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METHOD OF STRESS INTENSITY FACTOR CALCULATION BASED ON THE UNITARY WEIGHT FUNCTION

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A method of stress intensity factor evaluation is presented. Calculating procedure is based on the well known weight function or influence function concept. After significant modifications, a unit an weight function is proposed, which may be easily transformed to obtain a matrix data base adequate for computer use and subsequent storage in its memory. This data may be then applied to different loading functions, including residual stresses.
By separating the geometry of the body from the load applied, one geometric description covers many particular solutions for different loading functions.
Simplified integrating procedure is also described, which makes the computing time of K values very short.

1. Introduction

The stress intensity factor K is one of the most important parmeters, widely used in practice, to estimate failure conditions, fatigue crack growth rate, corrosion cracking etc. Many methods of stress intensity factor determination have been developed and published (cf Rice, 1972; Bueckner, 1973; Labbens et al., 1976) in last few decades. Several standard numerical techniques provide us with solutions to more complicated geometrical and loading problems.

A difficulty in the application of particular solutions found in the literature is that all results are usually expressed in many different ways: graphs, tables, polynomial approximations etc. These are often useless in computer techniques of design or need additional effort for data preparation.

2. Weight function

2.1. Conventional weight function method

Many existing methods of crack analysis (cf Rice, 1972; Bueckner, 1973; Labbens et al., 1976; Albrecht and Yamada, 1977) are based on continuous load symmetrically applied to both crack sides. From the mathematical point of view, if a crack length is a function of one coordinate only, the stress intensity factor value is an integral of a product of two functions: loading function $\sigma(x)$ and weight function $m(x)$, as follows

$$K = \int_0^a \sigma(x) m(x,r)\, dx \tag{2.1}$$

where
a – crack length
r – geometrical parameter
x – coordinate along the crack path.

Eq (2.1) is usually solved by numerical techniques, where special care is given to the singularity at the crack tip. This makes the calculations more difficult and longer.

2.2. Unitary weight function

The main idea of the present approach is to normalize the $m(x)$ function with respect to the crack length a and divide it by a correction function $F(r)$ for uniform stress. For example, assuming a single edge crack in an infinite strip of width b, Eq (2.1), after rearrangement, takes the following form

$$K = \sqrt{\pi a}\Big[\int_0^1 \sigma(t) w(t,r)\, dt\Big] F(r) \tag{2.2}$$

where $t = x/a$, $r = a/b$ and $F(r)$ is the finite width correction function for a uniform load.

Normalized in such a way, new $w(t,r)$ implicit function, which appears in the integral, is called "unitary weight function", and has an important feature useful in further analysis and expressed by

$$\Omega = \int_0^1 w(t,r)\, dt = 1 \tag{2.3}$$

Thus, the integral of Eq (2.2), which appears in square brackets, may be interpreted as a uniform load, called an equivalent stress σ_{eq}, which gives the same stress intensity factor value as a real one. Numerical integration of Eq (2.2) may be considerably simplified by using the procedure described below.

3. Procedure of integration

3.1. Linear loading function

The procedure of integration is based on a well known idea, frequently used in mechanics. It states that the definite integral of the product of two functions, where at least one of them is linear, is equal to the definite integral of the first (non-linear) function, multiplied by the value of the linear one at the centroid coordinate of the former area. Graphic interpretation of this rule is shown in Fig.1.

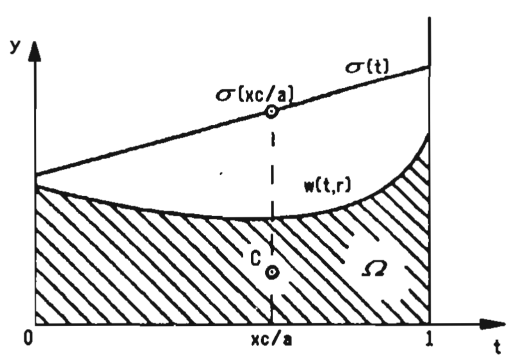

Fig. 1. Graphic interpretation of Eq (3.1)

