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ANALYSIS OF THE STOCHASTIC VELOCITY FIELD BY PROPER ORTHOGONAL DECOMPOSITION

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The Karhunen-Loeve expansion, known in fluid dynamics as the Proper Orthogonal Decomposition (POD), of random vector fields is determined. These fields are the velocity of nonstationary flow past the cylindrical contour and uniform at infinity. The velocity field has been calculated with the Vortex Blobs Method. The POD procedure is carried out on the assumption of ergodicity which leads to an algebraic eigenvalue problem.

1. Introduction

Turbulent flows are characterized by highly irregular dependence of kinematic and dynamical parameters on time and space. Currently it is generally believed that the nature of these phenomena is purely deterministic. The chaotic or "stochastic" evolution of a flow is implied by extremely complicated phase-space structure of the nonlinear dynamical system defined by the mathematical model of fluid motion. This structure and its bifurcations due to changes of flow characteristics are responsible for all known features like hydrodynamic instabilities, sensitive dependence on boundary and/or initial data, laminar-turbulent transition, etc.

The nature of turbulent flows makes the application of stochastic tools particularly advantageous. Leaving aside the problem of phenomenological models of turbulent flows we concentrate rather on the stochastic analysis of structures "hidden behind" a chaos.

The methods to perform this task have been known for a relatively long time. The mathematical foundation of them consists in the Karhunen-Loeve

theorem (cf Loeve (1955), Lumley (1970)) on the orthogonal decomposition of a random process. Lumley (1967) proposed the algorithm for extracting regular structures from turbulent flow fields. Since the Karhunen-Loeve expansions are in a way (to be explained later) optimal Lumley called his method the Proper Orthogonal Decomposition (POD). Practical application of this method had to be delayed to middle 80's. It seems that the main reason was the lack of data of appropriate quality. The heart of POD procedure is the space-correlation analysis of a velocity field. Reasonable results can be obtained only when the spatial resolution of the velocity in the whole flow domain is sufficient. It is difficult (and expensive) to satisfy this requirements in an experiment and only in the last decade the quality of numerical simulations reached the desired level. Since 1986 real "explosion" of interest in the POD technique has been noticeable. Probably the most remarkable result lays in realization that the structures generated via the POD can be used as basic functions in the construction of finite dimensional approximations of the Navier-Stokes equations. Such an approach has been successfully applied to boundary and mixing layers (cf Aubry et al.(1988), Glauser et al. (1991), Deliville et al. (1991) and references herein) where the existence of large structures with complicated dynamics has been strongly evidenced by experiments and direct numerical simulation of the full Navier-Stokes equations. In our opinion the POD provides also an effective method for validation and comparison between different numerical methods when applied to viscous flows.

In this paper we present the results of POD procedure applied to the random flow obtained in the random-vortex simulation. The vortex method is a "natural" source of stochastic velocity fields since diffusion of the vorticity is simulated by the random walks of vortex blobs (cf Chorin (1973), Styczek (1987) for details on random vortex algorithms).

It should be emphasized that the results of physically meaning can be achieved only after some averaging process and the comparison of an isolated realization to a flow obtained with the use of other method is generally meaningless. However, after the POD analysis comparison between the "internal structures" of flows is possible.

2. Mathematical foundations of the POD procedure

Here we present the general idea of POD procedure. Detailed exposition of the theory can is given be found by Loeve (1955) and Lumley (1970).

Let $\boldsymbol{U} = \boldsymbol{U}(t, \boldsymbol{x})$ denote a random velocity field defined in a bounded subset D of R^n ($n = 2$ or 3). We assume that all realizations of $\boldsymbol{U}$ are square-integrable in D at any fixed time t

$$\int_D \boldsymbol{U}^2(t, \boldsymbol{x})\, d\boldsymbol{x} < \infty \tag{2.1}$$

In other words, realizations of the random field are from $L^2(D)$.

Consider a deterministic field $\boldsymbol{F} = \boldsymbol{F}(\boldsymbol{x})$ such that $\boldsymbol{F} \in L^2(D)$ and

$$\int_D \boldsymbol{F}^2(\boldsymbol{x})\, d\boldsymbol{x} = 1 \tag{2.2}$$

Let the time t be fixed and define the following random value

$$r = \int_D \boldsymbol{U}\boldsymbol{F}\, d\boldsymbol{x} \tag{2.3}$$

Thus the value r is the orthogonal projection of the realizations of $\boldsymbol{U}$ on the deterministic field $\boldsymbol{F}$. The existence of the integral (2.3) is guaranted by Eqs (2.1) and (2.2).

