MECHANIKA TEORETYCZNA
I STOSOWANA
Journal of Theoretical
and Applied Mechanics
1, 32, 1994

STUDY OF FRACTURE TOUGHNESS AND FRACTURE ENERGY IN COMPOSITES

Mieczysław Jaroniek

Department of Strength of Materials

Technical University of Łódź

The aim of this work is to give a numerical formula for the ultimate strength under compression in terms of an energy approach ($G_c = \partial U/\partial A$) characterizing energy dissipation during the fracture process. The experimental and numerical models have been elaborated to investigate and to calculate a stress intensity approach (factor K) and an energy approach (crack resistance R, crack extension force G). We considered a two-phase system consisting of a matrix (phase 1) and cylindrical grains (phase 2) dispersed in it – according to the accepted composite (concrete) model.
A plane stress-state substitutes for the real three-dimensional stress-state. The 2D models were made of two components: mortar (based on the cement) and the cylinders of pyrex glass (grains).

1. Experimental and numerical models

Plane (two-dimensional) models of composite were made of two components: mortar (based on the cement) and cylinders of pyrex glass (grains) dispersed randomly in the matrix).

Specimens were subjected to the compression loadings. The loading experiments were performed in terms of the displacement control method applying an automatic measurement (MTS) system. Properties of the components of experimental model are given in Table 1 and Table 2.

Table 1. Properties of the experimental model components

No.	Component	Young modulus	Poisson ratio	Ultimate strength	
				compression	tension
		E [MPa]	ν	R_c [MPa]	R_m [MPa]
1	epoxy resin EP-51	$3.45 \cdot 10^3$	0.36	125	37
2	plexi	$4.21 \cdot 10^3$	0.35	130	42
4	mortar (matrix)	$2.67 \cdot 10^4$	0.171	55	8
5	grains (pyrex)	$8.25 \cdot 10^4$	0.213	1100	65

Table 2

Component	Photoelastic constants in ters of	
	strain f_ε	stresses k_σ [MPa/is.ord.]
epoxy resin EP-51	$66.2 \cdot 10^{-5}$	1.68
plexiglas	$23.6 \cdot 10^{-4}$	7.36
grains-pyrex	$37.6 \cdot 10^{-6}$	2.56

2. Strain energy release rate

Basing on the experimental and numerical results being obtained one can determine displacement of the compression forces as well as the crack and fracture zone propagation due to them. The stress intensity factors at the crack tip can be written as

$$K_I = \lim_{s \to 0} \sigma_x \sqrt{2\pi(x-s)} \qquad K_{II} = \lim_{s \to 0} \tau_{xy} \sqrt{2\pi(x-s)} \tag{2.1}$$

The ratio of the crack extension force or the strain energy release to the crack surface area can be calculated as (Farris and Keer, 1985)

$$G = \frac{\partial U}{\partial A} = \frac{\pi}{4\beta}(K_I^2 + K_{II}^2) \qquad \beta = \mu_1 \frac{\lambda_i - 1 - \Gamma(\chi_2 - 1)}{(\chi + \Gamma)(1 + \chi\Gamma)} \qquad \Gamma = \frac{\mu_1}{\mu_2} \tag{2.2}$$

where

$$\chi_i = \begin{cases} 3 - 4\nu_i & \text{– for a plane strain} \\ \dfrac{3-\nu_i}{1+\nu_i} & \text{– for a plane stress} \end{cases}$$
$$\mu_i = \frac{E_i}{2(1+\nu_i)} \tag{2.3}$$

and approximately

$$\phi = \frac{K_I}{K_{II}} = \frac{\sigma_x(s)}{\tau_{xy}(s)}$$
$$G = \frac{\pi(1+\phi^2)}{4\beta} K_{II}^2 \tag{2.4}$$