In the present case of stress intensity factor calculation, the following equation should be applied

$$\int_0^1 \sigma(t) w(t,r)\, dt = \Omega\sigma\Big(\frac{xc}{a}\Big) = \sigma\Big(\frac{xc}{a}\Big) \tag{3.1}$$

where, according to Eq (2.3), $\Omega = 1$. Eq (3.1) holds only for a linear loading function. It means that if the centroid coordinate xc of the unitary weight function is known, then the equivalent stress σ_{eq} for the linear load is directly obtained. One example of this situation is shown in Fig.2, where a single edge crack in a semi-infinite plate is considered. For both cases given in Fig.2, the stress intensity factor can be obtained from the following equation

$$K = \sqrt{\pi a}\ \sigma(0.609a)1.1215 \tag{3.2}$$

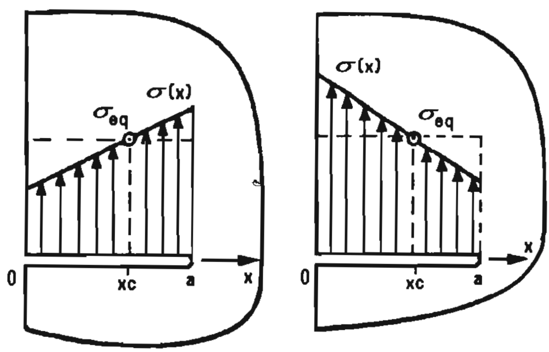

Fig. 2. Edge crack subjected to a linear load

3.2. Non-linear loading function

In general, the crack surface load is not a linear one and the problem becomes more difficult. Eq (3.1) may not be directly applied and some modifications are necessary. At this point, few possible solutions seem to be reasonable. The best one, in the author's opinion, is based on dividing the whole range of integration (crack length) into ten equal segments and calculating ten fragmentary integrals Ω_i of the unitary weight function, as well as ten centroid coordinates xc_i. The stress intensity factor is represented thus by a sum of ten products along the crack length as it is shown in Fig.3 and described by the following formula

$$K = \sqrt{\pi a}\Big[\sum_1^{10} \Omega_i(r)\sigma(xc_i)\Big]F(r) \tag{3.3}$$

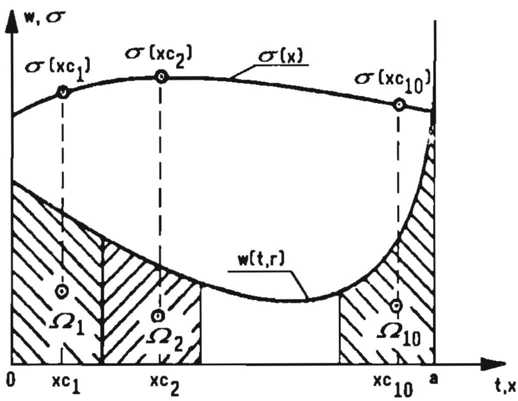

Fig. 3. Integration of two non-linear functions

Accuracy of this procedure is verified by comparing the real values of stress intensity factors for a case of a central, symmetrically loaded crack in an infinite plate, to the numerical results of Eq (3.3). Loading stress has the form

$$\sigma(t) = \sigma_0 t^n \tag{3.4}$$

where σ_0 is an arbitrary constant.

All results are listed in Table 1.

Table 1. Accuracy of the present method for an exponential load (Eq (3.4))

exponent n	1	2	3	4	5	6	7
error %	0	0.14	0.29	0.54	0.87	1.3	2.0

It is obvious that accuracy of the method depends on the shape of loading function, which may be interpreted now as ten straight-line segments. Maximum error (2%) occured when $n = 7$. Higher values of the exponent n were not investigated. For lower n values, the accuracy was much better.