We calculate the mean value (or expectation) of r^2

$$\begin{aligned} E\{r^2\} &= E\Big\{\int_D U(\boldsymbol{x})F(\boldsymbol{x})\, d\boldsymbol{x} \int_D U(\boldsymbol{y})F(\boldsymbol{y})\, d\boldsymbol{y}\Big\} = \\ &= \iint_D E\{U(\boldsymbol{x}) \otimes U(\boldsymbol{y})\}\boldsymbol{F}(\boldsymbol{x})\boldsymbol{F}(\boldsymbol{y})\, d\boldsymbol{x}d\boldsymbol{y} \end{aligned} \tag{2.4}$$

where $\otimes$ denotes the tensor product of two vectors.

It is important to note that the operator E concerns the realizations of the random field and thus it commutes with any operator dealing with $\boldsymbol{x}$ and t.

What we obtain is the bilinear functional which is induced by the integral operator

$$(\mathcal{K}\boldsymbol{F})(\boldsymbol{x}) = \int_D E\{U(\boldsymbol{x}) \otimes U(\boldsymbol{y})\}\boldsymbol{F}(\boldsymbol{y})\, d\boldsymbol{y} \tag{2.5}$$

The kernel of this operator

$$\mathsf{K}(\boldsymbol{x}, \boldsymbol{y}) = E\{U(\boldsymbol{x}) \otimes U(\boldsymbol{y})\} \tag{2.6}$$

is called the correlation tensor.

The operator $\mathcal{K}$ is symmetric, compact in $L^2(D)$ and nonnegative. Hence it possesses a denumerable set of real and positive eigenvalues $\lambda_1 \geq \lambda_2 \geq \lambda_3 ...$ The only accumulation point of this series is zero and the largest eigenvalue is equal to the norm of the operator $\mathcal{K}$

$$\lambda_1 = \|\mathcal{K}\| = \max_{\|f\|=1} |\mathcal{K}f|$$

The eigenspaces are finite dimensional and orthogonal for different eigenvalues. The denumerable and orthonormal set of eigenfunctions $\{\phi_1, \phi_2, ...\}$ is complete, i.e., any square-integrable field $\boldsymbol{H}$ in D can be expressed as the Fourier series

$$\boldsymbol{H} = \sum_j h_j \boldsymbol{\phi}_j \tag{2.7}$$

where scalar coefficients are given by

$$h_i = \int_D \boldsymbol{H}\boldsymbol{\phi}_j \, d\boldsymbol{x} \tag{2.8}$$

Using the notion of the scalar product in $L^2(D)$ we can write

$$E\{r^2\} = \int_D \mathcal{K}\boldsymbol{F}\boldsymbol{F} \, d\boldsymbol{x} \equiv <\mathcal{K}\boldsymbol{F}, \boldsymbol{F}> \equiv \mathcal{B}(\boldsymbol{F}) \tag{2.9}$$

Up to now $\boldsymbol{F}$ has been an arbitrary field satisfying the normalization condition (2.2). We want to choose $\boldsymbol{F}$ in the way that maximizes the functional $\mathcal{B}$ in the set of all normalized fields

$$S_0 = \left\{\boldsymbol{f}; \quad \int_D f^2 \, d\boldsymbol{x} = 1\right\} \tag{2.10}$$

The properties of $\mathcal{K}$ mentioned above enable drawing the conclusion that

$$\max_{\boldsymbol{F} \in S_0} \mathcal{B}(\boldsymbol{F}) = \mathcal{B}(\boldsymbol{\phi}_1) = \lambda_1 \tag{2.11}$$

i.e., the maximum is achieved for the eigenfunction of $\mathcal{K}$ corresponding to the largest eigenvalue λ_1. More general

$$\max_{\boldsymbol{F} \in S_m} \mathcal{B}(\boldsymbol{F}) = \mathcal{B}(\boldsymbol{\phi}_m) = \lambda_{m+1} \tag{2.12}$$

where

$$S_m = \left\{\boldsymbol{f}; \quad \int_D \boldsymbol{f}^2 \, d\boldsymbol{x} = 1 \wedge <\boldsymbol{f}, \boldsymbol{\phi}_j> = 0 \quad j = 1, .., m\right\} \tag{2.13}$$

We will call the eigenfunctions $\{\phi_1, \phi_2, ...\}$ the eigenstructures of the random field U (or "coherent" structures). At any fixed time U can be written as an expansion

$$U(x) = \sum a_i \phi_i(x) \tag{2.14}$$

where a_i are random values given by the scalar products $a_i = < U, \phi_i >$.

These values are uncorrelated (see the formula (2.5) defining the correlation operator $\mathcal{K}$)

$$E\{a_i a_j\} = E\Big\{ \int_D U(x)\phi_i(x)\, dx \int_D U(y)\phi_j(y)\, dy \Big\} = < \mathcal{K}\phi_i, \phi_j > = \lambda_i \delta_{ij} \tag{2.15}$$

i.e., $E\{a_i a_j\} = 0$ when $i \neq j$ and $E\{a_i^2\} = \lambda_i$.

