaotic behaviour can occur. Some hints about those regions of the parameters can be found in the previous works by Kurnik and Starczewski (1985) and Starczewski (1990). Numerical simulations which were performed in abundance have disclosed the regions of chaotic behaviour of the considered model. The results corresponding to various amplitudes of excitation and different external loads are displayed in the form of graphs presented in Fig.3 ÷ Fig.10. Each figure consists of five diagrams where the following is shown:

(a) – phase trajectory for the ξ direction

(b) – phase trajectory for the η direction

(c) – Poincaré map corresponding to phase coordinates $\xi - \dot{\xi}$

(d) – Poincaré map corresponding to phase coordinates $\eta - \dot{\eta}$

(e) – trajectories of the journal center $\eta = f(\xi)$.

The simulations were made with the help of: IBM PC 486 DX2 (double precission used) and IRYS-INDYGO.

5. Final remarks

According to the numerical analysis with the special emphasis put on the chaotic behaviour the following remarks can be formulated:

- The journal-bearing system can exhibit chaotic vibration

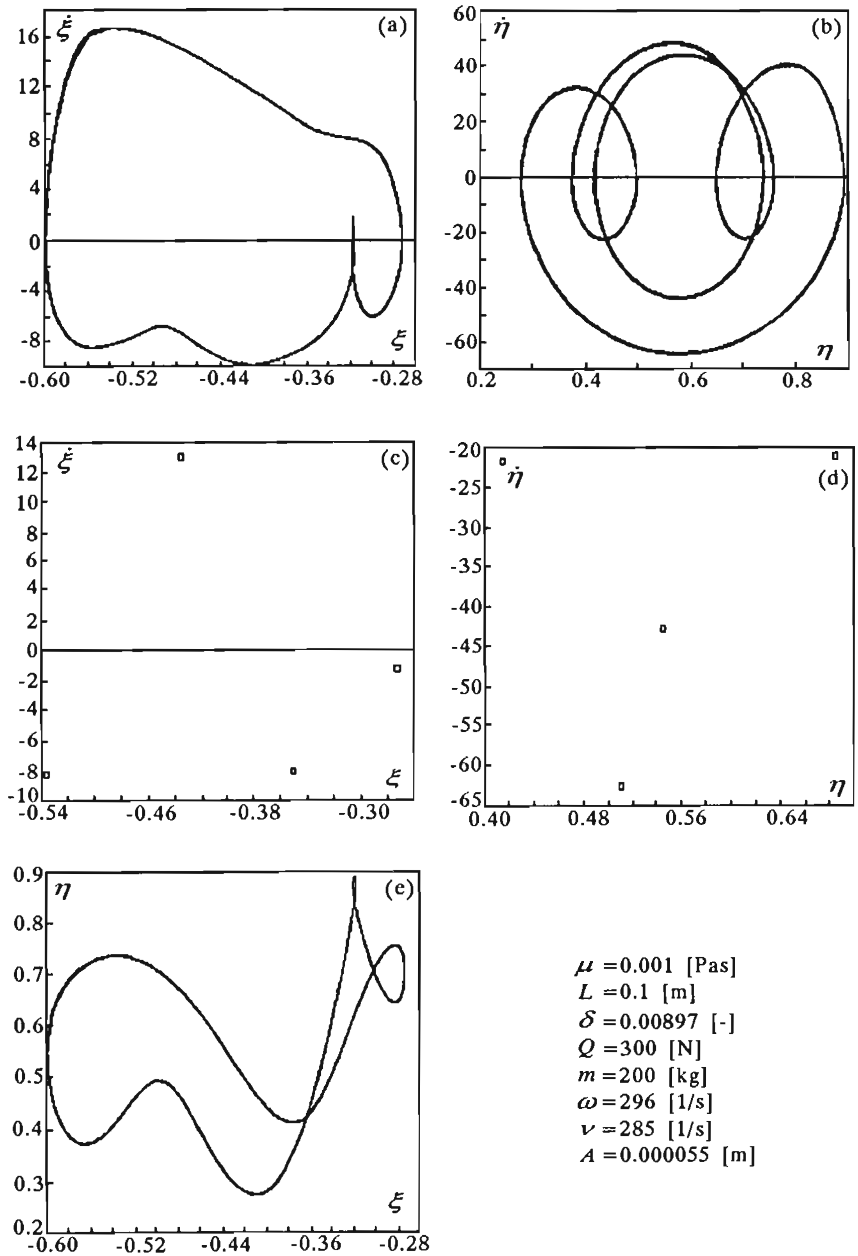

(a)
(b)
(c)
(d)
(e)
$\dot{\xi}$
ξ
$\dot{\eta}$
η
-0.60 -0.52 -0.44 -0.36 -0.28
0.2 0.4 0.6 0.8
-0.54 -0.46 -0.38 -0.30
0.40 0.48 0.56 0.64
μ =0.001 [Pas]
L =0.1 [m]
δ =0.00897 [-]
Q =300 [N]
m =200 [kg]
ω=296 [1/s]
ν =285 [1/s]
A =0.000055 [m]

Fig. 7.

