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APPLICATION OF A NEWMAN MODEL TO A FATIGUE LIFE PREDICTION OF FINITE-WIDTH STEEL SPECIMEN WITH A SINGLE EDGE CRACK

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In the paper an application of a Newman model to fatigue life predictions of steel specimens with single edge crack is presented. At the begining a short description of the Newman model is given. Then problems which have to be solved are pointed. Some initial modifications were introduced to the Newman model. Calibration of the model was performed using experimental data. To make a preliminarily verification of the model some predictions were done. On this stage the model was found as adequate for realization of this work aims.

1. Newman model

Fracture mechanic offers a large number of models for predicting fatigue crack growth. Most of them are of Paris type, so they are phenomenological in character. A large quantity of adequate data are available too. Investigations into a fatigue crack growth resulted in learning phenomena connected with the process e.g. crack-closure. Several analytical models accounting for such phenomena were developed. One of them is a Newman model.

Originally, the Newman model was developed for a central crack in a finite-width plate subjected to uniform applied stress, Fig.1a. The model was based on the Dugdale model, but was modified to leave plastically deformed material in the wake of the advancing crack, Fig.1c. A plastic zone size and crack surface displacements were calculated on the basis of superposition of two elastic problems

– a crack in a finite-width plate subjected to remote uniform stress, Fig.1a,

– a partially loaded crack, Fig.1b.

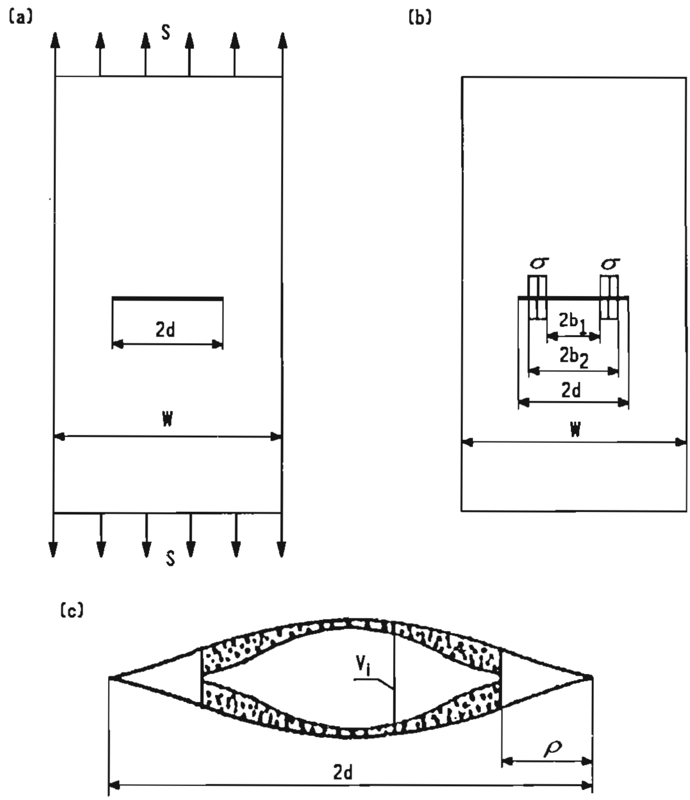

Fig. 1.

The model was composed of three regions (Fig.2)

– a linear elastic region containing a fictitious crack of half length $d = a + \rho$,

– a plastic region of length ρ,

– a residual deformation region of the width a.

Regions a and ρ were composed of rigid – perfectly plastic bar elements with the flow stress equal to an average value of the yield stress R_e and the ultimate tensile strength R_m. At any applied stress level, the bar elements are either intact like in the plastic region, or broken like in the residual plastic deformation region. The broken elements carry compresive loads, and then only

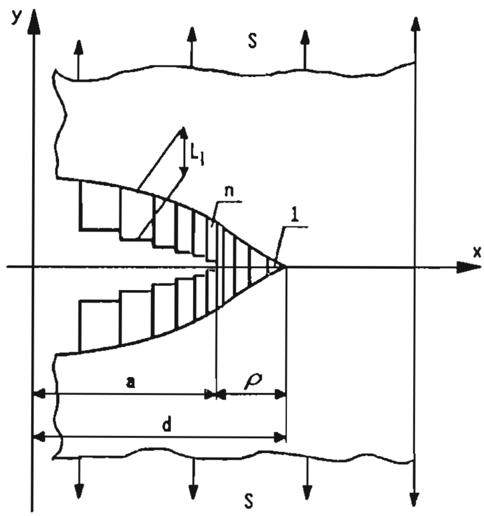

Fig. 2.

if they are in contact. In such situations they can (it depends on the applied stress level) yield in compression. Then their lengths change. Elements that are not in contact do not affect the calculation of crack surface displacements. To account for the effect of a state-of-stress a constraint factor was used (equal to 1 for a plane stress and 3 for plane strain conditions, respectively).