The correlation tensor can be expanded as follows

$$\mathbf{K}(x, y) = \sum \lambda_i \phi_i(x) \otimes \phi_i(y) \tag{2.16}$$

The series (2.16) is uniformly convergent (Mercer's theorem, cf Vladymirov (1976)).

The average energetic norm of U given by the followin integral

$$E\{\|U\|^2\} = \int_D E\{UU\}\, dx \tag{2.17}$$

can be calculated

$$\begin{aligned} E\{UU\}(x) &= \operatorname{trace} \mathbf{K}(x, x) = \sum \lambda_i \phi_i^2(x) \\ E\{\|U\|^2\} &= \int_D \sum \lambda_i \phi_i^2(x)\, dx = \sum \lambda_i \end{aligned} \tag{2.18}$$

As we see, each eigenstructure has its own contribution to the average kinetic energy of the flow U.

Assume that the random flow has been approximated by the truncated series

$$U \simeq U_{apr} = \sum_{i=1}^{M} a_i \phi_i \tag{2.19}$$

Then the energetic norm of U_{apr} is determined by

$$E\{\|U_{apr}\|^2\} = \int_D E\{U_{apr} U_{apr}\}\, dx = \int_D E\Big\{ \sum_{i=1}^{M} \sum_{j=1}^{M} a_i a_j \phi_i(x)\phi_j(x) \Big\}\, dx = \tag{2.20}$$

$$= \int_D \sum_{i=1}^{M} \sum_{j=1}^{M} E\{a_i a_j\} \phi_i(x)\phi_j(x)\, dx = \sum_{i=1}^{M} \lambda_i \int_D \phi_i^2(x)\, dx = \sum_{i=1}^{M} \lambda_i$$

We see that the portion of total kinetic energy contained in the finite linear combination of M leading eigenstructures is the largest possible one. In the sense of average energetic norm the expansion of U in the basis of its eigenstructures is optimal.

It should be again emphasized that the time has been treated as fixed parameter. This means that eigenvalues and eigenstructures of U are in general time-dependent. Providing that $U(t,x)$ is sufficiently regular one can expect the continuity of (λ_i, ϕ_i), $i = 1, 2, ...$ in time. If U is the flow field with "developed turbulence" then only weak dependence on time can be suspected. This is exactly the case we are going to consider farther on.

3. The ergodic assumption and the stroboscopic method

The basic problem is how to apply the general theory outlined above to calculation of the eigenstructures on the basis of finite set of realizations obtained from a certain numerical or experimental procedure. In order to determine the eigenstructures of the random flow U the correlation tensor $\mathbf{K}$ should be calculated with reasonable accuracy.

The crucial point in the POD procedure is ensemble averaging and thus sufficiently numerous sample of realizations of U ought to be available. Since the number of all realizations is infinite the probability of any single realization is exactly zero. However some realizations are, in certain sense, more probable than others.

In other words, a reasonable averaging process should take into account the variation in relative frequency of occurrence of different realizations. The naive arithmetic averaging is not likely to produce good results. This problem becomes more severe when the number of realization available is rather small. On the other hand, to collect the sufficiently large set of realizations usually strenuous work has to be done – experimental or numerical.

It seems that in some cases these difficulties can be removed. Many real flows can be regarded as stationary ones in statistical meaning. Some of them can be additionally assumed to be ergodic, at least within the acceptable accuracy. In such a case the ensemble averaging is known to be equivalent to time averaging with the use of single and arbitrary chosen realization.

Let $\boldsymbol{u}(t,x)$ denote the realization of the random flow U. The expectation

of the random value r^2 (cf Eq (2.4)) can be now expressed in the form of

$$E\{r^2\} = \lim_{T\to\infty} \frac{1}{T} \int_0^T < \boldsymbol{u}, F > \, dt \tag{3.1}$$

where the product in $L^2(D)$ is defined by Eq (2.3).