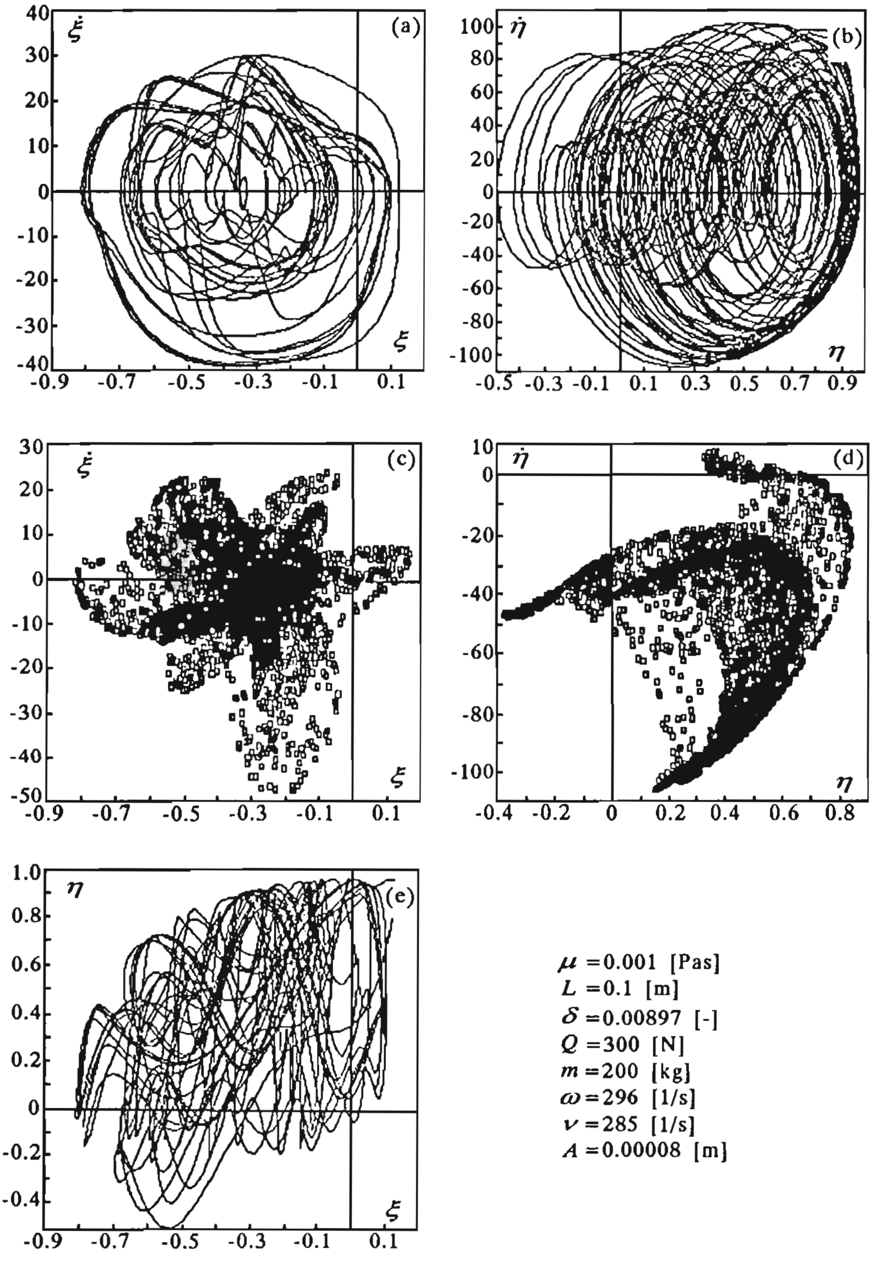

(a)
(b)
(c)
(d)
(e)
μ =0.001 [Pas]
L =0.1 [m]
δ =0.00897 [-]
Q =300 [N]
m =200 [kg]
ω=296 [1/s]
ν =285 [1/s]
A =0.00008 [m]

Fig. 8.