A breakdown of the model components and the coordinate system are shown in Fig.3. Because of the symmetry, only one quater of the plate was analyzed. The plate had the fictitious crack of a width d and was subjected to the uniform stress S. The bar element connected to point j was subjected to a compressive stress σ_j. This element is in contact when its length L_j is larger then the current crack surface displacement V_j. The displacement at any point was calculated from equation (1.1) as

$$V_i = Sf(x_i) - \sum_{j=1}^{n} \sigma_j g(x_i, x_j) \tag{1.1}$$

To solve the equation, functions describing crack surface displacements $f(x_i)$ and $g(x_i, x_j)$ are to be known. Function $f(x_i)$ describes the displacements for configuration presented in Fig.1a, and is given in form

$$f(x_i) = \frac{2(1-\eta^2)}{E}\sqrt{d^2 - x_i^2}F \tag{1.2}$$

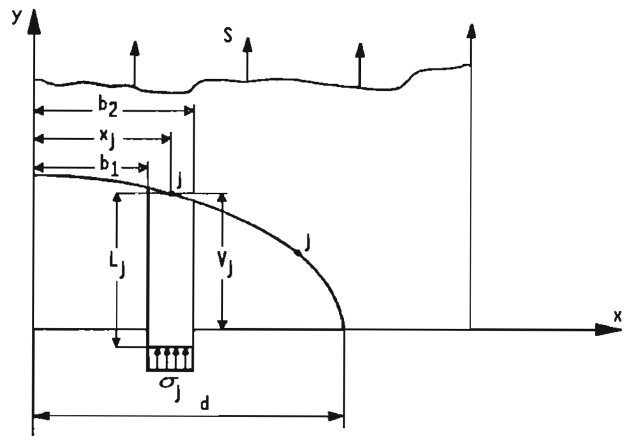

Fig. 3.

Function $g(x_i, x_j)$ characterizes the displacements for configuration shown in Fig.1b and has the following form

$$g(x_i, x_j) = \frac{2(1-\eta^2)}{\pi E}\left[(b - x_i)\cos^{-1}\frac{d^2 - bx_i}{d|b - x_i|} + \sqrt{d^2 - x_i^2}\sin^{-1}\frac{b}{d}\right]_{b=b_1}^{b=b_2} F \quad (1.3)$$

The equations were obtained from the Westergaard stress functions. In equations given above E is the Young modulus of elasticity, F is a correction function, η is a material constant (0 for a plane stress and Poisson ratio for plain strain conditions). For other symbols see Fig.3.

The compatibility equation for element i ($L_i = V_i$) is expressed as

$$\sum_{j=1}^{n} \sigma_j g(x_i, x_j) = Sf(x_i) - L_i \quad (1.4)$$

Solving it in consideration of contact stresses σ_j with S equal to $S_{\min}$, a stress distribution over the crack was obtained. Then crack opening stress S_0 was calculated.

In the presented model, a modified by Newman Elber's crack propagation equation was used

$$\frac{da}{dN} = C_1 \Delta K_{\text{eff}}^{C_2} \frac{1 - \left(\frac{\Delta K_0}{\Delta K_{\text{eff}}}\right)^2}{1 - \left(\frac{K_{\max}}{C_5}\right)^2} \quad (1.5)$$

where:

— threshold stress intensity factor range

$$\Delta K_0 = C_3\Big(1 - C_4\frac{S_0}{S_{\max}}\Big) \tag{1.6}$$

— effective stress intensity factor range

$$\Delta K_{\text{eff}} = \Delta S_{\text{eff}}\sqrt{\pi a}F \tag{1.7}$$

ΔS_{eff} – effective stress range

— maximum stress intensity factor

$$K_{\max} = S_{\max}\sqrt{\pi a}F \tag{1.8}$$

— material crack growth constants $C_1 \div C_5$.

Equation (1.5) gives characteristic sigmoidal shape of da/dN vs ΔK_{eff} curve.

The model provides no information about the amount of crack growth per cycle. The crack growth is simulated by extending the crack over an incremental value Δa^* at the moment of maximum stress applied. The amount of crack extension was an arbitrarily defined value. In the original model it was equal to five percent of maximum plastic zone size.

Number of cycles required to grow the crack the incremental value was calculated from Eq (1.5) and a cyclic load history. During that time, the cyclic load history was monitored to find the lowest applied stress before $S_{\min B}$ and after $S_{\min A}$ the highest applied stress level $S_{\max H}$, Fig.4.