Changing the order of integration we have

$$E\{r^2\} = \int_D \Big[\int_D \Big(\lim_{T\to\infty} \frac{1}{T} \int_0^T \boldsymbol{u}(t,\boldsymbol{x}) \otimes \boldsymbol{u}(t,\boldsymbol{y})\, dt \Big) F(\boldsymbol{y})\, d\boldsymbol{y} \Big] F(\boldsymbol{x});\, d\boldsymbol{x} \tag{3.2}$$

Again we introduce the correlation tensor

$$\mathsf{K}(\boldsymbol{x},\boldsymbol{y}) = \lim_{T\to\infty} \frac{1}{T} \int_0^T \boldsymbol{u}(t,\boldsymbol{x}) \otimes \boldsymbol{u}(t,\boldsymbol{y})\, dt \tag{3.3}$$

and the integral operator

$$(\mathcal{K}F)(\boldsymbol{x}) = \int_D \mathsf{K}(\boldsymbol{x},\boldsymbol{y}) F(\boldsymbol{y})\, d\boldsymbol{y} \tag{3.4}$$

which, in turn, induces the bilinear functional $\mathcal{B}(F)$ by Eq (2.9). Time averaging is performed over the infinite time interval. Obviously in practical situation this interval should be cut at some, possibly long, time T. The correlation tensor is then given by

$$\mathsf{K}(\boldsymbol{x},\boldsymbol{y}) = \frac{1}{T} \int_0^T \boldsymbol{u}(t,\boldsymbol{x}) \otimes \boldsymbol{u}(t,\boldsymbol{y})\, dt \tag{3.5}$$

This integral is evaluated with the use of some rule of numerical integration. For simplicity we assume that the interval $[0,T]$ is divided into N parts with the length τ. Then

$$\frac{1}{T} \int_0^T \boldsymbol{u}(t,\boldsymbol{x}) \otimes \boldsymbol{u}(t,\boldsymbol{y})\, dt \cong \frac{1}{N} \sum_{j=0}^{N} \alpha_j \boldsymbol{u}_j(\boldsymbol{x}) \otimes \boldsymbol{u}_j(\boldsymbol{y}) \tag{3.6}$$

where $u_j(\cdot) = u(j\tau,\cdot)$ are instantaneous velocities at sample instants $j\tau$ (the "strobes" of velocity field) and the coefficients α_j are prescribed by the integration rule.

In result we obtain the integral operator

$$(\mathcal{K}_N F)(x) = \int_D \mathsf{K}_N(x,y)F(y)\,dy \tag{3.7}$$

where the kernel K_N is given by the right-hand side of Eq (3.6). This operator is finite dimensional since the kernel has the form of finite sum with separated variables $\boldsymbol{x}$ and $\boldsymbol{y}$. Indeed

$$\int_D \mathsf{K}_N(x,y)F(y)\,dy = \frac{1}{N}\sum_{j=0}^{N} \alpha_j < F, \boldsymbol{u}_j > \boldsymbol{u}_j \tag{3.8}$$

i.e., $\mathcal{K}_N \boldsymbol{F}$ belongs to the finite dimensional subspace spanned by the "strobes" $\{\boldsymbol{u}_0, ..., \boldsymbol{u}_N\}$. We assume that the "strobes" are linearly independent, so the dimension of this subspace is $N+1$.

The eigenfunctions of $\mathcal{K}_N$ are sought in the following form

$$\phi = \sum_{j=0}^{N} a_j \boldsymbol{u}_j \tag{3.9}$$

where the constants a_j, $j = 0, .., N$ should be found. After substitution of Eq (3.8) into Eq (3.7) we have

$$\mathcal{K}_N \phi = \sum_{i=0}^{N} \Big(\sum_{j=0}^{N} \frac{1}{N} \alpha_j < \boldsymbol{u}_i, \boldsymbol{u}_j > a_j \Big) \boldsymbol{u}_i \tag{3.10}$$

Then the eigenvalue problem $\mathcal{K}_N \phi = \lambda \phi$ yields

$$\sum_{i=0}^{N} \Big(\sum_{j=0}^{N} \frac{1}{N} \alpha_j < \boldsymbol{u}_i, \boldsymbol{u}_j > a_j \Big) \boldsymbol{u}_i = \sum_{i=0}^{N} \lambda a_i \boldsymbol{u}_i \tag{3.11}$$

Since the "strobes" are assumed to be linearly independent we have

$$\sum_{j=0}^{N} \frac{1}{N} \alpha_j < \boldsymbol{u}_i, \boldsymbol{u}_j > a_j = \lambda a_i \qquad i = 0, ..., N \tag{3.12}$$

or, in vector notation

$$\mathsf{K}\boldsymbol{a} = \lambda \boldsymbol{a} \tag{3.13}$$

where

$$[\mathsf{K}]_{ij} = \frac{1}{N} \alpha_j < \boldsymbol{u}_i, \boldsymbol{u}_j > \qquad \boldsymbol{a} = [a_0, .., a_N]^{\mathsf{T}}$$

The matrix $\mathbf{K}$ is symmetric and nonnegative. The eigenvalues are real and positive; the eigenvectors can be chosen to form the orthonormal basis in R^{N+1}. Components of the eigenvectors are the coefficients of linear combinations of $\boldsymbol{u}_j$, $j = 0,..,N$ which determine the set of $N+1$ eigenstructures $\boldsymbol{\phi}_j$

$$\boldsymbol{\phi}_0 = \sum_{i=0}^{N}(a_0)_i \boldsymbol{u}_i \qquad ... \qquad \boldsymbol{\phi}_N = \sum_{i=0}^{N}(a_N)_i \boldsymbol{u}_i \tag{3.14}$$