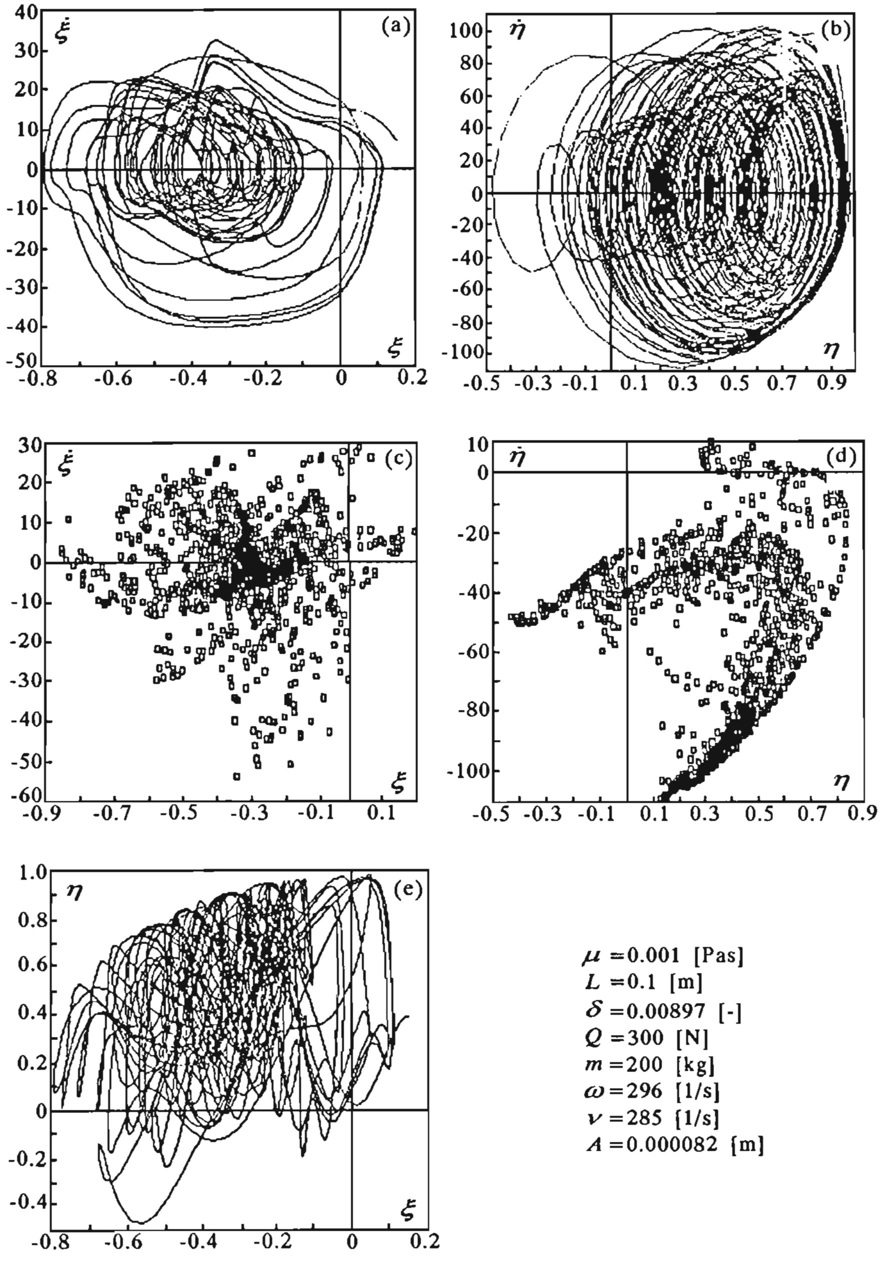

(a)
(b)
(c)
(d)
(e)
μ =0.001 [Pas]
L =0.1 [m]
δ =0.00897 [-]
Q =300 [N]
m =200 [kg]
ω =296 [1/s]
ν =285 [1/s]
A =0.000082 [m]

Fig. 9.