Many tests made for different plane crack problems showed that fragmentary quantities Ω_i strongly depended on the ratio of crack length to finite width. This relation is shown in Fig.4 for a single edge crack in a finite width plate, Mode I and II, and in Fig.5 for two symmetrically loaded semi-infinite cracks in an infinite plate, Mode I, II and III. All fragmentary values with the same index number were then approximated with 0.5% accuracy by ten biquadratic polynomials, expressed by

$$\Omega_i(r) = A_{0i} + A_{1i}r + ... + A_{4i}r^4 \tag{3.5}$$

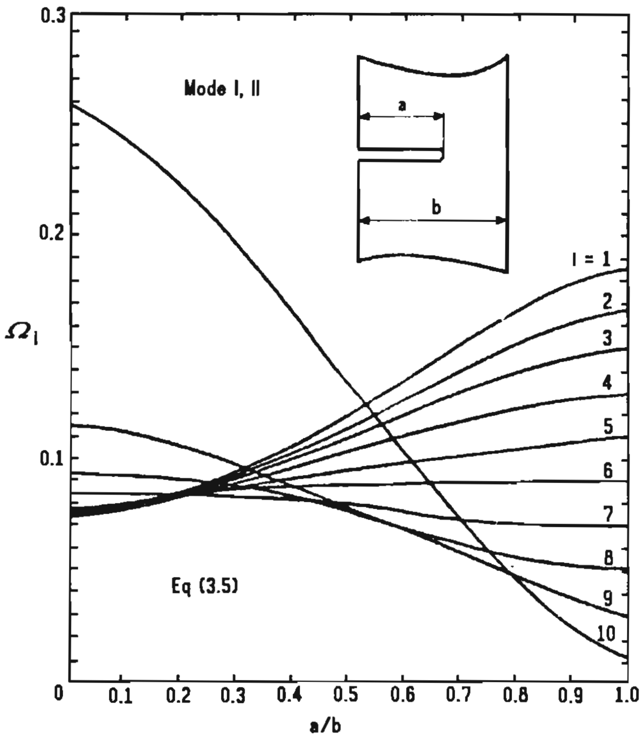

Fig. 4. Fragmentary values of the unitary weight function integrals vs a/b ratio, for a single edge crack in an infinite strip

Suprisingly, while Ω_i values changed strongly with the a/b ratio, the centroid coordinates xc_i remained almost fixed. The explanation of this phenomenon is as follows: although the unitary weight function changes, its form remains similar.

4. Data pattern

From the above analysis one may conclude that every plane crack problem can be represented in the form of a matrix, given in Table 2.