The eigenstructures exhibit two important properties. First, $\mathrm{div}\boldsymbol{\phi}_i = 0$ $i = 0,..,N$ since they are linear combinations of divergent-free instantaneous velocity fields $\boldsymbol{u}_i$. Secondly, they are orthogonal in $L^2(D)$. Indeed

$$\begin{aligned} <\boldsymbol{\phi}_m, \boldsymbol{\phi}_n> &= \sum_{j=0}^{N}\sum_{i=0}^{N}(a_m)_i(a_n)_j \int_D \boldsymbol{u}_i \boldsymbol{u}_j \, d\boldsymbol{x} = N\sum_{j=0}^{N}\sum_{i=0}^{N}(a_m)_i[\mathbf{K}]_{ij}(a_n) = \\ &= N\boldsymbol{a}_m^{\top}\mathbf{K}\boldsymbol{a}_n = N\lambda_m \boldsymbol{a}_m^{\top}\boldsymbol{a}_n = N\lambda_m\delta_{mn} \end{aligned} \tag{3.15}$$

We have used the orthogonality of the eigenvectors of $\mathbf{K}$. The set of the eigenstructures can be orthonormalized by taking the eigenvectors $\boldsymbol{a}_i$, $i = 0,..,N$ such that $\|\boldsymbol{a}_i\| = 1/(N\lambda_i)$.

It is also interesting to ask how the projections $<\boldsymbol{U}, \boldsymbol{\phi}_n>$, $i = 1,..,N$ evolve in time. If we consider the set of instants when the velocity is sampled then

$$<\boldsymbol{u}_j, \boldsymbol{\phi}_n> = N\lambda_n(a_n)_j \qquad j = 1,..,N$$

This means that the discrete temporal evolution of $<\boldsymbol{U}, \boldsymbol{\phi}_n>$ on the eigenstructure $\boldsymbol{\phi}_n$ is just the rescaled eigenvector $\boldsymbol{a}_n$.

4. Application of the POD to random flow analysis

The procedure described in the previous paragraphs has been applied to extract "coherent" structures out of the random velocity field of the two dimensional flow past a cylindrical contour. This field has beenobtained by the stochastic vortex method. Details of the numerical algorithm are presented by Styczek and Wald [9]. The flows at two different Reynolds numbers have been considered: (a) Re = 100, (b) Re = 100000.

The eigenvalue problem for the correlation matrix is solved by the Jacobi iterations. The spectra put into decreasing order are shown in Fig.1.

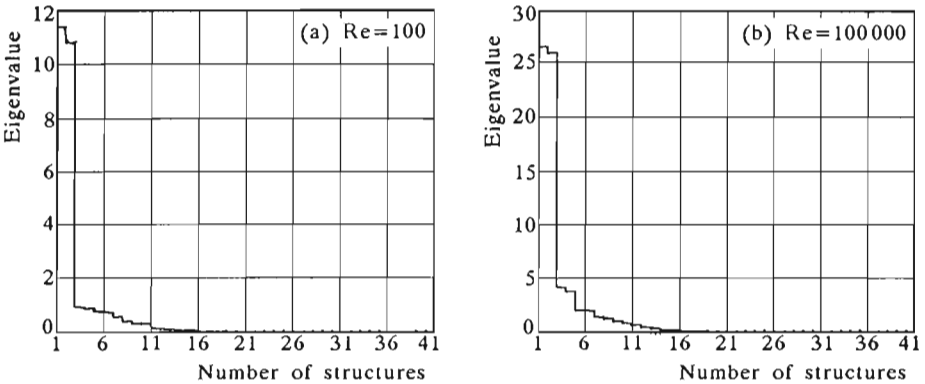

Fig. 1.

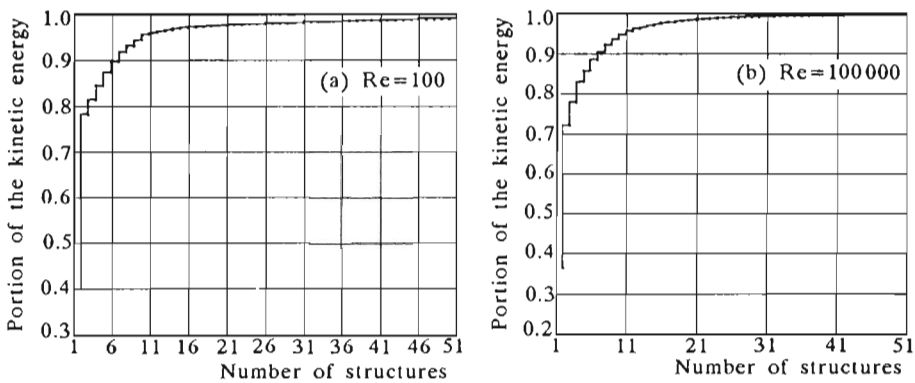

Fig. 2.