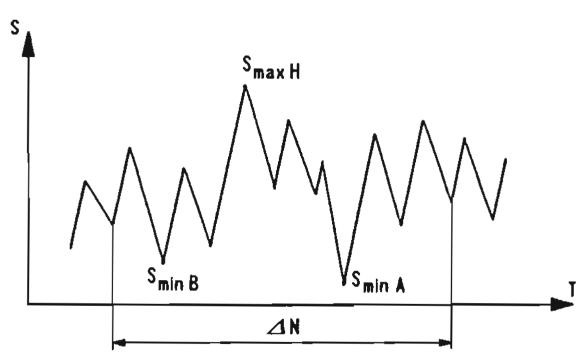

Fig. 4.

The procedure of application of the Newman model consists of

– applying minimum stress $S_{\min B}$ at a crack length a,

- applying maximum stress $S_{\max H}$ at a crack length a,
- extending crack by an incremental value Δa^*,
- applying minimum stress $S_{\min A}$ at the new crack length,
- calculating the new crack opening stress S_0,
- continuing the cyclic load history,
- repeating process when the crack extension reaches a new incremental value or number of cycles reaches 300 cycles.

2. Newman model modification

In the following a modification of the Newman model for a finite-width plate with a single edge crack (Fig.5) is presented.

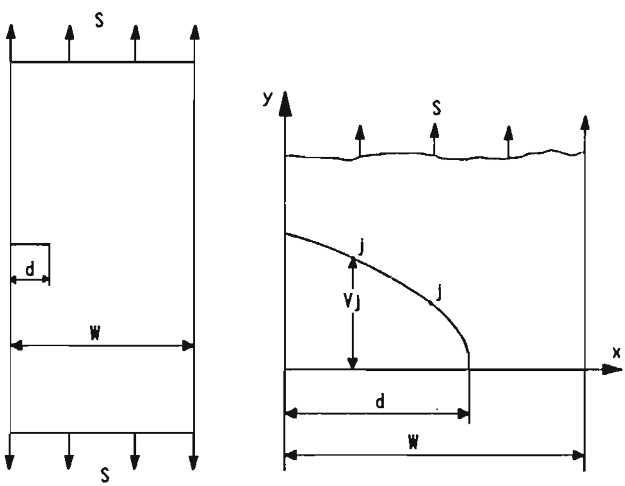

Fig. 5.

The model requires following modifications:

- Because of a configuration (geometry and load) in a crack section besides compression-tension also bending exists. Geometry function F changes its form to

$$F = F_t + F_b \frac{S_b}{S_t} \tag{2.1}$$

where

F_t, S_t – geometry function and stresses, respectively, for compression-tension,

F_b, S_b – geometry function and stresses, respectively, for bending.

The S_b/S_t ratio is obtained from experiments. F_t and F_b are attainable in adequate handbooks.

- Originally the Newman model was developed for aluminium alloys. To modificate it for a steel specimen it is necessary to change the formula for plastic zone size calculation.

- Because of the new configuration, formulas for stress intesity factors were changed.

- Crack surface displacements can't be now obtained from the Westergaard stress functions. On the present stage it is proposed to apply a Petroski-Achenbach equation. They assumed that crack surface displacements at any point of the crack approximately can be calculated from equation

$$V(x,d) = \frac{(1-\eta^2)\sigma}{E\sqrt{2}}\Big[4F\sqrt{d}\sqrt{d-x} + \frac{G(d)}{\sqrt{d}}\sqrt{(d-x)^3}\Big] \tag{2.2}$$

- Constants $C_1 \div C_5$ are calculated according to the procedure proposed by Newman.

Literature offers a small number of test results for such a type of steel specimen subjected to cyclic compression-tension loading. They make impossible any verification of the correspondingly modified Newman model. The adequate tests are being performed now. Since experimental works are in progress, only the preliminary verification is possible.

The presented above modified model was realized in the form of a computer code. Predictions of specimens' lifes were too optimistic. It has become clear that further tests are indispensable. Also results obtained from FEM analyses are required. On the present stage it can be said that probably the Newman model can be applied to prediction of a fatigue crack growth in the case of the finite-width plate with the single edge crack subjected to a cyclic compression-tension loading. But further modifications seem to be necessary.

An attractiveness of the model from an engineering point of view is based on its following features: simulations can be performed at any load conditions, overload and underload effects are taken into account, crack-closure phenomenon is considered. Moreover, clarity of the algorithm of computation and possibility of making use of it on PC computers raises its advantages.

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Zastosowanie modelu Newmana do przewidywania trwałości próbek stalowych z pojedyńczą szczeliną brzegową

Streszczenie

W pracy przedstawiono zastosowanie modelu Newmana do przewidywania trwałości próbek stalowych z pojedyńczą szczeliną brzegową. W pierwszej części pracy podano krótki opis modelu Newmana. Wskazano także na problemy, które należy rozwiązać. Do modelu Newmana wprowadzono kilka modyfikacji. Wykorzystując dane doświadczalne przeprowadzono kalibrację modelu. Na obecnym etapie uznano, że model jest odpowiedni do realizacji dalszych celów pracy.

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