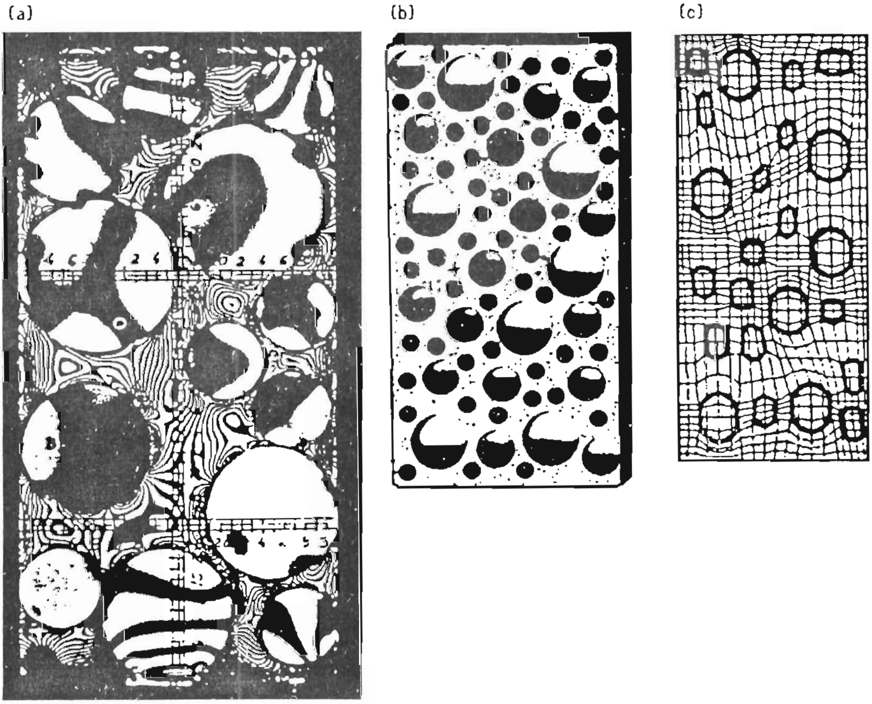

Fig. 1. Two-dimensional model (plane model) of composite; (a) the photoelastic model of concrete and isochromatic fringes obtained from experiment, (b) the model of concrete, (c) the finite element mesh

The critical value of strain energy release rate G_c has been calculated in terms of experimentally obtained values of the work and the crack surface area.

Knowing the displacements of compression forces together with the crack propagation way due to them one can determine the stress factor K and the crack resistance R, respectively, from the following formula

$$K = \sqrt{E''G_c} \qquad\qquad G_c = R = \frac{\Delta U}{\Delta A_i} = \frac{P_i \Delta v_i}{aB} \tag{2.5}$$

where

G_c – crack propagation energy
R – crack resistance
ΔU – dissipated energy
$\Delta A_i = aB$ – fractured area along the interphase
P_i – force corresponding to the crack propagation
a – crack length, $a = \sum\limits_i \sum\limits_j a_{ij}$
Δv_i – displacement corresponding to the cracks length a
E'' – Young modulus, corresponding to P_i.

3. Experimental tests of the models

3.1. Photoelastic model of concrete

The optical properties of plexiglass and epoxy resins allow us to determine stresses in the grains and in the matrix using the photoelastic method in which isochromatic patterns represent the stress distribution. Since an analytical solution to this problem is not available, the distribution of stresses and displacements has been calculated in terms of the finite element method. The crack initiates within the interface region between the components of the specimen. Relation between the critical value of strain energy release rate G_c and the ultimate strength R_c of concrete (obtained experimentally) has been formulated. The concentration of stresses along the interphase boundary and the direction of the crack propagation has been put forth. After examination of the fractures in specimens failed under compression it occured thatthe directions of crack propagation calculated numerically overlapped the expeimental ones.