If we calculate the fraction of the kinetic energy contained in first M most significant eigenstructures

$$e(M) = \frac{\sum_{j=1}^{M} \lambda_j}{\sum_{j=1}^{N} \lambda_j} \qquad M = 1, .., N$$

then we obtain the distributions presented in Fig.2.

The POD analysis is carried out on the flow history sample, covering large number of time steps and begining far enough from the initial transient stage of simulation. Then the average has been calculated and subtracted from each intantaneous field in the recorded flow. In both cases the dimension N of the

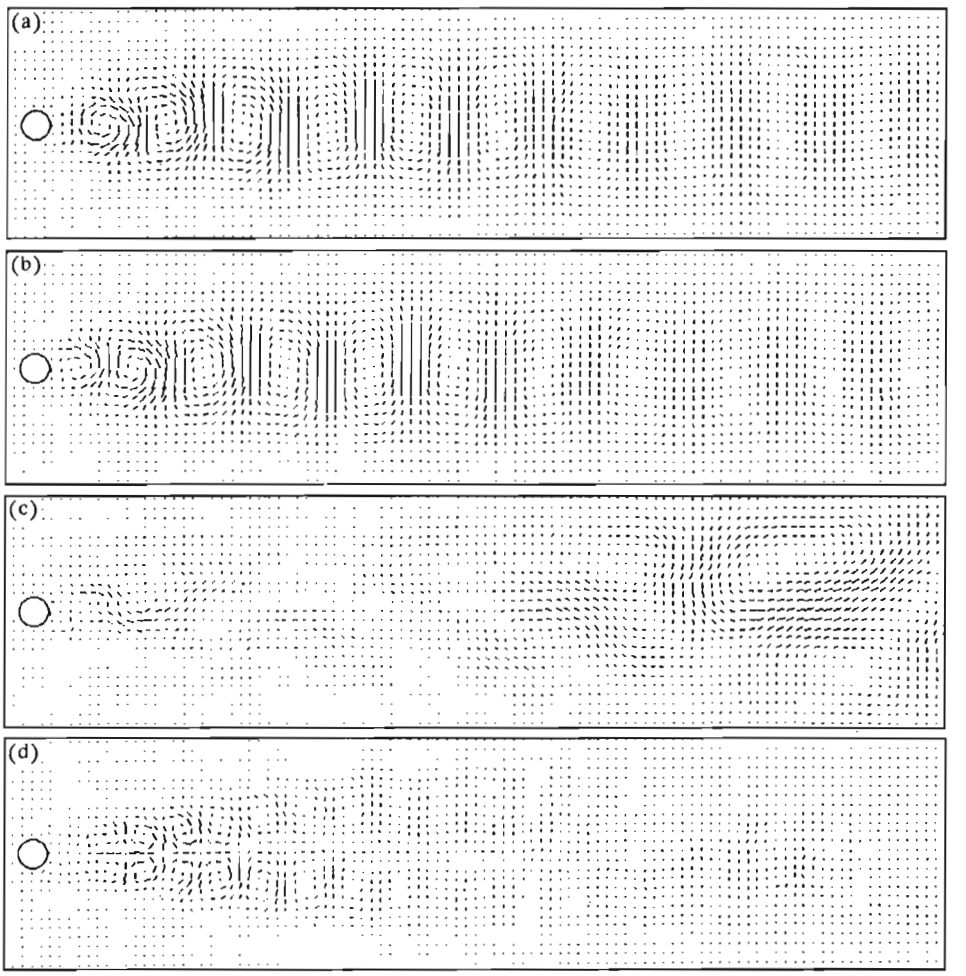

Fig. 3. The velocity field of selected eigenstructures for Re=100; (a) – no.1, (b) – no.2, (c) – no.6, (d) – no.10

eigenvalue problem is 200. The eigenstructures are numbered in decreasing order with respect to the magnitudes of corresponding eigenvalues. We show the results in the following order:

- Re = 100

  In Fig.3 the velocity fields of the structures 1, 2, 6 and 10 are presented. First two of them are the most energetic and the rest are shown to give an idea how other structures can look like. Fig.4 shows the temporal evolution of the instantaneous field projections on these four structures. As

it was mentioned above we see in fact the components of corresponding eigenvectors of the matrix **K**.