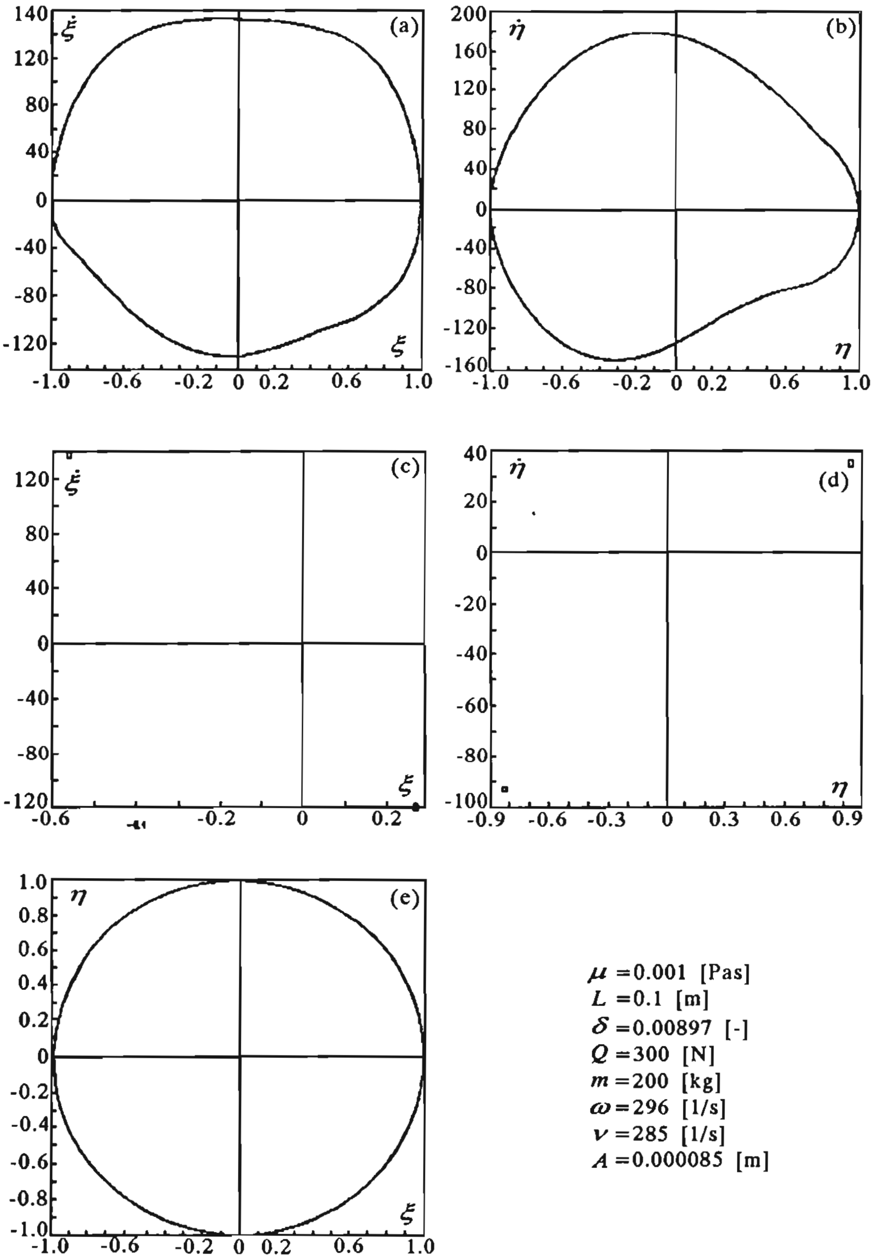

(a)
140
120
80
40
0
-40
-80
-120
-1.0 -0.6 -0.2 0 0.2 0.6 1.0
(b)
200
160
120
80
40
0
-40
-80
-120
-160
-1.0 -0.6 -0.2 0 0.2 0.6 1.0
(c)
120
80
40
0
-40
-80
-120
-0.6 -0.2 0 0.2
(d)
40
20
0
-20
-40
-60
-80
-100
-0.9 -0.6 -0.3 0 0.3 0.6 0.9
(e)
1.0
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1.0
-1.0 -0.6 -0.2 0 0.2 0.6 1.0
μ =0.001 [Pas]
L =0.1 [m]
δ =0.00897 [-]
Q =300 [N]
m =200 [kg]
ω=296 [1/s]
ν =285 [1/s]
A =0.000085 [m]

Fig. 10.

- The appearance of chaotic motion is conditioned by some structural parameters of the system; such as, low oil viscosity μ and high values of the relative clearance δ

- In the case of excitation brought about by the harmonically-varying externally applied transverse force (of constant amplitude) chaos occurs only within the parameters characterized by low values of μ and very short bearings but considerable mass of the rotor, thus the parameters unrecommended while designing a rotor-bearing system (unlikely in practice)

- High sensitivity to initial conditions has been found in the system. Even small differences of initial conditions result in completely different positions and velocities of the journal center

- Presented graphs distinctly show the transition to chaos and return again to periodic motion consistently with the changes of applied load Q and excitation amplitude A.

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Drgania chaotyczne w układzie łożyskowanym ślizgowo

Streszczenie

W pracy zbadano możliwość występowania ruchów chaotycznych w ukladzie symetrycznego wirnika łożyskowanego ślizgowo. Określono wplyw obciążenia zewnętrznego oraz amplitudy wymuszenia kinematycznego na obszary występowania i zanikania ruchów chaotycznych.

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