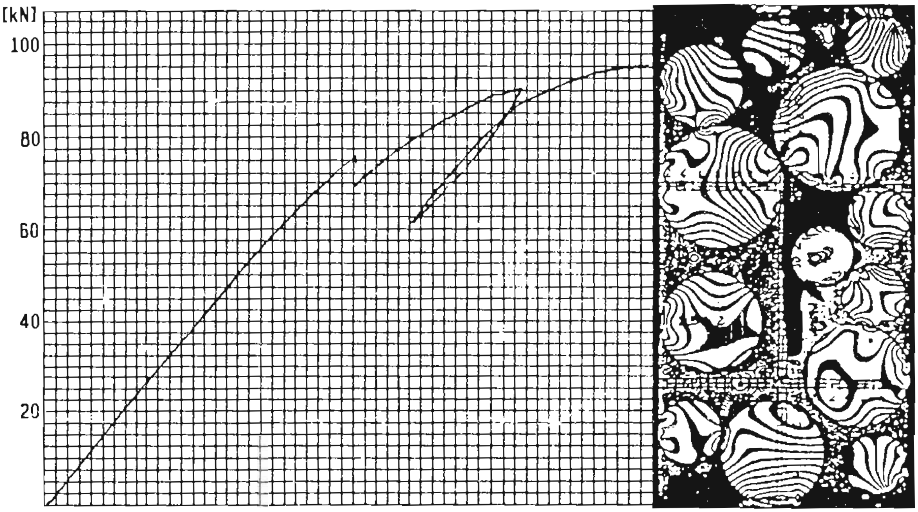

Fig. 2. Force-displacement diagram for the photoelastic model subject to compression and isochromatic fringes corresponding to cracking

3.2. Experimental model of concrete

Optical properties of pyrex glass allow us to determine the stresses in the grains using the photoelastic method. The stress distributions overthe grain were represented by isochromatic patterns. Some numerical and experimental results of investigations into the model of concrete under compression are given in Fig.3. A series of tests was carried out to examine the failure mechanism in the model of concrete and to find out the relation between the work of compression forces and the crack surface area.

4. Numerical analysis of stress and strain distribution

The numerical calculations were done on an IBM-AT computer with the aid of the finite element program (Szmelter, 1973; Zienkiewicz, 1971), applying the substructure technique as well as rectangular isoparametric elements. Finite element calculations were done in order to predict the branching phenomenon observed in experiments and find the isochromatic distribution representing the interface crack propagation along debonded parts of the circular grains.

The geometry and material parameters were chosen in the way showing

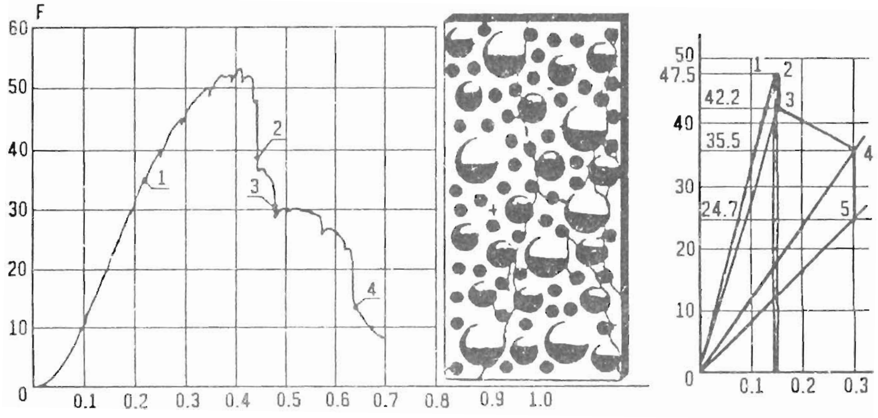

Fig. 3. Force-displacement diagram for the plane model of concrete obtained experimentally and numerically using FEM (points corresponding to the crack propagation process are given in figure)

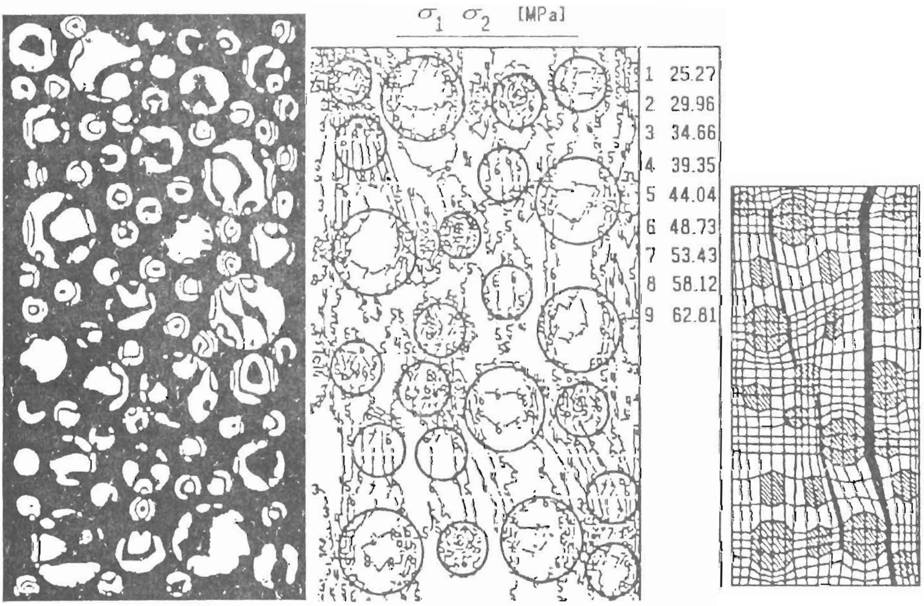

Fig. 4. Experimental and numerical results of the model subjected to compression loading. Isochromatic fringes obtained from experiments and numerical calculations and finite element mesh representin the cracking