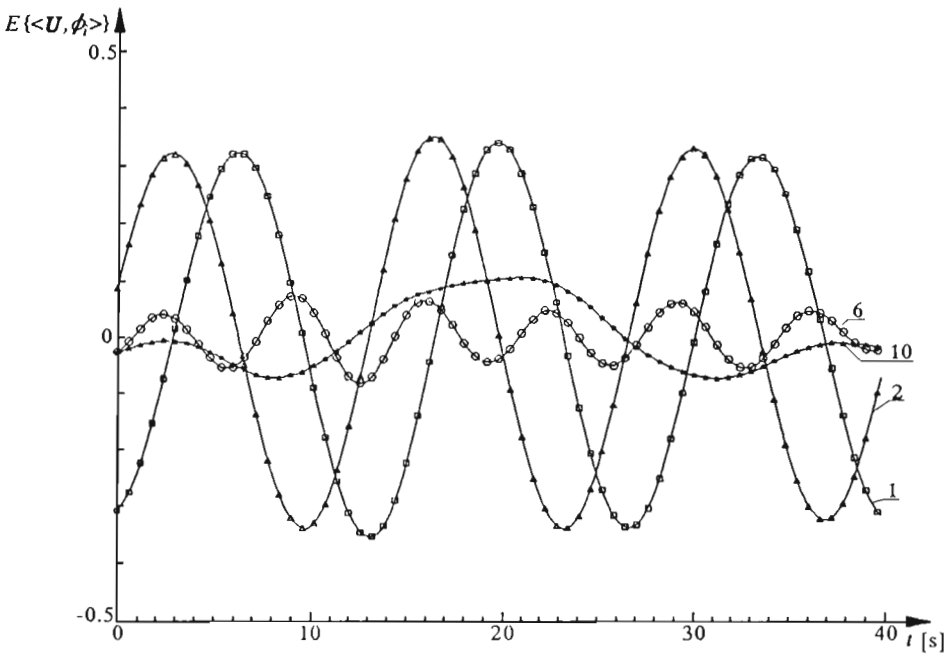

Fig. 4. Evolution in time of the projections of an instantaneous velocity field on the selected eigenstructures for Re=100

- Re = 100000

  In Fig.5 the velocity fields of the structures 1, 2, 4 and 12 are presented. The reason for this choice is the same as for Fig.3. Also Fig.6 contains the information analogous to that gven in Fig.4.

5. Conclusions

The most vivid feature of the POD results is that the spectrum of correlation matrix is more concentrated near most significant structures in case of lower Reynolds number. This behavior is consistent with the expectations. From the other side even at Re = 100 000 the number of structures containing most of the kinetic energy (say 99%) is rather low (say twenty). This suggests