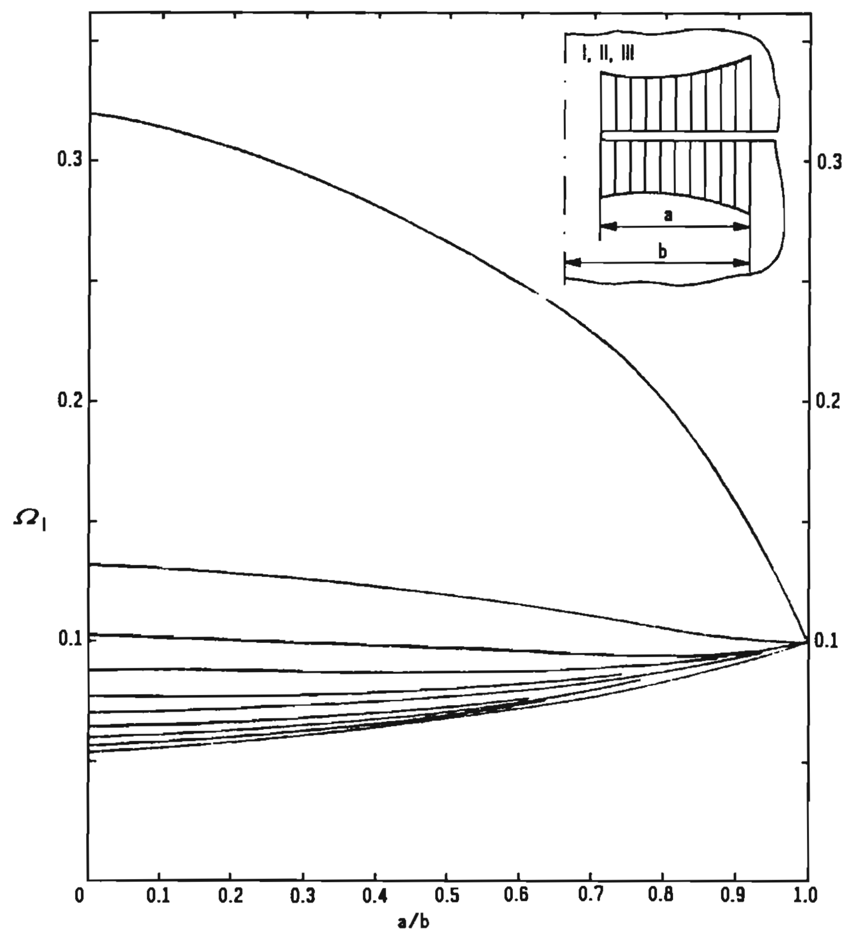

Fig. 5. Fragmentary values of the unitary weight function integrals vs a/b ratio, for two symmetrically loaded cracks in an infinite plate

Table 2. General pattern of data matrix for plane crack problems

Code	$A_{0,1}$	$A_{1,1}$	$A_{2,1}$	$A_{3,1}$	$A_{4,1}$	$(xc/a)_1$
Mode	$A_{0,2}$	$A_{1,2}$	$A_{2,2}$	$A_{3,2}$	$A_{4,2}$	$(xc/a)_2$
Type	$A_{0,3}$	$A_{1,3}$	$A_{2,3}$	$A_{3,3}$	$A_{4,3}$	$(xc/a)_3$
F_0	$A_{0,4}$	$A_{1,4}$	$A_{2,4}$	$A_{3,4}$	$A_{4,4}$	$(xc/a)_4$
F_1	$A_{0,5}$	$A_{1,5}$	$A_{2,5}$	$A_{3,5}$	$A_{4,5}$	$(xc/a)_5$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
F_6	$A_{0,10}$	$A_{1,10}$	$A_{2,10}$	$A_{3,10}$	$A_{4,10}$	$(xc/a)_{10}$

In the first column, starting from the top, there should appear: Code – case number, Mode – I-st, II-nd or III-d Mode of loading, Type – singularity type of the correction function. The remaining seven entries of the first column should contain polynomial coefficients of the F function, which is then expressed by

$$F(r) = \text{Type}\Big(F_0 + F_1 r + F_2 r^2 + ... + F_6 r^6\Big) \tag{4.1}$$

Next five columns are ocuppied by ten groups of coefficients of biquadratic polynomials, described by Eq (3.5), which represent fractional values of the unitary weight function. Thus, the sum of all elements of the second column of the matrix must equal one, but for the next four columns the result of adding them up should be zero.

The last column includes ten normalized centroid coordinates, which are considered to be constants.

5. Conclusions

Any form of weight function may be easily transformed into the unitary weight function and partially integrated by numerical procedures to obtain a matrix data base adequate for computer use and subsequent storage in its memory. This data may be then applied to different loading functions, which are considered as ten straight-line segments along the crack length. Thus, numerical calculations of stress intensity factors K need only ten steps, making the computing time very short.

By separating the geometry of the body from the load applied, one geometric description covers many particular solutions for different loading functions. For plane problems, only one width correction function F, assumed for constant load, is needed.

Accuracy of the method depends mainly on the loading function and no more than a two percentage error may be expected for engineering applications.

Approximate values of classic weight functions can be found using the data base described above. One example of such a transformation can be found in the paper by Shen and Glinka (1990).

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Automatyzacja obliczeń współczynnika K w oparciu o zmodyfikowaną funkcję wagową

Streszczenie

Przedstawiono metodę obliczania wartości współczynnika intensywności naprężenia K w oparciu o zmodyfikowaną funkcję wagową, nazwaną jednostkową funkcją wagową. Podano definicję, budowę i sposób zapisu jednostkowych funkcji wagowych w formie zunifikowanej, dogodnej do budowy komputerowych baz danych dla różnych geometrii elementu i konfiguracji szczelin. Omówiono procedurę obliczeniową współczynnika K oraz uproszczony sposób całkowania. Podano dokładność oraz zalety metody.

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