good correlation with the specimens used in the experiments. A finite element mesh of the model (used for numerical simulation), the stress distribution and the isochromatic fringes are shown in Fig.4 (for comparison the isochromatics obtained experimentally in the grains of pyrex glass are given in Fig.4). The experimental and numerical results are given in Table 3.

Table 3. Numerical results

	Force F [kN]	Displacement v [mm]	Crack extension energy G_{c1}/G_{c2} [kN/m]	Crack extension energy A [mm^2]	Stress intens. factor approach K [MN/m$^{3/2}$]
1	47.46	0.1448			
2	47.46	0.150	0.221	952.95	2.815
3	42.17	0.15	0.194	4100.5	2.63
4	35.13	0.30			
5	24.71	0.30	0.5725	5460.0	2.48

where:

A – fractured area

$1 \div 5$ – number point in curve force-displacement.

5. Conclusions

Using linear fracture mechanics and numerical results it is possible to determine the energy approach ($G_c = \partial U/\partial A$) charactrerizing the energy dissipation during the fracture process and the stress intensity approach (factor K). The ultimate strength under compression corresponds to the longitudinal propagation of the cracks due to normal and shear stresses in the interface between the mortar and the grains. Mixed mode of fracture is affected by the normal (perpendicular) and tangential (shear) stresses (characterized by K_I and K_{II}), respectively. In the case of brittle composites the ultimate strength under compression should be considered as the stress value affecting the longitudinal fracturein the direction overlapping the compression loading one. The compression stress influences the critical values of crack resistance R and stress intensity approach K.

Acknowledgements

This resarch was supported by Laboratoire Central des Ponts et Chaussees, Paris, France. The autor is most grateful to Prof. Francois de Larrard and Prof. Jean Michel Torrenti for many valuable comments on the content of this paper.

References

1. Brandt A.M., Kasperkiewicz J., 1979, *Crack propagation energy in SFRC,* Proc. of Delft., 1979 Symposium, Stevin Lab. Delft 5, 1-31
2. Deukman A.N., 1987, *Weight function theory for a rectilinear anisotropic body,* Int.J.of Fract., 34, 85-109
3. Erdogan F., 1963, *Stress distribution in a nonhomogeneous elastic plane with cracks,* J.Appl.Mech., E30, 232-236
4. Farris T.N., Keer L.M., 1985, *Williams' blister test analyzed as an interface crack problem,* Int.J.of Fract., 27, 91-103
5. Rice J.R., Sih G.C., 1965, *Plane problem of cracks in dissimilar media,* J.Appl.Mech., E32, 418-423
6. Szmelter J., 1973, *The Finite Element Programs,* Arkady, Warsaw
7. Zienkiewicz O.C., 1971, *The Finite Element Method in Engineering Science,* Mc Graw-Hill, London

Analiza mechanizmów pękania i zniszczenia ściskanych elementów kompozytowych

Streszczenie

Przedmiotem pracy jest zagadnienie propagacji rys ściskanych elementów kompozytowych przy obciążeniu zbliżonym do niszczącego w ujęciu mechaniki zniszczenia. Na podstawie badań modelowych oraz obliczeń numerycznych określono stan naprężenia i energię zniszczenia odpowiadającą powstawaniu mikrorys i ich rozwoju aż do zniszczenia elementu. Określono energię zniszczenia potraktowaną jako wytrzmalość na zarysowanie w przypadku kruchego pękania. Ze względu na skomplikowaną strukturę materialów kompzytowych badania przeprowadzono dla "dwufazowych" i dwuwymiarowych modeli betonu potraktowanego jako material kompozytowy.

Manuscript received October 1, 1993; accepted for print October 14, 1993