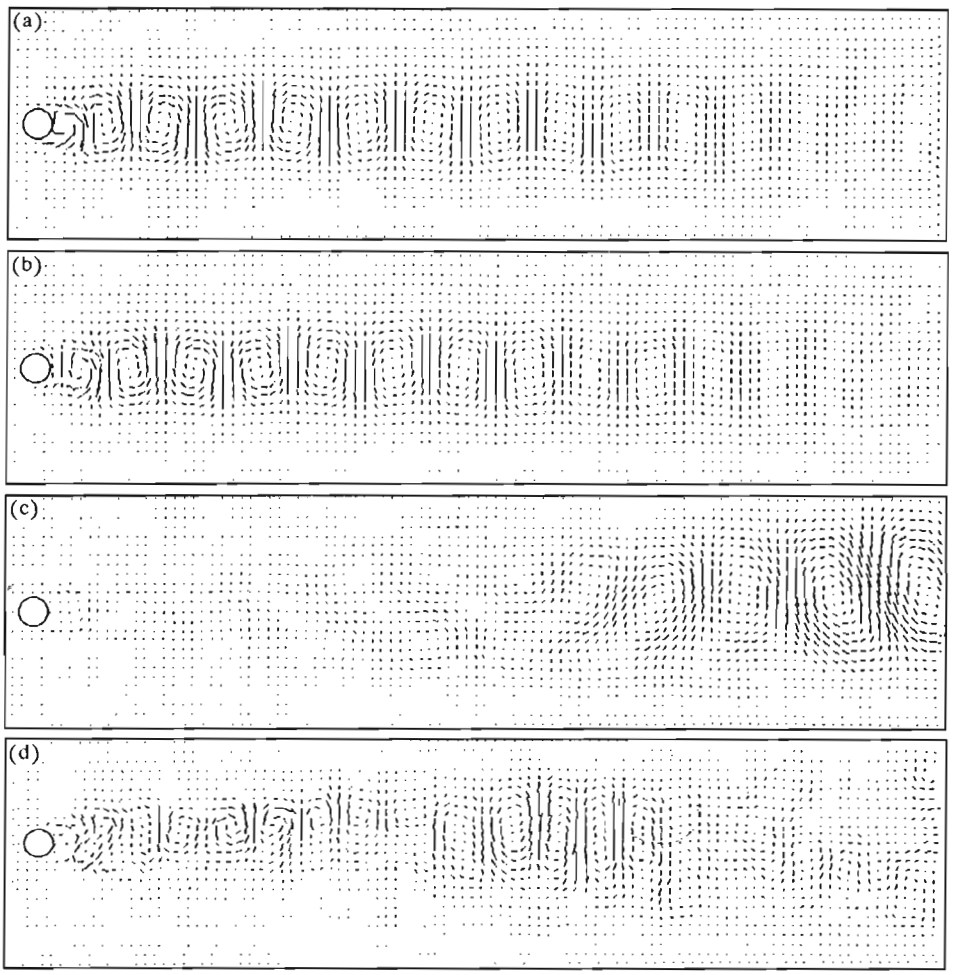

Fig. 5. The velocity field of selected eigenstructures for $\mathrm{Re} = 10^5$; (a) – no.1, (b) – no.2, (c) – no.4, (d) – no.12

the possibility of approximation of this flow by a relatively low dimensional set of ordinary differential equations. The selected structures exhibit three typical patterns: regular path of large vortices rotating in alternating directions, q pair of parallel vortex paths and spatially irregular system of weak vortex structures. The evolution of the flow projections onto selected structures reveal the oscillatory nature of the flow. An approximate value of the basic frequency can be easily established on the basis of two most energetic structures – it corresponds to the vortex shedding frequency. The structures,

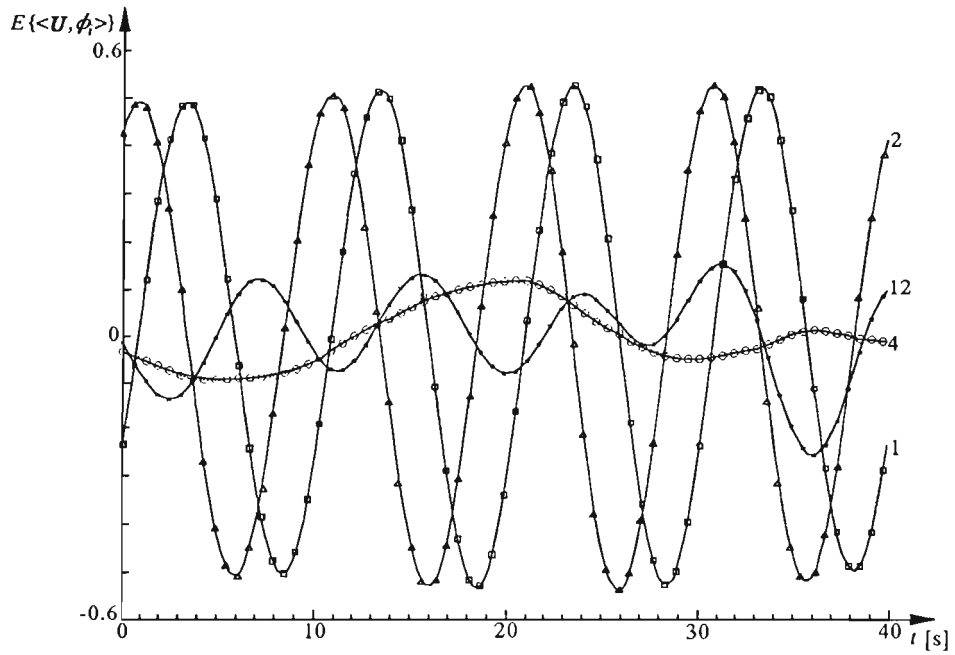

Fig. 6. Evolution in time of the projections of an instantaneous velocity field on the selected eigenstructures for $\mathrm{Re} = 10^5$

contribution of which to mean kinetic energy is lower present more irregular time behavior. In order to draw more information the substantial increase of the number of time steps considered is necessary.

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Analiza stochastycznego pola przepływu metodą dekompozycji ortogonalnej

Streszczenie

W artykule opisano metodę Karhunena-Loeve rozkladu pola losowego zastosowaną do analizy struktury pola prędkości niestacjonarnego oplywu profilu kolowego. Pole prędkości otrzymano w wyniku symulacji numerycznej metodą liniową. Przyjęto założenie ergodyczności przeplywu, co pozwolilo analizować jedną, dostatecznie długą realizację pola prędkości. Struktury pola wyznaczono numerycznie przy użyciu standardowych procedur dla algebraicznego zagadnienia wlasnego.

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